



復旦大學

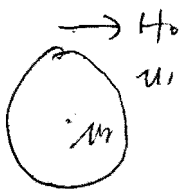
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半径为  $R$  的无限长圆柱，磁导率  $\mu_2$ ，放入  $\mu_1$  的无限介质中，在  $z$  方向施加均匀磁感  $B_0$ ，求等效偶极矩。

标势法：



$$\varphi_1 = -H_0 \rho \cos\theta + \frac{B}{\rho} \cos\theta$$

$$\varphi_2 = A \rho \cos\theta$$

$$\varphi_1 = \varphi_2 |_{\rho=R} \Rightarrow -H_0 R + \frac{B}{R} = AR$$

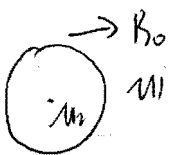
$$\mu_1 \frac{\partial \varphi_1}{\partial \rho} = \mu_2 \frac{\partial \varphi_2}{\partial \rho} |_{\rho=R} \Rightarrow -H_0 - \frac{B}{R^2} = \frac{\mu_2}{\mu_1} A$$

$$\Rightarrow A = \frac{-2\mu_1 H_0}{\mu_1 + \mu_2}, \quad B = H_0 R^2 \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}$$

$$\varphi = \frac{1}{2\pi} \frac{m \cdot \vec{r}}{r^3} \quad (= \text{均匀磁偶极子})$$

$$\Rightarrow \vec{m} = 2\pi H_0 R^2 \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \hat{z}$$

矢势法：



$$A_1 = B_0 \rho \sin\theta + \frac{A}{\rho} \sin\theta$$

$$A_2 = B \rho \sin\theta$$

$$A_1 = A_2 |_{\rho=R} \Rightarrow B_0 R + \frac{A}{R} = BR$$

$$\frac{1}{\mu_1} \frac{\partial A_1}{\partial \rho} = \frac{1}{\mu_2} \frac{\partial A_2}{\partial \rho} |_{\rho=R} \Rightarrow \frac{1}{\mu_1} \left[ B_0 - \frac{A}{R^2} \right] = \frac{1}{\mu_2} B$$

$$\Rightarrow B = \frac{2B_0 \mu_2}{\mu_1 + \mu_2}, \quad A = B_0 R^2 \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}$$

$$\Rightarrow \vec{m} = \frac{2\pi B_0}{\mu_0} R^2 \frac{(\mu_2 - \mu_1)}{\mu_2 + \mu_1} \hat{z} \quad ???$$

前后不一致！但所得的  $B$  场， $H$  场是一致的。