Philosophy of Logics

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Chap5 Quantifiers

- I. The quantifiers and their interpretation
- II. Quantification and ontology
- III. The Choice of interpretation

Reading Texts

- Susan Haack, *Philosophy of Logics*. London : Cambridge, 1978.
- Chap 4. Quantifiers

I. The quantifiers and their interpretation

- 1. the formal readings of quantifiers
- 2. the difficulties of formal readings
- 3. different interpretations

1.The formal readings

- （x）F(x)is usually read long the lines of "for all x, F(x)"
- （$x）along the lines of "for some x, F(x)", or more accurately,
- "for at least one x, F(x)",
- （...）is generally known as the universal, （$...）as the existential, quantifier.
- A variable inside the scope of a quantifier, such as （$x）Fx, is said to be bound, a variable not bound by any quantifier, such as x in Fx, or y in ($x) Rxy, to be free.

Frege invented quantification theory

- Frege 1879, Peirce 1885, Mitchell
- The importance of shifting attention from the subject-predicate distinction to the function-argument distinction.
- One consequence of this is to allow for relations, to allow for second level functions.
- To say that three-legged dog does exist, is to say that the concept three-legged dog is not empty;
- The quantifier （$...）is a concept which applies to concepts, a second-level function.

2. the difficulties of formal readings

- Only the existential and universal quantifiers are included in standard first-order predicate logic.
- The existential quantifier is commonly used to capture the logical properties of 'some' and 'a' and the universal quantifier those of 'every''each,' and 'all'
- 'any' is a tricky case because it seems to function sometimes as a universal and sometimes as an existential quantifier.

differences

- But there are differences between 'some' and 'a' and between 'every,' 'each,' and 'all' that are not captured by their formal symbolizations.
- For example, only 'some' and 'all' can combine with plural nouns. Also, 'some' but not 'a' can be used with mass terms, as in 'Max drank some milk' as opposed to 'Max drank a milk' (' Max drank a beer' is all right, but only because the mass term 'beer' is used here as a count noun, as in 'Max drank three beers').
- But these differences are superficial as compared with two deeper difficulties with the symbolization of quantifiers in first-order predicate logic.

Difficulty -existential

- A simple sentence like (4) is standardly symbolized with existential quantification, as in (4PL): (4_{PL}) $(\exists x)(Qx \,\&\, Sx)$
- (4) Some quarks are strange.
- The difficulty is that there is nothing in (4) corresponding to the connnective '&' in (4PL) or to the two open sentences it conjoins. There is no constituent of (4PL) that corresponds to the quantified noun phrase 'some quarks' in (4).

Difficulty –universal

- The situation with universal quantification is similar, illustrated by the symbolization of a sentence like (35) as (35PL): (35_{PL}) $(\forall x)(Fx \supset Gx)$
- (35) All fish are garish.
- In fact, not only is there is nothing in (35) that corresponds to the connective '⊃' in (35PL), but (35PL) is true if there are no Fs, as with (36),
- (36) All four-legged fish are gymnasts.
- This is not a difficulty only if (36) is equivalent to (37),
- (37) Anything that is a four-legged fish is a gymnast.
- and intuitions differ on that.

All Fs are G

- In standard predicate logic, universal sentences of the form All Fs are G' are true if there are no Fs, and, according to Russell's theory of descriptions, sentences of the form 'The F is G' are true if there is no unique F.
- Of course, one would not assert such a sentence if one believed there to be no F or no unique F, but logic need not concern itself with that.
- In any case, clearly the forms of (4_{PL}) (35_{PL}) $(\forall x)(Fx \supset Gx)$ (4_{PL}) $(\exists x)(Qx \,\&\, Sx)$ and (35_{PL}) do not correspond to the grammatical forms of the sentences they symbolize.

a problem with English

- These discrepancies might be thought to reveal a problem with English rather than with predicate logic. Indeed, Russell regarded it as a virtue of his theory of descriptions that the structure of the formal rendering of a description sentence does not mirror that of the sentence it symbolizes.
- A sentence like (38),

  (38) The director of *Star Wars* is rich.
- should not be symbolized with 'Rd,' where 'R' stands for 'is rich' and 'd' stands for 'the director of *Star Wars*,' but with the more complex but logically revealing (38PL):

(38_{PL}) $(\exists x)(Dx \,\&\, (y)(Dy \supset (y = x) \,\&\, Rx)$

the grammatical form

- Whereas (38) has 'the director of Star Wars' as its grammatical subject and 'is rich' as its grammatical predicate, it is revealed by logical analysis not to be of subject-predicate logical form.
- Hence the grammatical form of a sentence like (38) is "misleading as to logical form," as Russell was paraphrased by Strawson (1952: 51). The definite description 'the director of Star Wars' does not correspond to any constituent of the proposition expressed by (38).
- Definite descriptions "disappear on analysis." The contribution they make to the propositions in which they occur is a complex quantificational structure of the sort contained in (38_{PL}).

the notation is not adequate

- Indeed, as Barwise and Cooper (1981) have shown, the notation of first-order logic is not adequate for symbolizing such quantificational expressions as 'most,''many''several,''few.'
- And there are numerical quantifiers to contend with, like 'eleven' and 'a dozen,' and more complex quantificational expressions, such as 'all but one,''three or four,''fewer than ten,''between ten and twenty''at most ninety-nine,' and 'infinitely many.'

The formalization of numerical statements

- numerical statements
- There are n xs which are F
- There is at least one x which is F

  $(\$x)F(x)$,
- There is at most one x which is F

  $(x)(y)(F(y) \equiv x=y)$,
- There is exactly one x which is F

  $(\$x)(y)(F(y) \equiv x=y)$,
- There are exactly two xs which are F

  $(\$x)(\$y)(z)(F(z) \equiv x=z \vee y=z)$

3. different interpretations

- It is often observed that the universal quantifier is analogous to conjunction:
- And the existential quantifier to disjunctions:
- $(x)Fx \equiv Fa \wedge Fb \wedge Fc \wedge$...etc.
- For a theory for which the domain is finite a universally quantified formula is equivalent to a finite conjunction and existentially quantified formula to a finite disjunction.
- $(\$x)Fx \equiv Fa \vee Fb \vee Fc \vee$ ...etc.
- However for a theory with an infinite domain, the '...etc' is in-eliminable.
- An acceptable interpretation will have to supply the requisite generality.

Objectual interpretation

- Two distinct styles of interpretation have been offered for the quantifiers.
- Objectual interpretation appeals to the values of the variables, the objects over which the variables range:
- （x）F(x) is interpreted as 'For all objects, x, in the domain D, F(x)'.
- （$x）F(x) is interpreted as 'For at least one object, x, in the domain D, F(x)'.
- On the 'model-theoretic' approach, the domain D, is a set of objects assigned as the range of variables-as it might be, the natural numbers, persons, fictional characters, or whatever; the 'absolute' approach however requires D to be the 'universe', i.e. all the objects there are.

Substitutional interpretation

- Substitutional interpretation appeals, not to the values, but to the substituends for the variables, the expressions, that is , that can be sbustututed for the variables:
- （x）F(x) is interpreted as 'All substitution instances of 'F...' are true'
- （$x）F(x) is interpreted as 'At least one substitution instance of 'F...' is true'

Two camps

- The objectual interpretation is championed by Quine and Davidson; the substitutional interpretation by Mates and Marcus. Both interpretations have a pretty long history;
- The objectual interpretation is generally thought of as standard, the substututuional as a challenger whose credentials stands in need of scrutiny.
- There are two possible views about the status of the two styles of interpretation: that they are rivals, only one of which can be 'right'; or that they may both have their uses.
- The later tolerant view was supported by Susan Hacck, Belnap and Dunn 1968, Linsky 1972, Kripke 1976.

II. Quantification and ontology

- 1. the question of ontology
- 2. Quine's two slogans
- 3. the criterion of ontological commitment

1. the question of ontology

- Ontology may be characterized as that part of metaphysics which concerns the question, what kinds of thing there are.
- Aristotle was the founder of logic and ontology. The first discipline is concerned with the validity of arguments irrespective of their subject-matter.
- The second discipline, called 'first philosophy' by Aristotle (and 'ontologia' by Rudolphus Goclenius in the *Lexicum Philosophicum* (1613)) investigates being in its own right, that is the categorial aspects of entities in general, and the modes and aspects of being. It can be traced back to Aristotle's *Categories* and *Metaphysics*.

logic and ontology

- The interplay between logic and ontology has inspired major philosophical works of the twentieth century such as Russell's *Philosophy of Logical Atomism* (1918) and Wittgenstein's *Tractatus logico-philosophicus* (1921).
- Though both works now belong to the history of the subject, the issue they address, that is whether a logical language could be designed which would depict the main ontological structures of reality, remains a live issue.

Quine's *Word and Object*

- With Quine's *Word and Object* (1960), a major shift of emphasis occurred. The mirror of the most important traits of reality is no longer to be sought in *language* as such, but in the *theories* about the world which scientists hold to be true, and only derivatively in the language needed to formulate them.
- According to Quine, the ontological work incumbent on philosophers consists of the critical scrutiny of the realm of objects introduced into scientific theories by scientists. It is "the task of making explicit what had been tacit, and precise what had been vague; of exposing and resolving paradoxes, smoothing kinds, lopping off vestigial growths, clearing ontological slums"

2. Quine's two slogans

- Ockham's razor, "*Entia non sunt multiplicanda praeter necessitatem*" ``entities should not be multiplied unnecessarily''
- the preliminary question: 'what are unnecessary entities?'
- One possible answer is: entities are unnecessary if we can abstain from countenancing them without sacrificing *scientific truth*.
- That answer is controversial. One might argue that besides preserving the *set of truths* of a given science, we should also be concerned about preserving the *explanatory power* of our theories.

No entity without identity

- Quine has also contributed to the *methodology of ontology* by imposing a constraint encapsulated in the motto: "No entity without identity" .
- This slogan introduces his standard of ontological admissibility – only those entities should be tolerated for which adequate criteria of identity can be supplied.
- Such a requirement is fulfilled by *sets*: two sets are identical if and only if they have the same members.

To be is to be the value of a variable

- This slogan introduces his criterion of ontological commitment, a test of what kinds of thing a theory says there are.
- Quine provided a definite criterion: "in general, *entities of a given sort are assumed by a theory if and only if some of them must be counted among the values of the variables in order that the statements affirmed in the theory be true*" (Quine 1953, 1961: 103).
- This is primary relevant to his support for objectual quantification.

3. the criterion of ontological commitment

- to say that a given existential quantification presupposes objects of a given kind is to say simply that the open sentence which follows the quantifier is true of some objects of that kind and none not of that kind. (1953a p.131)
- One tells what a theory says there is by putting it in predicate calculus notation, and asking what kinds of thing are required as values of variables if theorems beginning '($x)...' are to be true.

An example

- So a theory in which '($x)(x is prime and x>1,000,000) is a theorem is committed to the existence of prime numbers greater than a million, and a fortiori to the existence of prime numbers and to the existence of numbers.
- The criterion applies only to interpreted theories, only when the theory is expressed in primitive notation;
- If quantification over numbers is only an abbreviation for quantification over classes, then the theory is committed to classes but not to numbers.

Intensional entities

- Quine's criterion is a test of what a theory says there is , not of what there is.
- What there is is what a ture theory says there is.
- The refusal to admit intensional entities acts as a sort of preliminary filter; theories which say there are intensional entities are not, in Quine's view, really intelligible, so a fortiori they are not true.

the status of possible objects

- The demand for clear *identification criteria* has far-reaching consequences in ontology. It has a bearing on another burning issue under discussion today: that of the status of possible objects.
- By Quine's standards, possible objects are not eligible as *entities*. They lack criteria of identification.
- Nobody, Quine complains, can decide whether "the possible fat man in that doorway" and "the possible bald man in that doorway" denote the same individual.

Eliminability of singular terms

- Eliminability of singular terms shows that, the ontological commitment of a theory cannot reside in its names.
- Quine's proposal has two stages: first, singular terms are replaced by definite descriptions, and then the definite descriptions are eliminated in favour of quantifiers and variables.
- The x which is F is G df = there is exactly one F, and whatever is F is G,
- i.e. in symbols,

G((ιx)F(x)) df= ($x)((y)(F(y)≡x=y)&G(x))

- 'Socrates took poison'- 'the x which socratises took poison' , -there is just one x which socratises and whatever socratises took poision.

III. The Choice of interpretation

- 1.the ontological commitment of objectual interpretaion
- 2.the substitutional interpretaion postpones the question
- 3.Quine's refusal to second-order quantification

1.the ontological commitment of objectual interpretaion

- As to the objectual interpretation, if one takes the domain D to be the universe – everything there is, as Quine assumed, then '(∃x) Fx' means that there is an (existent, real) object which is F.
- Then if it is a theorem of a theory that '(∃x) Fx, then that theory says that there is an object which is F, and if one says that there are Fs, one is committed to there being Fs.
- The objectual reading of the quantifier does indeed locate ontological commitment in the bound variables of a theory.
- Quine's slogan: to be said to be is to be the value of a variable bound by an objectual quantifier.

2.the substitutional interpretaion postpones the question

- The substitutional interpretation does not give a negative answer to ontological questions, rather, it postpones them.
- If 'Fa' is true only if 'a' is a singular term which denotes an (existent) object, then there will have to be an object which is F if (∃x) F(x) is to come out true; but it is not inevitable that the truth-conditions for the appropriate substitution instances will bring an ontological commitment.
- (∃x) (F(x)∨¬F(x))
- At least one substitution instance of 'F...∨¬F...' is true.

Quine: a deplorable evasion of metaphysical responsibility.

3.Quine's refusal to second-order quantification

- On the objectual interpretation, the appropriate substituends for bound variables should be expressions whose role is to denote objects, singular terms.
- Quine sometimes defines a singular term as an expression which can take the position of a bound variable.
- On the substitutional interpretation, quantification is related directly not to objects, but to substituends, and so there is no particular need to insist that only expressions of the category of singular terms may be bound by quantifiers.

(∃p) (p→¬p)

- On the objectual interpretation, a first-order quantification like (∃x) F(x)says that there is an object (individual) which is F; a second-order quantification like (∃F) F(x)says that there is an object (property) which x has.
- (∃p) (p→¬p)says that there is an object(proposition) which materially implies its own negation.
- the objectual interpretation to allow second-order quantification would commit him to their existence, in which Quine prefers not to indulge at all, but to confine himself to first-order theories.

Fig.2 –Susan Hacck, p44

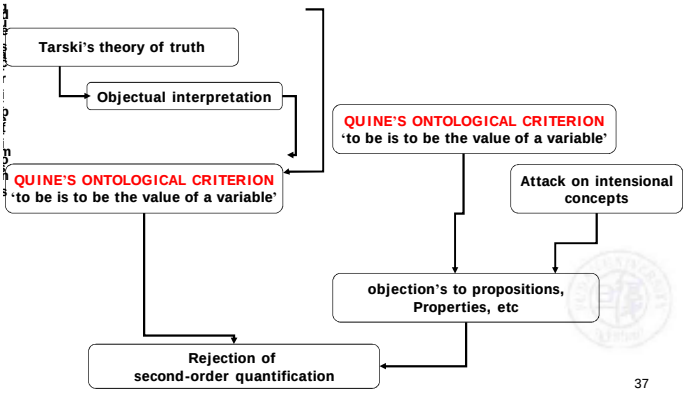
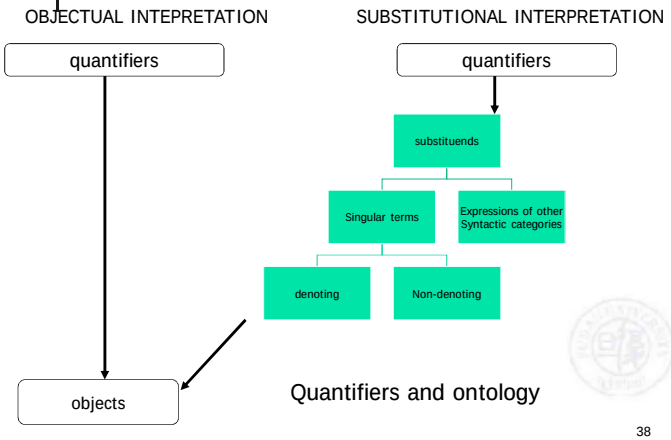


Fig.3 –Susan Hacck, p54



Thanks

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