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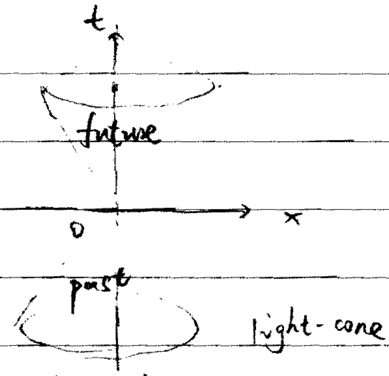
① retard potential



$x^\mu = (ct, x, y, z)$.

$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

$\eta_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

null: $ds^2 = 0$.

$\Rightarrow -c^2(t'-t)^2 + R^2 = 0$.

$\Rightarrow t' = t - \frac{R}{c}$

retard potential.

② Liénard - Wiechert potential

i) notation: Inertial reference frame I

Non-inertial reference frame N^a (acceleration)

N^g (gravitation)

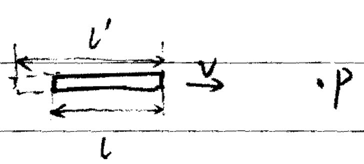
ii) Two facts.

a) The charge is Lorentz invariant

b). At every instant all physical quantities determined in non-inertial reference frame N^a (or N^g) are equal to those determined in the co-moving (or local) inertial reference frame I.

iii) Origin of Liénard - Wiechert potential: an apparent change in the volume of a charge as seen at the observation point. (Griffiths, Intro. to Electrodynamics)

IV) Now consider a charge $e = \rho V$ of length l and volume V. Assume the charge constantly emits signal to "update" the field. (think of QED)



The potential at P at t in I is determined by all signals originating from different points of the charge at different moments and reaching P at the same moment t.

(But $ds^2 = 0$ is always true for this condition. so we need retard time)

For simplicity, consider the potential of a charge moving towards P.

$\Delta t = \frac{l + v\Delta t}{c} \Rightarrow \Delta t = \frac{l/c}{1 - v/c}$ The vector notation is easily obtained:

$$\Delta t = \frac{l/c}{1 - \frac{\vec{v}\cdot\vec{n}}{c}} \Rightarrow l' = c\Delta t = \frac{l}{1 - \frac{\vec{v}\cdot\vec{n}}{c}}$$

so $V' = \frac{V}{1 - \frac{\vec{v}\cdot\vec{n}}{c}}$, then the potential at P is

$$\varphi = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho v'}{r} = \frac{e}{4\pi\varepsilon_0 r}\cdot\frac{1}{(1-\frac{\vec{v}\cdot\vec{n}}{c})}$$ (note: never forget retard time)

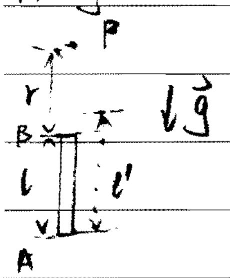
the vector Liénard-Wiechart potential is

$$\vec{A} = \frac{e}{4\pi\varepsilon_0 r c^2}\left[\frac{\vec{v}}{1-\frac{\vec{v}\cdot\vec{n}}{c}}\right]$$

The scalar and vector potential do not depend on the size of the size.
so they are valid for a point charge.

③ A rest charge in non-inertial reference frame

i) gravitational field N^g



$$\varphi^g = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho V^g}{r^g}$$

~~Note: P is in the local inertial reference frame~~. it doesn't matter.

$$c^g_{AP} = c\left(1-\frac{g(r+l)}{2c^2}\right),\quad c^g_{BP} = c\left(1-\frac{gr}{2c^2}\right)$$

$$r^g = \frac{c^g_{BP}}{c}\cdot r,\quad \Delta t^g = \frac{r+l}{c^g_{AP}} - \frac{r}{c^g_{BP}}$$

$$l^g = c^g_{AP}\,\Delta t^g = r+l-\frac{c^g_{AP}}{c^g_{BP}}\cdot r \Rightarrow V^g = \frac{l^g}{l}\cdot V$$

$$\therefore \varphi^g = \frac{\rho V}{4\pi\varepsilon_0 r}\cdot\frac{c}{c^g_{BP}}\cdot\frac{r+l-\frac{c^g_{AP}}{c^g_{BP}}\cdot r}{l} = \frac{\rho V}{4\pi\varepsilon_0 r}\left(1+\frac{g\cdot r}{c^2}\right) + \underbrace{o(l)}_{\text{point charge}}$$

the vector notation is

$$\varphi^g = \frac{\rho V}{4\pi\varepsilon_0 r}\left(1-\frac{\vec{g}\cdot\vec{r}}{c^2}\right)$$

ii) accelerated N^a

the same proccesion $$\varphi^a = \frac{\rho V}{4\pi\varepsilon_0 r}\left(1+\frac{\vec{a}\cdot\vec{r}}{c^2}\right)$$

(note: no retarted time appears here)

$\vec{g} \leftrightarrow -\vec{a}$ Equivalance principle.

Ref. E. Fermi Nuovo Cimento (1921). $\varphi^g_{Fermi} = \frac{\rho V}{4\pi\varepsilon_0 r}\left(1-\frac{\vec{g}\cdot\vec{r}}{2c^2}\right)$

But he get a factor $\frac{1}{2}$ by some mistakes.

④ Charge moving in non-inertial reference frame.

We can just use V^g, r^g instead of V, r in the Liénard - Wiechert potential.

$$\varphi^g = \frac{e}{4\pi\varepsilon_0 \cdot r} \cdot \left[\frac{1}{1-\frac{\vec{v}\cdot\vec{n}}{c}} \cdot \left(1 - \frac{\vec{g}\cdot\vec{r}}{c^2}\right)\right], \quad \vec{A}^g = \frac{e}{4\pi\varepsilon_0 r} \cdot \left[\frac{\vec{v}}{1-\frac{\vec{v}\cdot\vec{n}}{c}} \cdot \left(1 - \frac{\vec{g}\cdot\vec{r}}{c^2}\right)\right]$$

for N^g and $\vec{a} = -\vec{g}$ for N^a

Let the charge moving with $\vec{a} = \vec{g}$ in N^g (but with $\vec{v} = 0$)

then $\vec{E} = -\nabla\varphi^g - \frac{\partial\vec{A}}{\partial t} = \frac{\rho V}{4\pi\varepsilon_0}\left[\left(\frac{\vec{n}}{r^2} + \frac{\vec{g}\cdot\vec{n}}{c^2 r} - \frac{\vec{g}}{c^2 r}\right) + \left(-\frac{\vec{g}\cdot\vec{n}}{c^2 r}\vec{n} + \frac{\vec{g}}{c^2 r}\right)\right]$

$= \frac{e}{4\pi\varepsilon_0} \cdot \frac{\vec{n}}{r^2}$, which reduce to Coulomb field.

so No Radiation is obtened for a free falling charge.