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# THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

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What is the "cost of capital" to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be obtained by many different media, ranging from pure debt instruments, representing money-fixed claims, to pure equity issues, giving holders only the right to a pro-rata share in the uncertain venture? This question has vexed at least three classes of economists: (1) the corporation finance specialist concerned with the techniques of financing firms so as to ensure their survival and growth; (2) the managerial economist concerned with capital budgeting; and (3) the economic theorist concerned with explaining investment behavior at both the micro and macro levels.[1]

In much of his formal analysis, the economic theorist at least has tended to side-step the essence of this cost-of-capital problem by proceeding as though physical assets—like bonds—could be regarded as yielding known, sure streams. Given this assumption, the theorist has concluded that the cost of capital to the owners of a firm is simply the rate of interest on bonds; and has derived the familiar proposition that the firm, acting rationally, will tend to push investment to the point

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[1] The literature bearing on the cost-of-capital problem is far too extensive for listing here. Numerous references to it will be found throughout the paper though we make no claim to completeness. One phase of the problem which we do not consider explicitly, but which has a considerable literature of its own is the relation between the cost of capital and public utility rates. For a recent summary of the "cost-of-capital theory" of rate regulation and a brief discussion of some of its implications, the reader may refer to H. M. Somers [20].

where the marginal yield on physical assets is equal to the market rate of interest.[2] This proposition can be shown to follow from either of two criteria of rational decision-making which are equivalent under certainty, namely (1) the maximization of profits and (2) the maximization of market value.

According to the first criterion, a physical asset is worth acquiring if it will increase the net profit of the owners of the firm. But net profit will increase only if the expected rate of return, or yield, of the asset exceeds the rate of interest. According to the second criterion, an asset is worth acquiring if it increases the value of the owners' equity, *i.e.*, if it adds more to the market value of the firm than the costs of acquisition. But what the asset adds is given by capitalizing the stream it generates at the market rate of interest, and this capitalized value will exceed its cost if and only if the yield of the asset exceeds the rate of interest. Note that, under either formulation, the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.

It must be acknowledged that some attempt is usually made in this type of analysis to allow for the existence of uncertainty. This attempt typically takes the form of superimposing on the results of the certainty analysis the notion of a "risk discount" to be subtracted from the expected yield (or a "risk premium" to be added to the market rate of interest). Investment decisions are then supposed to be based on a comparison of this "risk adjusted" or "certainty equivalent" yield with the market rate of interest.[3] No satisfactory explanation has yet been provided, however, as to what determines the size of the risk discount and how it varies in response to changes in other variables.

Considered as a convenient approximation, the model of the firm constructed via this certainty—or certainty-equivalent—approach has admittedly been useful in dealing with some of the grosser aspects of the processes of capital accumulation and economic fluctuations. Such a model underlies, for example, the familiar Keynesian aggregate investment function in which aggregate investment is written as a function of the rate of interest—the same riskless rate of interest which appears later in the system in the liquidity-preference equation. Yet few would maintain that this approximation is adequate. At the macroeconomic level there are ample grounds for doubting that the rate of interest has

[2] Or, more accurately, to the marginal cost of borrowed funds since it is customary, at least in advanced analysis, to draw the supply curve of borrowed funds to the firm as a rising one. For an advanced treatment of the certainty case, see F. and V. Lutz [13].

[3] The classic examples of the certainty-equivalent approach are found in J. R. Hicks [8] and O. Lange [11].

as large and as direct an influence on the rate of investment as this analysis would lead us to believe. At the microeconomic level the certainty model has little descriptive value and provides no real guidance to the finance specialist or managerial economist whose main problems cannot be treated in a framework which deals so cavalierly with uncertainty and ignores all forms of financing other than debt issues.[4]

Only recently have economists begun to face up seriously to the problem of the cost of capital *cum* risk. In the process they have found their interests and endeavors merging with those of the finance specialist and the managerial economist who have lived with the problem longer and more intimately. In this joint search to establish the principles which govern rational investment and financial policy in a world of uncertainty two main lines of attack can be discerned. These lines represent, in effect, attempts to extrapolate to the world of uncertainty each of the two criteria—profit maximization and market value maximization—which were seen to have equivalent implications in the special case of certainty. With the recognition of uncertainty this equivalence vanishes. In fact, the profit maximization criterion is no longer even well defined. Under uncertainty there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has become a random variable and as such its maximization no longer has an operational meaning. Nor can this difficulty generally be disposed of by using the mathematical expectation of profits as the variable to be maximized. For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes.

Under these conditions the profit outcomes of alternative investment and financing decisions can be compared and ranked only in terms of a *subjective* "utility function" of the owners which weighs the expected yield against other characteristics of the distribution. Accordingly, the extrapolation of the profit maximization criterion of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form.[5]

The utility approach undoubtedly represents an advance over the certainty or certainty-equivalent approach. It does at least permit us

[4] Those who have taken a "case-method" course in finance in recent years will recall in this connection the famous Liquigas case of Hunt and Williams, [9, pp. 193–96] a case which is often used to introduce the student to the cost-of-capital problem and to poke a bit of fun at the economist's certainty-model.

[5] For an attempt at a rigorous explicit development of this line of attack, see F. Modigliani and M. Zeman [14].

to explore (within limits) some of the implications of different financing arrangements, and it does give some meaning to the "cost" of different types of funds. However, because the cost of capital has become an essentially subjective concept, the utility approach has serious drawbacks for normative as well as analytical purposes. How, for example, is management to ascertain the risk preferences of its stockholders and to compromise among their tastes? And how can the economist build a meaningful investment function in the face of the fact that any given investment opportunity might or might not be worth exploiting depending on precisely who happen to be the owners of the firm at the moment?

Fortunately, these questions do not have to be answered; for the alternative approach, based on market value maximization, can provide the basis for an operational definition of the cost of capital and a workable theory of investment. Under this approach any investment project and its concomitant financing plan must pass only the following test: Will the project, as financed, raise the market value of the firm's shares? If so, it is worth undertaking; if not, its return is less than the marginal cost of capital to the firm. Note that such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well. If any current stockholder disagrees with management and the market over the valuation of the project, he is free to sell out and reinvest elsewhere, but will still benefit from the capital appreciation resulting from management's decision.

The potential advantages of the market-value approach have long been appreciated; yet analytical results have been meager. What appears to be keeping this line of development from achieving its promise is largely the lack of an adequate theory of the effect of financial structure on market valuations, and of how these effects can be inferred from objective market data. It is with the development of such a theory and of its implications for the cost-of-capital problem that we shall be concerned in this paper.

Our procedure will be to develop in Section I the basic theory itself and to give some brief account of its empirical relevance. In Section II, we show how the theory can be used to answer the cost-of-capital question and how it permits us to develop a theory of investment of the firm under conditions of uncertainty. Throughout these sections the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given. We have chosen to focus at this level rather than on the economy as a whole because it is at the level of the firm and the industry that the interests of the various specialists concerned with the cost-of-capital problem come most closely together. Although the emphasis has thus been placed on partial-equilibrium analysis, the results obtained also provide the essential building blocks for a general equilibrium model which shows how those prices which are here taken as given, are themselves determined. For reasons of space, however, and because the material is of interest in its own right, the presentation of the general equilibrium model which rounds out the analysis must be deferred to a subsequent paper.

## I. *The Valuation of Securities, Leverage, and the Cost of Capital*

### A. *The Capitalization Rate for Uncertain Streams*

As a starting point, consider an economy in which all physical assets are owned by corporations. For the moment, assume that these corporations can finance their assets by issuing common stock only; the introduction of bond issues, or their equivalent, as a source of corporate funds is postponed until the next part of this section.

The physical assets held by each firm will yield to the owners of the firm—its stockholders—a stream of "profits" over time; but the elements of this series need not be constant and in any event are uncertain. This stream of income, and hence the stream accruing to any share of common stock, will be regarded as extending indefinitely into the future. We assume, however, that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution. We shall refer to the average value over time of the stream accruing to a given share as the return of that share; and to the mathematical expectation of this average as the expected return of the share.[6] Although individual investors may have different views as to the shape of the probability distri-

[6] These propositions can be restated analytically as follows: The assets of the $i$th firm generate a stream:

$$X_i(1),\ X_i(2)\ \cdots\ X_i(T)$$

whose elements are random variables subject to the joint probability distribution:

$$\chi_i[X_i(1),\ X_i(2)\ \cdots\ X_i(t)].$$

The return to the $i$th firm is defined as:

$$X_i = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} X_i(t).$$

$X_i$ is itself a random variable with a probability distribution $\Phi_i(X_i)$ whose form is determined uniquely by $\chi_i$. The expected return $\bar{X}_i$ is defined as $\bar{X}_i = E(X_i) = \int_{X_i} X_i \Phi_i(X_i) dX_i$. If $N_i$ is the number of shares outstanding, the return of the $i$th share is $x_i = (1/N)X_i$ with probability distribution $\phi_i(x_i)dx_i = \Phi_i(Nx_i)d(Nx_i)$ and expected value $\bar{x}_i = (1/N)\bar{X}_i$.

bution of the return of any share, we shall assume for simplicity that they are at least in agreement as to the expected return.[7]

This way of characterizing uncertain streams merits brief comment. Notice first that the stream is a stream of profits, not dividends. As will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail. Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two totally different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. It can be shown, furthermore, that whether the elements of a stream are sure or uncertain, the effect of variability per se on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too).[8]

The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the *ratio* of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return.

The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another. We have, thus, an analogue to the familiar concept of the industry in which it is the commodity produced by the firms that is taken as homogeneous. To complete this analogy with Marshallian price theory, we shall assume in the

[7] To deal adequately with refinements such as differences among investors in estimates of expected returns would require extensive discussion of the theory of portfolio selection. Brief references to these and related topics will be made in the succeeding article on the general equilibrium model.

[8] The reader may convince himself of this by asking how much he would be willing to rebate to his employer for the privilege of receiving his annual salary in equal monthly installments rather than in irregular amounts over the year. See also J. M. Keynes [10, esp. pp. 53–54].

analysis to follow that the shares concerned are traded in perfect markets under conditions of atomistic competition.[9]

From our definition of homogeneous classes of stock it follows that in equilibrium in a perfect capital market the price per dollar's worth of expected return must be the same for all shares of any given class. Or, equivalently, in any given class the price of every share must be proportional to its expected return. Let us denote this factor of proportionality for any class, say the $k$th class, by $1/\rho_k$. Then if $p_j$ denotes the price and $\bar{x}_j$ is the expected return per share of the $j$th firm in class $k$, we must have:

$$p_j = \frac{1}{\rho_k} \bar{x}_j; \tag{1}$$

or, equivalently,

$$\frac{\bar{x}_j}{p_j} = \rho_k \text{ a constant for all firms } j \text{ in class } k. \tag{2}$$

The constants $\rho_k$ (one for each of the $k$ classes) can be given several economic interpretations: (a) From (2) we see that each $\rho_k$ is the expected rate of return of any share in class $k$. (b) From (1) $1/\rho_k$ is the price which an investor has to pay for a dollar's worth of expected return in the class $k$. (c) Again from (1), by analogy with the terminology for perpetual bonds, $\rho_k$ can be regarded as the market rate of capitalization for the expected value of the uncertain streams of the kind generated by the $k$th class of firms.[10]

### B. *Debt Financing and Its Effects on Security Prices*

Having developed an apparatus for dealing with uncertain streams we can now approach the heart of the cost-of-capital problem by dropping the assumption that firms cannot issue bonds. The introduction of debt-financing changes the market for shares in a very fundamental way. Because firms may have different proportions of debt in their capi-

[9] Just what our classes of stocks contain and how the different classes can be identified by outside observers are empirical questions to which we shall return later. For the present, it is sufficient to observe: (1) Our concept of a class, while not identical to that of the industry is at least closely related to it. Certainly the basic characteristics of the probability distributions of the returns on assets will depend to a significant extent on the product sold and the technology used. (2) What are the appropriate class boundaries will depend on the particular problem being studied. An economist concerned with general tendencies in the market, for example, might well be prepared to work with far wider classes than would be appropriate for an investor planning his portfolio, or a firm planning its financial strategy.

[10] We cannot, on the basis of the assumptions so far, make any statements about the relationship or spread between the various $\rho$'s or capitalization rates. Before we could do so we would have to make further specific assumptions about the way investors believe the probability distributions vary from class to class, as well as assumptions about investors' preferences as between the characteristics of different distributions.

tal structure, shares of different companies, even in the same class, can give rise to different probability distributions of returns. In the language of finance, the shares will be subject to different degrees of financial risk or "leverage" and hence they will no longer be perfect substitutes for one another.

To exhibit the mechanism determining the relative prices of shares under these conditions, we make the following two assumptions about the nature of bonds and the bond market, though they are actually stronger than is necessary and will be relaxed later: (1) All bonds (including any debts issued by households for the purpose of carrying shares) are assumed to yield a constant income per unit of time, and this income is regarded as certain by all traders regardless of the issuer. (2) Bonds, like stocks, are traded in a perfect market, where the term perfect is to be taken in its usual sense as implying that any two commodities which are perfect substitutes for each other must sell, in equilibrium, at the same price. It follows from assumption (1) that all bonds are in fact perfect substitutes up to a scale factor. It follows from assumption (2) that they must all sell at the same price per dollar's worth of return, or what amounts to the same thing must yield the same rate of return. This rate of return will be denoted by $r$ and referred to as the rate of interest or, equivalently, as the capitalization rate for sure streams. We now can derive the following two basic propositions with respect to the valuation of securities in companies with different capital structures:

*Proposition I.* Consider any company $j$ and let $\overline{X}_j$ stand as before for the expected return on the assets owned by the company (that is, its expected profit before deduction of interest). Denote by $D_j$ the market value of the debts of the company; by $S_j$ the market value of its common shares; and by $V_j \equiv S_j + D_j$ the market value of all its securities or, as we shall say, the market value of the firm. Then, our Proposition I asserts that we must have in equilibrium:

$$V_j \equiv (S_j + D_j) = \overline{X}_j/\rho_k, \text{ for any firm } j \text{ in class } k. \tag{3}$$

That is, the *market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate $\rho_k$ appropriate to its class.*

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\overline{X}_j/V_j$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$\frac{\overline{X}_j}{(S_j + D_j)} \equiv \frac{\overline{X}_j}{V_j} = \rho_k, \text{ for any firm } j, \text{ in class } k. \tag{4}$$

That is, *the average cost of capital to any firm is completely independent of* *its capital structure and is equal to the capitalization rate of a pure equity stream of its class.*

To establish Proposition I we will show that as long as the relations (3) or (4) do not hold between any pair of firms in a class, arbitrage will take place and restore the stated equalities. We use the term arbitrage advisedly. For if Proposition I did not hold, an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price. The exchange would therefore be advantageous to the investor quite independently of his attitudes toward risk.[11] As investors exploit these arbitrage opportunities, the value of the overpriced shares will fall and that of the underpriced shares will rise, thereby tending to eliminate the discrepancy between the market values of the firms.

By way of proof, consider two firms in the same class and assume for simplicity only, that the expected return, $\bar{X}$, is the same for both firms. Let company 1 be financed entirely with common stock while company 2 has some debt in its capital structure. Suppose first the value of the levered firm, $V_2$, to be larger than that of the unlevered one, $V_1$. Consider an investor holding $s_2$ dollars' worth of the shares of company 2, representing a fraction $\alpha$ of the total outstanding stock, $S_2$. The return from this portfolio, denoted by $Y_2$, will be a fraction $\alpha$ of the income available for the stockholders of company 2, which is equal to the total return $X_2$ less the interest charge, $rD_2$. Since under our assumption of homogeneity, the anticipated total return of company 2, $X_2$, is, under all circumstances, the same as the anticipated total return to company 1, $X_1$, we can hereafter replace $X_2$ and $X_1$ by a common symbol $X$. Hence, the return from the initial portfolio can be written as:

$$Y_2 = \alpha(X - rD_2). \tag{5}$$

Now suppose the investor sold his $\alpha S_2$ worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount $\alpha S_2$ realized from the sale of his initial holding and borrowing an additional amount $\alpha D_2$ on his own credit, pledging his new holdings in company 1 as a collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares and earnings of company 1. Making proper allowance for the interest payments on his personal debt $\alpha D_2$, the return from the new portfolio, $Y_1$, is given by:

[11] In the language of the theory of choice, the exchanges are movements from inefficient points in the interior to efficient points on the boundary of the investor's opportunity set; and not movements between efficient points along the boundary. Hence for this part of the analysis nothing is involved in the way of specific assumptions about investor attitudes or behavior other than that investors behave consistently and prefer more income to less income, *ceteris paribus*.

$$(6)\qquad Y_1 = \frac{\alpha(S_2 + D_2)}{S_1}X - r\alpha D_2 = \alpha\frac{V_2}{V_1}X - r\alpha D_2.$$

Comparing (5) with (6) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of company 2's shares to sell their holdings, thereby depressing $S_2$ and hence $V_2$; and to acquire shares of company 1, thereby raising $S_1$ and thus $V_1$. We conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account.

Consider now the other possibility, namely that the market value of the levered company $V_2$ is less than $V_1$. Suppose an investor holds initially an amount $s_1$ of shares of company 1, representing a fraction $\alpha$ of the total outstanding stock, $S_1$. His return from this holding is:

$$Y_1 = \frac{s_1}{S_1}X = \alpha X.$$

Suppose he were to exchange this initial holding for another portfolio, also worth $s_1$, but consisting of $s_2$ dollars of stock of company 2 and of $d$ dollars of bonds, where $s_2$ and $d$ are given by:

$$(7)\qquad s_2 = \frac{S_2}{V_2}s_1, \qquad d = \frac{D_2}{V_2}s_1.$$

In other words the new portfolio is to consist of stock of company 2 and of bonds in the proportions $S_2/V_2$ and $D_2/V_2$, respectively. The return from the stock in the new portfolio will be a fraction $s_2/S_2$ of the total return to stockholders of company 2, which is $(X - rD_2)$, and the return from the bonds will be $rd$. Making use of (7), the total return from the portfolio, $Y_2$, can be expressed as follows:

$$Y_2 = \frac{s_2}{S_2}(X - rD_2) + rd = \frac{s_1}{V_2}(X - rD_2) + r\frac{D_2}{V_2}s_1 = \frac{s_1}{V_2}X = \alpha\frac{S_1}{V_2}X$$

(since $s_1 = \alpha S_1$). Comparing $Y_2$ with $Y_1$ we see that, if $V_2 < S_1 \equiv V_1$, then $Y_2$ will exceed $Y_1$. Hence it pays the holders of company 1's shares to sell these holdings and replace them with a mixed portfolio containing an appropriate fraction of the shares of company 2.

The acquisition of a mixed portfolio of stock of a levered company $j$ and of bonds in the proportion $S_j/V_j$ and $D_j/V_j$ respectively, may be regarded as an operation which "undoes" the leverage, giving access to an appropriate fraction of the unlevered return $X_j$. It is this possibility of undoing leverage which prevents the value of levered firms from being consistently less than those of unlevered firms, or more generally prevents the average cost of capital $\overline{X}_j/V_j$ from being systematically higher for levered than for nonlevered companies in the same class.

Since we have already shown that arbitrage will also prevent $V_2$ from being larger than $V_1$, we can conclude that in equilibrium we must have $V_2 = V_1$, as stated in Proposition I.

*Proposition II.* From Proposition I we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, $i$, on the stock of any company $j$ belonging to the $k$th class is a linear function of leverage as follows:

$$i_j = \rho_k + (\rho_k - r)D_j/S_j. \tag{8}$$

That is, *the expected yield of a share of stock is equal to the appropriate capitalization rate $\rho_k$ for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between $\rho_k$ and $r$.* Or equivalently, the market price of any share of stock is given by capitalizing its expected return at the continuously variable rate $i_j$ of (8).[12]

A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results were not applicable to the actual capital markets.[13] Proposition II, however, so far as we have been able to discover is new.[14] To establish it we first note that, by definition, the expected rate of return, $i$, is given by:

$$i_j \equiv \frac{\bar{X}_j - rD_j}{S_j}. \tag{9}$$

From Proposition I, equation (3), we know that:

$$\bar{X}_j = \rho_k(S_j + D_j).$$

Substituting in (9) and simplifying, we obtain equation (8).

[12] To illustrate, suppose $\bar{X} = 1000$, $D = 4000$, $r = 5$ per cent and $\rho_k = 10$ per cent. These values imply that $V = 10{,}000$ and $S = 6000$ by virtue of Proposition I. The expected yield or rate of return per share is then:

$$i = \frac{1000 - 200}{6000} = .1 + (.1 - .05)\frac{4000}{6000} = 13\tfrac{1}{3} \text{ per cent.}$$

[13] See, for example, J. B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their "riskiness." This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial. Regarding Proposition I as resting on investors' attitudes toward risk leads inevitably to a misunderstanding of many factors influencing relative yields such as, for example, limitations on the portfolio composition of financial institutions. See below, esp. Section I.D.

[14] Morton does make reference to a linear yield function but only ". . . for the sake of simplicity and because the particular function used makes no essential difference in my conclusions" [15, p. 443, note 2].

C. *Some Qualifications and Extensions of the Basic Propositions*

The methods and results developed so far can be extended in a number of useful directions, of which we shall consider here only three: (1) allowing for a corporate profits tax under which interest payments are deductible; (2) recognizing the existence of a multiplicity of bonds and interest rates; and (3) acknowledging the presence of market imperfections which might interfere with the process of arbitrage. The first two will be examined briefly in this section with some further attention given to the tax problem in Section II. Market imperfections will be discussed in Part D of this section in the course of a comparison of our results with those of received doctrines in the field of finance.

*Effects of the Present Method of Taxing Corporations.* The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that the market values of firms in each class must be proportional in equilibrium to their expected return net of taxes (that is, to the sum of the interest paid and expected net stockholder income). This means we must replace each $\bar{X}_j$ in the original versions of Propositions I and II with a new variable $\bar{X}_j^\tau$ representing the total income net of taxes generated by the firm:

$$\bar{X}_j^\tau \equiv (\bar{X}_j - rD_j)(1 - \tau) + rD_j \equiv \bar{\pi}_j^\tau + rD_j, \tag{10}$$

where $\bar{\pi}_j^\tau$ represents the expected net income accruing to the common stockholders and $\tau$ stands for the average rate of corporate income tax.[15]

After making these substitutions, the propositions, when adjusted for taxes, continue to have the same form as their originals. That is, Proposition I becomes:

$$\frac{\bar{X}_j^\tau}{V_j} = \rho_k^\tau, \text{ for any firm in class } k, \tag{11}$$

and Proposition II becomes

$$i_j \equiv \frac{\bar{\pi}_j^\tau}{S_j} = \rho_j^\tau + (\rho_k^\tau - r) D_j/S_j \tag{12}$$

where $\rho_k^\tau$ is the capitalization rate for income net of taxes in class $k$.

Although the form of the propositions is unaffected, certain interpretations must be changed. In particular, the after-tax capitalization rate

[15] For simplicity, we shall ignore throughout the tiny element of progression in our present corporate tax and treat $\tau$ as a constant independent of $(X_j - rD_j)$.

$\rho_k{}^\tau$ can no longer be identified with the "average cost of capital" which is $\rho_k = \bar{X}_j/V_j$. The difference between $\rho_k{}^\tau$ and the "true" average cost of capital, as we shall see, is a matter of some relevance in connection with investment planning within the firm (Section II). For the description of market behavior, however, which is our immediate concern here, the distinction is not essential. To simplify presentation, therefore, and to preserve continuity with the terminology in the standard literature we shall continue in this section to refer to $\rho_k{}^\tau$ as the average cost of capital, though strictly speaking this identification is correct only in the absence of taxes.

*Effects of a Plurality of Bonds and Interest Rates.* In existing capital markets we find not one, but a whole family of interest rates varying with maturity, with the technical provisions of the loan and, what is most relevant for present purposes, with the financial condition of the borrower.[16] Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio of the borrowing firm (or individual). If so, and if we can assume as a first approximation that this yield curve, $r = r\ (D/S)$, whatever its precise form, is the same for all borrowers, then we can readily extend our propositions to the case of a rising supply curve for borrowed funds.[17]

Proposition I is actually unaffected in form and interpretation by the fact that the rate of interest may rise with leverage; while the average cost of *borrowed* funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage (apart from the tax effect). This conclusion follows directly from the ability of those who engage in arbitrage to undo the leverage in any financial structure by acquiring an appropriately mixed portfolio of bonds and stocks. Because of this ability, the ratio of earnings (*before* interest charges) to market value—*i.e.*, the average cost of capital from all

[16] We shall not consider here the extension of the analysis to encompass the time structure of interest rates. Although some of the problems posed by the time structure can be handled within our comparative statics framework, an adequate discussion would require a separate paper.

[17] We can also develop a theory of bond valuation along lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be a nonlinear one in contrast to the linear function of leverage developed for common shares. However, we would also expect that the rate of increase in the yield on new issues would not be substantial in practice. This relatively slow rise would reflect the fact that interest rate increases by themselves can never be completely satisfactory to creditors as compensation for their increased risk. Such increases may simply serve to raise $r$ so high relative to $\rho$ that they become self-defeating by giving rise to a situation in which even normal fluctuations in earnings may force the company into bankruptcy. The difficulty of borrowing more, therefore, tends to show up in the usual case not so much in higher rates as in the form of increasingly stringent restrictions imposed on the company's management and finances by the creditors; and ultimately in a complete inability to obtain new borrowed funds, at least from the institutional investors who normally set the standards in the market for bonds.

sources—must be the same for all firms in a given class.[18] In other words, the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock. This seemingly paradoxical result will be examined more closely below in connection with Proposition II.

A significant modification of Proposition I would be required only if the yield curve $r=r(D/S)$ were different for different borrowers, as might happen if creditors had marked preferences for the securities of a particular class of debtors. If, for example, corporations as a class were able to borrow at lower rates than individuals having equivalent personal leverage, then the average cost of capital to corporations might fall slightly, as leverage increased over some range, in reflection of this differential. In evaluating this possibility, however, remember that the relevant interest rate for our arbitrage operators is the rate on brokers' loans and, historically, that rate has not been noticeably higher than representative corporate rates.[19] The operations of holding companies and investment trusts which can borrow on terms comparable to operating companies represent still another force which could be expected to wipe out any marked or prolonged advantages from holding levered stocks.[20]

Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If $r$ increases with leverage, the yield $i$ will still tend to

[18] One normally minor qualification might be noted. Once we relax the assumption that all bonds have certain yields, our arbitrage operator faces the danger of something comparable to "gambler's ruin." That is, there is always the possibility that an otherwise sound concern—one whose long-run expected income is greater than its interest liability—might be forced into liquidation as a result of a run of temporary losses. Since reorganization generally involves costs, and because the operation of the firm may be hampered during the period of reorganization with lasting unfavorable effects on earnings prospects, we might perhaps expect heavily levered companies to sell at a slight discount relative to less heavily indebted companies of the same class.

[19] Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form, not of having the arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock. Margin requirements are also somewhat less of an obstacle to maintaining any desired degree of leverage in a portfolio than might be thought at first glance. Leverage could be largely restored in the face of higher margin requirements by switching to stocks having more leverage at the corporate level.

[20] An extreme form of inequality between borrowing and lending rates occurs, of course, in the case of preferred stocks, which can not be directly issued by individuals on personal account. Here again, however, we would expect that the operations of investment corporations plus the ability of arbitrage operators to sell off their holdings of preferred stocks would act to prevent the emergence of any substantial premiums (for this reason) on capital structures containing preferred stocks. Nor are preferred stocks so far removed from bonds as to make it impossible for arbitrage operators to approximate closely the risk and leverage of a corporate preferred stock by incurring a somewhat smaller debt on personal account.

rise as $D/S$ increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to fall.[21] The relation between $i$ and $D/S$ could conceivably take the form indicated by the curve $MD$

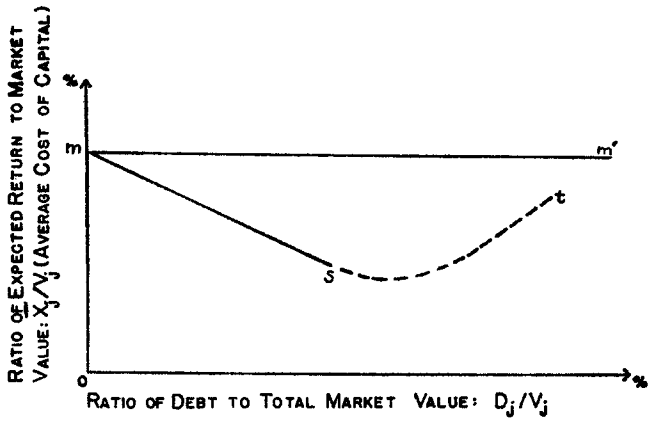


FIGURE 1

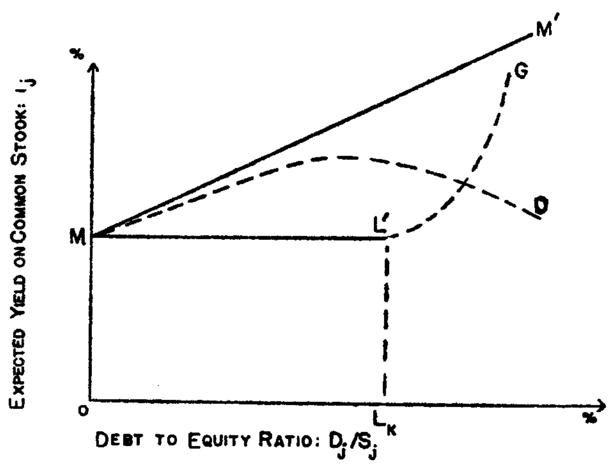


FIGURE 2

in Figure 2, although in practice the curvature would be much less pronounced. By contrast, with a constant rate of interest, the relation would be linear throughout as shown by line $MM'$, Figure 2.

The downward sloping part of the curve $MD$ perhaps requires some

[21] Since new lenders are unlikely to permit this much leverage (*cf.* note 17), this range of the curve is likely to be occupied by companies whose earnings prospects have fallen substantially since the time when their debts were issued.

comment since it may be hard to imagine why investors, other than those who like lotteries, would purchase stocks in this range. Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve $MD$, this demand would be reinforced by the action of arbitrage operators. The latter would find it profitable to own a pro-rata share of the firm as a whole by holding its stock *and* bonds, the lower yield of the shares being thus offset by the higher return on bonds.

### D. *The Relation of Propositions I and II to Current Doctrines*

The propositions we have developed with respect to the valuation of firms and shares appear to be substantially at variance with current doctrines in the field of finance. The main differences between our view and the current view are summarized graphically in Figures 1 and 2. Our Proposition I [equation (4)] asserts that the average cost of capital, $\bar{X}_j^\tau / V_j$, is a constant for all firms $j$ in class $k$, independently of their financial structure. This implies that, if we were to take a sample of firms in a given class, and if for each firm we were to plot the ratio of expected return to market value against some measure of leverage or financial structure, the points would tend to fall on a horizontal straight line with intercept $\rho_k^\tau$, like the solid line $mm'$ in Figure 1.[22] From Proposition I we derived Proposition II [equation (8)] which, taking the simplest version with $r$ constant, asserts that, for all firms in a class, the relation between the yield on common stock and financial structure, measured by $D_j/S_j$, will approximate a straight line with slope $(\rho_k^\tau - r)$ and intercept $\rho_k^\tau$. This relationship is shown as the solid line $MM'$ in Figure 2, to which reference has been made earlier.[23]

By contrast, the conventional view among finance specialists appears to start from the proposition that, other things equal, the earnings-price ratio (or its reciprocal, the times-earnings multiplier) of a firm's common stock will normally be only slightly affected by "moderate" amounts of debt in the firm's capital structure.[24] Translated into our no-

[22] In Figure 1 the measure of leverage used is $D_j/V_j$ (the ratio of debt to market value) rather than $D_j/S_j$ (the ratio of debt to equity), the concept used in the analytical development. The $D_j/V_j$ measure is introduced at this point because it simplifies comparison and contrast of our view with the traditional position.

[23] The line $MM'$ in Figure 2 has been drawn with a positive slope on the assumption that $\rho_k^\tau > r$, a condition which will normally obtain. Our Proposition II as given in equation (8) would continue to be valid, of course, even in the unlikely event that $\rho_k^\tau < r$, but the slope of $MM'$ would be negative.

[24] See, *e.g.*, Graham and Dodd [6, pp. 464–66]. Without doing violence to this position, we can bring out its implications more sharply by ignoring the qualification and treating the yield as a virtual constant over the relevant range. See in this connection the discussion in Durand [3, esp. pp. 225–37] of what he calls the "net income method" of valuation.

tation, it asserts that for any firm $j$ in the class $k$,

$$\frac{\overline{X}_j^\tau - rD_j}{S_j} \equiv \frac{\bar{\pi}_j^\tau}{S_j} = i_k^*, \text{ a constant for } \frac{D_j}{S_j} \leq L_k \tag{13}$$

or, equivalently,

$$S_j = \bar{\pi}_j^\tau / i_k^*. \tag{14}$$

Here $i_k^*$ represents the capitalization rate or earnings-price ratio on the common stock and $L_k$ denotes some amount of leverage regarded as the maximum "reasonable" amount for firms of the class $k$. This assumed relationship between yield and leverage is the horizontal solid line $ML'$ of Figure 2. Beyond $L'$, the yield will presumably rise sharply as the market discounts "excessive" trading on the equity. This possibility of a rising range for high leverages is indicated by the broken-line segment $L'G$ in the figure.[25]

If the value of shares were really given by (14) then the over-all market value of the firm must be:

$$V_j \equiv S_j + D_j = \frac{\overline{X}_j^\tau - rD_j}{i_k^*} + D_j = \frac{\overline{X}_j^\tau}{i_k^*} + \frac{(i_k^* - r)D_j}{i_k^*}. \tag{16}$$

That is, for any given level of expected total returns after taxes ($\overline{X}_j^\tau$) and assuming, as seems natural, that $i_k^* > r$, the value of the firm must tend to *rise* with debt;[26] whereas our Proposition I asserts that the value of the firm is completely independent of the capital structure. Another way of contrasting our position with the traditional one is in terms of the cost of capital. Solving (16) for $\overline{X}_j^\tau / V_j$ yields:

$$\overline{X}_j^\tau / V_j = i_k^* - (i_k^* - r) D_j / V_j. \tag{17}$$

According to this equation, the average cost of capital is not independent of capital structure as we have argued, but should tend to *fall* with increasing leverage, at least within the relevant range of moderate debt ratios, as shown by the line *ms* in Figure 1. Or to put it in more familiar terms, debt-financing should be "cheaper" than equity-financing if not carried too far.

When we also allow for the possibility of a rising range of stock yields for large values of leverage, we obtain a U-shaped curve like *mst* in

[25] To make it easier to see some of the implications of this hypothesis as well as to prepare the ground for later statistical testing, it will be helpful to assume that the notion of a critical limit on leverage beyond which yields rise rapidly, can be epitomized by a quadratic relation of the form:

$$\bar{\pi}_j^\tau / S_j = i_k^* + \beta(D_j/S_j) + \alpha(D_j/S_j)^2, \qquad \alpha > 0. \tag{15}$$

[26] For a typical discussion of how a promoter can, supposedly, increase the market value of a firm by recourse to debt issues, see W. J. Eiteman [4, esp. pp. 11–13].

Figure 1.[27] That a yield-curve for stocks of the form $ML'G$ in Figure 2 implies a U-shaped cost-of-capital curve has, of course, been recognized by many writers. A natural further step has been to suggest that the capital structure corresponding to the trough of the U is an "optimal capital structure" towards which management ought to strive in the best interests of the stockholders.[28] According to our model, by contrast, no such optimal structure exists—all structures being equivalent from the point of view of the cost of capital.

Although the falling, or at least U-shaped, cost-of-capital function is in one form or another the dominant view in the literature, the ultimate rationale of that view is by no means clear. The crucial element in the position—that the expected earnings-price ratio of the stock is largely unaffected by leverage up to some conventional limit—is rarely even regarded as something which requires explanation. It is usually simply taken for granted or it is merely asserted that this is the way the market behaves.[29] To the extent that the constant earnings-price ratio has a rationale at all we suspect that it reflects in most cases the feeling that moderate amounts of debt in "sound" corporations do not really add very much to the "riskiness" of the stock. Since the extra risk is slight, it seems natural to suppose that firms will not have to pay noticeably higher yields in order to induce investors to hold the stock.[30]

A more sophisticated line of argument has been advanced by David Durand [3, pp. 231–33]. He suggests that because insurance companies and certain other important institutional investors are restricted to debt securities, nonfinancial corporations are able to borrow from them at interest rates which are lower than would be required to compensate

[27] The U-shaped nature of the cost-of-capital curve can be exhibited explicitly if the yield curve for shares as a function of leverage can be approximated by equation (15) of footnote 25. From that equation, multiplying both sides by $S_j$ we obtain: $\bar{\pi}_j^\tau = \bar{X}_j^\tau - rD_j = i_k^* S_j + \beta D_j + \alpha D_j^2 / S_j$ or, adding and subtracting $i_k^* D_k$ from the right-hand side and collecting terms,

$$\bar{X}_j^\tau = i_k^*(S_j + D_j) + (\beta + r - i_k^*)D_j + \alpha D^2{}_j/S_j. \tag{18}$$

Dividing (18) by $V_j$ gives an expression for the cost of capital:

$$\bar{X}_j^\tau/V_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha D_j^2/S_jV_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha(D_j/V_j)^2/(1 - D_j/V_j) \tag{19}$$

which is clearly U-shaped since $\alpha$ is supposed to be positive.

[28] For a typical statement see S. M. Robbins [16, p. 307]. See also Graham and Dodd [6, pp. 468–74].

[29] See *e.g.*, Graham and Dodd [6, p. 466].

[30] A typical statement is the following by Guthmann and Dougall [7, p. 245]: "Theoretically it might be argued that the increased hazard from using bonds and preferred stocks would counterbalance this additional income and so prevent the common stock from being more attractive than when it had a lower return but fewer prior obligations. In practice, the extra earnings from 'trading on the equity' are often regarded by investors as more than sufficient to serve as a 'premium for risk' when the proportions of the several securities are judiciously mixed."

creditors in a free market. Thus, while he would presumably agree with our conclusions that stockholders could not gain from leverage in an unconstrained market, he concludes that they can gain under present institutional arrangements. This gain would arise by virtue of the "safety superpremium" which lenders are willing to pay corporations for the privilege of lending.[31]

The defective link in both the traditional and the Durand version of the argument lies in the confusion between investors' subjective risk preferences and their objective market opportunities. Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle).

An analogy may he helpful at this point. The relations between $1/\rho_k$, the price per dollar of an unlevered stream in class $k$; $1/r$, the price per dollar of a sure stream, and $1/i_j$, the price per dollar of a levered stream $j$, in the $k$th class, are essentially the same as those between, respectively, the price of whole milk, the price of butter fat, and the price of milk which has been thinned out by skimming off some of the butter fat. Our Proposition I states that a firm cannot reduce the cost of capital—*i.e.*, increase the market value of the stream it generates—by securing part of its capital through the sale of bonds, even though debt money appears to be cheaper. This assertion is equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming the milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk. Similarly our Proposition II—that the price per dollar of a levered stream falls as leverage increases—is an ex-

[31] Like Durand, Morton [15] contends "that the actual market deviates from [Proposition I] by giving a changing over-all cost of money at different points of the [leverage] scale" (p. 443, note 2, inserts ours), but the basis for this contention is nowhere clearly stated. Judging by the great emphasis given to the lack of mobility of investment funds between stocks and bonds and to the psychological and institutional pressures toward debt portfolios (see pp. 444–51 and especially his discussion of the optimal capital structure on p. 453) he would seem to be taking a position very similar to that of Durand above.

act analogue of the statement that the price per gallon of thinned milk falls continuously as more butter fat is skimmed off.[32]

It is clear that this last assertion is true as long as butter fat is worth more per unit weight than whole milk, and it holds even if, for many consumers, taking a little cream out of the milk (adding a little leverage to the stock) does not detract noticeably from the taste (does not add noticeably to the risk). Furthermore the argument remains valid even in the face of instituional limitations of the type envisaged by Durand. For suppose that a large fraction of the population habitually dines in restaurants which are required by law to serve only cream in lieu of milk (entrust their savings to institutional investors who can only buy bonds). To be sure the price of butter fat will then tend to be higher in relation to that of skimmed milk than in the absence such restrictions (the rate of interest will tend to be lower), and this will benefit people who eat at home and who like skim milk (who manage their own portfolio and are able and willing to take risk). But it will still be the case that a farmer cannot gain by skimming some of the butter fat and selling it separately (firm cannot reduce the cost of capital by recourse to borrowed funds).[33]

Our propositions can be regarded as the extension of the classical theory of markets to the particular case of the capital markets. Those who hold the current view—whether they realize it or not—must as-

[32] Let $M$ denote the quantity of whole milk, $B/M$ the proportion of butter fat in the whole milk, and let $p_M$, $p_B$ and $p_\alpha$ denote, respectively, the price per unit weight of whole milk, butter fat and thinned milk from which a fraction $\alpha$ of the butter fat has been skimmed off. We then have the fundamental perfect market relation:

$$\text{(a)} \qquad p_\alpha(M - \alpha B) + p_B \alpha B = p_M M, \qquad 0 \leq \alpha \leq 1,$$

stating that total receipts will be the same amount $p_M M$, independently of the amount $\alpha B$ of butter fat that may have been sold separately. Since $p_M$ corresponds to $1/\rho$, $p_B$ to $1/r$, $p_\alpha$ to $1/i$, $M$ to $\bar{X}$ and $\alpha B$ to $rD$, (a) is equivalent to Proposition I, $S+D=\bar{X}/\rho$. From (a) we derive:

$$\text{(b)} \qquad p_\alpha = p_M \frac{M}{M - \alpha B} - p_B \frac{\alpha B}{M - \alpha B}$$

which gives the price of thinned milk as an explicit function of the proportion of butter fat skimmed off; the function decreasing as long as $p_B > p_M$. From (a) also follows:

$$\text{(c)} \qquad 1/p_\alpha = 1/p_M + (1/p_M - 1/p_B) \frac{p_B \alpha B}{p_\alpha(M - \alpha B)}$$

which is the exact analogue of Proposition II, as given by (8).

[33] The reader who likes parables will find that the analogy with interrelated commodity markets can be pushed a good deal farther than we have done in the text. For instance, the effect of changes in the market rate of interest on the over-all cost of capital is the same as the effect of a change in the price of butter on the price of whole milk. Similarly, just as the relation between the prices of skim milk and butter fat influences the kind of cows that will be reared, so the relation between $i$ and $r$ influences the kind of ventures that will be undertaken. If people like butter we shall have Guernseys; if they are willing to pay a high price for safety, this will encourage ventures which promise smaller but less uncertain streams per dollar of physical assets.