復旦大學
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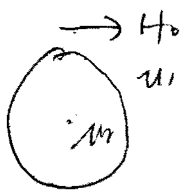
半径为R的无限长圆柱，磁导率$\mu_2$，放入$\mu_1$的无限介质中，在某方向施加磁场$B_0$，求等效偶极矩。

标势法：




$$\varphi_1 = -H_0\rho\cos\theta + \frac{B}{\rho}\cos\theta$$

$$\varphi_2 = A\rho\cos\theta$$

$$\varphi_1=\varphi_2\big|_{\rho=R} \Rightarrow -H_0R + \frac{B}{R} = AR$$

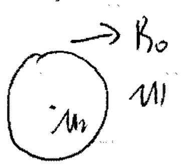
$$\mu_1\frac{\partial\varphi_1}{\partial\rho} = \mu_2\frac{\partial\varphi_2}{\partial\rho}\Big|_{\rho=R} \Rightarrow -H_0 - \frac{B}{R^2} = \frac{\mu_2}{\mu_1}A$$

$$\Rightarrow A = \frac{-2\mu_1H_0}{\mu_1+\mu_2}, \quad B = H_0R^2\frac{\mu_2-\mu_1}{\mu_2+\mu_1}$$

$$\varphi = \frac{1}{2\pi}\frac{\vec{m}\cdot\vec{r}}{r^2} \quad (\text{二维磁偶极子})$$

$$\Rightarrow \vec{m} = 2\pi H_0R^2\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\hat{x}$$

矢势法：




$$A_1 = B_0\rho\sin\theta + \frac{A}{\rho}\sin\theta$$

$$A_2 = B\rho\sin\theta$$

$$A_1=A_2\big|_{\rho=R} \Rightarrow B_0R + \frac{A}{R} = BR$$

$$\frac{1}{\mu_1}\frac{\partial A_1}{\partial\rho} = \frac{1}{\mu_2}\frac{\partial A_2}{\partial\rho}\Big|_{\rho=R} \Rightarrow \frac{1}{\mu_1}\left[B_0 - \frac{A}{R^2}\right] = \frac{1}{\mu_2}B$$

$$\Rightarrow B = \frac{2B_0\mu_2}{\mu_1+\mu_2}, \quad A = B_0R^2\frac{\mu_2-\mu_1}{\mu_2+\mu_1}$$

$$\Rightarrow \vec{m} = \frac{2\pi B_0}{\mu_0}R^2\frac{(\mu_2-\mu_1)}{\mu_2+\mu_1} \quad ???$$

前后不一致？但所求得的B场，H场是一致的。