Philosophy of Logic

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Chap2 Brief History of Logic

- I. A Map of Logic
- II. Logic in Western
- III. Logic in Ancient India
- IV. Logic in Ancient China
- V . The status of Logic in contemporary academics

I. A Map of Logic

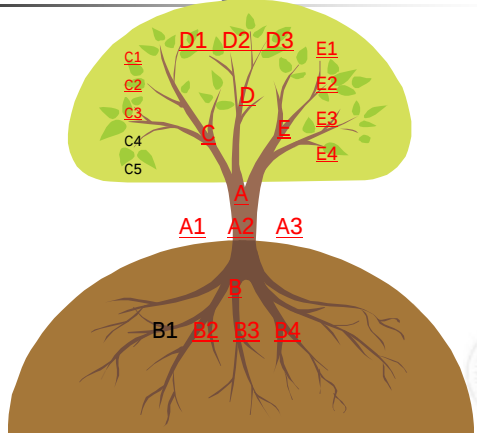
- Nicholas Rescher

Topics in Philosophical Logic

A Map of Logic

- A. Basic Logic
- B. Metalogic
- C. Mathematical Developments
- D. Scientific Developments
- E. Philosophical Developments

A tree of logic



A. Basic Logic

- 1.Traditional Logic
- 2.Orthodox Modern Logic
- 3. Unorthodox Modern Logic

1. Traditional Logic

- a. Aristotelian logic
  - i. Theory of categorical propositions
  - ii. Immediate inference
  - iii. Syllogistic logic
- b. other developments
  - i. The medieval theory of consequentiae
  - ii. Discussions of the 'laws of thought' in idealistic logic

2. Orthodox Modern Logic

- a. Propositional logic
- b. Quantificational logic
  - i. Predicate logic
  - ii. Logic of relations

3. Unorthodox Modern Logic

- a. Modal logic
  - i. Alethic modalities
  - ii. Physical modalities (see D1b)
  - iii. Deontic modalities (see E1b)
  - iv. Epistemic modalities (see E3b)
- b. Many-valued logic
- c. Nonstandard systems of implication
  - i. Strict implication
  - ii. Intuitionistic propositional logic
  - iii. Entailemnt and relevant implication
  - iv. Connexive implication
- d. Nonstandard systems of quantification (see B2cv, E2a)

B. Metalogic

- 1. Logical Syntax
- 2. Logical Semantics
- 3. Logical Pragmatics
- 4. Logical Linguistics

2.Logical Semantics

- a. Basic semantics (denotation, extension/intension, truth, satisfiability, validity, completeness)
- b. Theory of models
- c. Special topics
  - i . Theory of definition
  - ii. Theory of terms(abstraction)
  - iii. Theory of descriptions
  - iv. Theory of identity
  - v. Logic of exsistence(exsitents and nonexistents) (see A3d, E2a)
  - vi. Logic of information and information-processing(see E3d)

3.Logical pragmatics

- a. Logical linguistics and the logical theory of natural languages (see B4)
- b. Rhetorical analysis(Aristotelian 'topics'; the 'New Rhetoric' of Chaim Perelman)
- c. 'Contextual implication'
- d. Theory of informal (or material) fallacies
- e. Unorthodox applications of logic

4. Logical linguistics (see B3a)

- a. Theory of structure(morphology)
- b. Theory of meaning
- c. Theory of validity

C. Mathematical Developments

- 1. Arithmetical
- 2. Algebraic
- 3. Function-Theoretical
- 4. Proof Theory (Theory of axiomatizability, Gentzenization)
- 5. Probabilistic Logic (see E4b)
- [6. Theory of sets]
- [7. Foundations of Mathematics]

Mathematics

- 1. Arithmetical
  - a. Algorithms
  - b. Theory of computability
  - c. Computer programming
- 2. Algebraic
  - a. Boolean algebra
  - b. Lattice-theoretic logic
- 3. Function-Theoretical
  - a. Recursive functions
  - b. Lambda conversion
  - c. Combinatory logic

D. Scientific Developments

- 1. Physical Applications
  - a. Quantum-theoretic logic
  - b. Theory of 'physical' or 'causal' modalities (see A3aii)
- 2. Biological Applications
  - a. Woodger-style developments
  - b. cybernetic logic
- 3. Social-Science Applications
  - a. Logic of norms (see E1b)
  - b. Logic of valuation
  - c. Legal applications

E. Philosophical Developments

- 1. Ethical Applications
- 2. Metaphysical Applications
- 3. Epistemological Applications
- 4. Inductive Logic (see E3e)

1. Ethical Applications

- a. Logic of action
- b. Deontic logic (see D3a)
- c. Logic of commands (Logic of imperatives)
- d. Logic of preference and choice (utility, cost, logical issues in the theory of games and decisions)

2. Metaphysical Applications

- a. Logic of existence (see B2cv, A3d)
- b. Chronological logic (tense logic, change-logic, logic of process)
- c. Logic of part/whole (mereology, the calculus of individuals)
- d. Leśniewski's 'ontology'
- e. Constructivistic logic (logical reductionism, Aufbau-ism)
- f. Ontology (in the sense of the nominalism-realism debate)

3. Epistemological Applications

- a. Logic of questions (and answers)
- b. Epistemic logic (belief, assertion, knowledge, relevance, 'about', and other intentional concepts)
- c. Logic of supposition (hypothetical reasoning, counterfactuals)
- d. Logic of information and information-processing (see B2cvi)
- e. Inductive logic (see B4)

4. Inductive Logic (see E3e)

- a. Logic of evidence and confirmation, acceptance (rules of acceptance)
- b. Probabilistic logic (see C5)

S·Haack: Philosophy of Logics

- Formal Logic :
- Traditional logic -Aristotelian syllogistic
- Classical logic -2-valued sentence calculus
  Predicate calculus
- Extended logics -modal, tense, deontic, epistemic,preference,imperative, erotetic(interrogative) logics
- Deviant logics - many-valued logics
  Intuitionist logics, quantum logics, free logics
- Inductive logics

II. Logic in Western

- 1. Ancient Greek Philosophical Logic
- 2. History of Logic: Medieval
- 3. The Rise of Modern Logic

1. Ancient Greek Philosophical Logic

- Ancient Greek logic was inseparable from ancient Greek philosophy.
- The formal theories developed by major logicians such as Aristotle, Diodorus Cronus, and Chrysippus were in large part influenced by metaphysical and epistemological concerns.

Geometrical Demonstration

- The notion of demonstration attracted attention first in connexion with geometry.
- The Egyptians had discovered some truths if geometry empirically, and the name geometry originally meat the same as land measurement.
- Some stories give Thales(640-546,B.C.)the credit for proving the first theorem in geometry, but the systematic study of the science seems to have begun in the Pythagorean school.

Parmenides

- Greek philosophical logic originates with Parmenides (c. 510-c. 440 bce).
- Though Parmenides cannot be said to have had a logic, or even an interest in studying the validity of arguments, his views did much to set the agenda out of which many things in Greek philosophy, including logic, later arose.
- His philosophical position is both simple and mystifying: being is, whereas not being is not and cannot either be thought or said.

Zeno of Elea

- According to Plato's *Parmenides*, Zeno's goal was to defend Parmenides' views from the objection that they were absurd or in contradiction to our ordinary beliefs.
- In response, Zeno argued that the beliefs that there is motion and that there is a multiplicity of entities have consequences that are even more absurd because self-contradictory.
- This was the point of his celebrated arguments against motion and multiplicity.

motion is impossible

- In order to move from point A to point B, you must first reach the point halfway between them. But before you can reach that point, you must reach the point halfway to it. Continuing in this way, we see that before you can reach any point, you must already have reached an infinity of points, which is impossible. Therefore, motion is impossible.
- This argument rests only on the assumptions that motion is possible, that in order to move from one point to another one must first pass through the point halfway between, and that there is a point halfway between any two points.

originator of 'dialectic'

- Zeno's arguments take a particular form: beginning with premises accepted by his opponent, they derive conclusions that the opponent must recognize as impossible.-- reductio *ad* absurdum
- Aristotle says that in introducing this form of argument, Zeno was the originator of 'dialectic.'.

'Sophists'

- We know that a number of the Sophists had interesting (and quite divergent) views on philosophical matters.
- Teaching oratory was a profitable occupation, and several Sophists seem to have amassed fortunes from it.
- The content of their instruction, to judge by later treatises on rhetoric, would have included such things as style and diction, but it would also have included some training in argumentation.
- That could have ranged from teaching set pieces of argument useful for specific situations, all the way to teaching some kind of method for devising arguments according to principles.

Protagoras

- relativism about truth
- This is most forcefully put by Protagoras (c. 485–415 bce), who began his treatise entitled *Truth* with the line, "Man is the measure of all things; of things that are, that they are, and of things that are not, that they are not."
- Plato tells us in his *Theaetetus* that this meant "whatever seems to be true to anyone is true to that person": he denied that there is any truth apart from the opinions of individuals.

Socrates

- Contemporary with the Sophists was Socrates (469–399 bce), whose fellow Athenians probably regarded him as another Sophist.
- Socrates did not teach oratory (nor indeed does he appear to have taught anything for a fee). Instead, he engaged people he encountered in a distinctive type of argument: beginning by asking them questions about matters they claimed to have knowledge of, he would lead them, on the basis of their own answers to further questions, to conclusions they found absurd or to contradictions of their earlier admissions.
- This process, which Plato and Aristotle both saw as a form of dialectical argument, usually goes by the name of 'Socratic refutation.'

Plato

- Plato (428/7–348/7 bce) did not develop a logical theory in any significant sense. However, he did try to respond to some of the issues raised by Parmenides, Protagoras, and others.
- In his *Theaetetus*, he argues that Protagoras' relativistic conception of truth is self-refuting in the sense that if Protagoras intends it to apply universally then it must apply to opinions about Protagoras' theory of truth itself; moreover, it implies that the same opinions are both true and false simultaneously.

theory of Forms or Ideas

- Plato's most celebrated philosophical doctrine, his **theory of Forms or Ideas**, can be seen as a theory of predication, that is, a theory of what it is for a thing to have a property or attribute.
- In very crude outline, Plato's response is that **what it is for x (e.g. Socrates) to be F (e.g. tall)** is for x to **stand in a certain relation (usually called 'participation') to an entity**, '**the tall itself**,' which just is tall.

Aristotle

- Aristotle (384–322 bce), Plato's student, developed the **first logical theory of which we know**. He follows Plato in analyzing simple sentences into **noun and verb**, or **subject and predicate**,
- but he develops it in far greater detail and extends it to sentences which have **general or universal** (***katholou***, 'of a whole': the term seems to originate with Aristotle) subjects and predicates.

the principle of non-contradiction

- the principle of non-contradiction: "**it is impossible for the same thing to be both affirmed and denied of the same thing at the same time and in the same way**" (***Met***. IV.3, 1005b19–20).
- He argues that it follows from this principle itself that **no one can disbelieve it**. At the same time, since it is prior to every other truth, it cannot itself be proved.
- However, Aristotle holds that anyone who claims to deny it (or indeed claims anything at all) already presupposes it, and he undertakes to show this through what he calls a "**refutative demonstration**" (***Met***. IV.4)

demonstrative sciences

- A **demonstrative science** is a body of knowledge organized into demonstrations (proofs), which in turn are deductive arguments from premises already established.
- If a truth is demonstrable, then for Aristotle to know it just is to possess **its demonstration: proofs** are **neither** a means of finding out new truths **nor** an expository or pedagogical device for presenting results, **but rather** are **constitutive of knowledge**.
- Though he does not limit demonstrative sciences to mathematics, it is clear that he regards **arithmetic and geometry as the clearest examples of them**.

necessarily so

- A demonstration, for Aristotle, is a deduction that shows **why something is necessarily so**.
- This at once imposes two critical limits on demonstrations: nothing can be demonstrated except what is necessarily so, and nothing can be demonstrated except that which has a cause or explanation (the force of the latter restriction will be evident shortly).
- Since demonstrations are **valid arguments**, whatever holds of valid arguments in general will hold of them.
- Therefore, a natural place to begin the discussion of demonstrations would be with a **general account of validity**.

syllogism

- ***Prior Analytics***, the principal subject of which is the '**syllogism**', a term defined by Aristotle as "**an argument in which, some things being supposed, something else follows of necessity because of the things supposed**."
- This is obviously a general definition of '**Valid argument**.'
- However, Aristotle thought that all valid arguments could be '**reduced**' to a **relatively limited set of valid forms** which he usually refers to as '**arguments in the figures**' (modern terminology refers to these forms as 'syllogisms'; this can lead to confusion in discussing Aristotle's theory).

affirmation or *denial*

- Aristotle maintained that a single proposition was always either the ***affirmation*** or the ***denial*** of a single **predicate** of a single **subject**: 'Socrates is sitting' affirms 'sitting' of Socrates, 'Plato is not flying' denies 'flying.'

	Affirmed(affirmative)	*Denied(negative)*
Universal	'Every human is mortal'	'No human is mortal'
Particular	'Some human is mortal'	'Not every human is mortal'

Sentential Logic

- Aristotle never developed an account of sentential logic (the inferences that rest on **sentential operators** such as '**and,**' '**or,**' '**if,**' '**not**').
- Subsequent logicians, including Aristotle's own close associate **Theophrastus**, did not follow him in this and instead offered analyses of the role of **sentential composition** in arguments.

The Stoics:five 'indemonstrable' forms

- With Chrysippus, this develops into a full-fledged sentential logic, resting on five 'indemonstrable' forms of inference. The Stoics stated these using ordinal numbers as place-holders for propositions:
- 1. If the first, then the second; the first; therefore the second.
- 2. If the first then the second; not the first; therefore not the second.
- 3. Not both the first and the second; the first; therefore not the second.
- 4. Either the first or the second; the first; therefore not the second.
- 5. Either the first or the second; not the first; therefore the second.

logikè

- Aristotle did not use the Greek *logikè* for the logical art, but preferred *ta analytika* (from the verb *analuo*: to resolve (into premises or principles), from which the names of his 'sweet Analytics,' that is *Analytica priora and posteriora* derive.
- The Greek *logos* can be found in the writings of both Plato and Aristotle, where it stands for (the smallest meaningful parts of) 'speech' whereby something can be said.
- The Greek logical terminology was latinized by Cicero and Boethius, and the honour of having named the subject belongs to the former who coined *Logica*.

different aspects of logic

- These names, under which the discipline has been known, relate to different aspects of logic, or of how the subject should be seen.
- 'Logic,' thus, would be the study of (the use of words for making) reasoned claims,
- and 'Analytics' resolves reasoning into simpler parts in order to provide grounds.
- 'Dialectics' grounds reasoning in (eternal) relations between logical entities, whereas when logic is thought of as an organon, it serves as the tool for multiplying knowledge through the use of reasoning.
- The purely *formal* logic of today is regularly confined to theory of (logical) consequence between well-formed formulas (WFFs).

3. The Rise of Modern Logic

- The Dark Ages of Logic
- In 1543 the French humanist and logician Peter Ramus (1515–72), who had made a name for himself with his dissertation *Whatever Aristotle Has Said is False*, published his *Dialectic*, a slim book that went through 262 editions in several countries and became a model for many other textbooks.
- Ramus gratified the taste of the times by writing an elegant Latin, drawing his examples from Cicero and other classical authors, and by neglecting most of the finer points of medieval logic and the associated 'barbarous' technical vocabulary.

2. History of Logic: Medieval

- Seven 'liberal arts' constituted the curriculum at a medieval arts faculty. The three 'trivial' arts Grammar, Logic (*Dialectica*), and Rhetoric deal with the use of words rather than with (real) things. These are dealt with in the four mathematical arts-Geometry, Arithmetic, Astronomy, and Harmony (Music)-that comprise the *quadrivium*.
- The specific logical art is concerned with *reasoning*. The logical tradition is as old as Aristotle and history knows periods of intense logical activity.

Organon

- 'Dialectica', the alternative Platonic and Stoic name for logic as part of the *trivium*, derives from the Greek for conversation, since, in this tradition, thinking is seen as the soul's conversation with itself.
- The dialectician investigates relations between (eternal) ideas which have to be respected if the thinking were to be proper.
- In the sixth century the logical works of Aristotle-*Categories*, *On Interpretation*, the two *Analytics*, the *Topics*, and *On Fallacies* — came to be seen as an *Organon* (instrument, tool), and the term has stuck, for example in *Novum Organon* (1620), that is, Francis Bacon's attempt to emend Aristotle's instruments for reasoning.

A Timeline of Medieval Logicians

Before XI
Porphyry (232–305)
Augustinus (354–430)
Boethius (480–524)

XI
Abbo of Fleury
Garlandus Compotista
Anselm of Canterbury (d.1109)

XII
Peter Abailard, 1079–1142
Adam Parvipontanus
Gilbert of Poitiers, 1080–1154
Alberic van Reims
John of Salisbury, c. 1120–1180

XIII
Peter of Spain (d.1277)
William of Sherwood (1210?–66/70)
Robert Kilwardby (d. 1279)
Albert the Great (1200–80)
Roger Bacon (1215–94)

XIII (*cont.*)
Boethius of Dacia (c. 1270)
Henry of Ghent (c. 1217–93)
Ralph Brito (c. 1290–1330)
Siger of Kortrijk (d. 1341)
Simon of Faversham (c. 1300)
John Duns Scotus (1265–1308/9)

XIV
Walter Burleigh (c.1275–1344/5)
William of Ockham (1285–1347)
Robert Holkot (c.1290–1349)
William of Heytesbury (d.1272/3)
Gregory of Rimini (c.1300–1358)
John Buridan (c.1300–after 1358)
Nicholas of Autrecourt (c.1300–after 1358)
Richard Billingham, (c.1350–60)
Albert of Saxony (1316–1390)
Marsilius of Inghen (c.1340–1396)
Vincent Ferrer (c.1350–1420)
Peter of Ailly (1350–1420/1)
Paul of Venice (1369–1429)
Paul of Pergola (1380–1455)
Peter of Mantua (d. 1400)

24-valid syllogistic forms

- AAA-1 AII-1 EAE-1 EIO-1 AAI-1* EAO-1*
  Barbara Darii Celarent Ferio Barbari Celaront
- EAE-2 AEE-2 EIO-2 AOO-2 AEO-2* EAO-2*
  Cesare Camestres Festino Baroco Cesaro Camestros
- AII-3 EIO-3 IAI-3 OAO-3 AAI-3^ EAO-3^
  Datisi Feriso Disamis Bocardo Darapti Felapton
- AEE-4 EIO-4 IAI-4 AEO-4* AAI-4^ EAO-4^
  Calemes Fresison Dimatis Calemos Bamalip Fesapo

Francis Bacon（1561—1626）

- This was an age also of discovery in the sciences and mathematics. The textbook logic 'of the schools' played no role in this.

Francis Bacon claimed in the *Novum Organum* that the "logic we now have" does not help us to *discover* new things, but "has done more to … fasten errors upon us, than to open the way to truth" (Book 1, Aphorism xii). He advocated instead rules of induction, a methodology of scientific investigation.

an exposition of logic

- Traditionally an exposition of logic followed the sequence: theory of terms or concepts, their combination into judgments, and the composition of syllogisms from judgments.
- This was now commonly prefaced by a discussion of the origin of concepts, as inherent in the mind or deriving from sensation and perception.
- In the end, many logic books contained more of these epistemological preliminaries than logic.
- There was, further, especially in England, an ongoing emphasis on logic as the art of disputation.

John Stuart Mill

- In his *System of Logic* (1843) Mill did not contribute to the development of logic as formal science, but like Bacon, attacked it. He claimed that formal principles, especially the syllogism, are a *petitio principii* since they can generate no new knowledge.
- One can know that the major premise 'All men are mortal' is true only if one knows the truth of the conclusion 'Socrates is mortal.' If that is still doubtful, the "same degree of uncertainty must hang over the premiss" (*System of Logic*, 2.3.2).
- The *System of Logic* is best known for formulating rules for the discovery of causes, his famous 'canons': the methods of agreement, difference, residues, and concomitant variation.

A proof of the syllogism *Barbara*

The syllogism	*Boolean computation*	*Comment*
All M are P	1. m(1 − p) = 0	the intersection of m and non-p = 0
All S are M	2. s(1 − m) = 0	the intersection of s and non-m = 0
	3. m = mp	algebraically from 1.
	4. s = sm	algebraically from 2.
	5. s = smp	mp for m in 4, licensed by 3.
	6. s = sp	s for sm in 5, licensed by 4.
	7. s − sp = 0	algebraically from 6.
All S are P	8. s(1 − p) = 0	algebraically from 7. QED.

Gottfried Wilhelm Leibniz(1646-1716)

- Gottfried Wilhelm Leibniz was the great exception to the logic bashing of the seventeenth and eighteenth centuries. He saw the general outline of what logic would much later become, but left only fragments of a 'universal characteristic' through which it would become possible, he thought, to settle philosophical disputes through calculation.
- In the *New Essays Concerning Human Understanding*, a dialogue in which he responded to Locke, the latter's representative Philateles eventually admits "I regarded [logic] as a scholar's diversion, but I now see that, in the way you understand it, it is like a universal mathematics" (*New Essays* 4.17.9).

Bernard Bolzano

- At about the same time, Bernard Bolzano (1781–1848), "one of the greatest Logicians of all time" (Edmund Husserl), published his four-volume *Theory of Science* (*Wissenschaftslehre* (WL) 1837). It is the finest original contribution to logic since Aristotle, and a rich source for the history of the subject.
- In WL no formal calculus or system is developed; it is, rather, a treatise on the semantic concepts of logic. It was celebrated for its resolute avoidance of psychology in the development of these concepts.
- Bolzano defines a spoken or written sentence as a speech act that is either true or false.

George Boole

- George Boole (1815–64) formulated his algebraic logic in conscious opposition to Mill's approach.
- In his *Mathematical Analysis of Logic* of 1847 Boole introduced the notion of an 'elective symbol,'

for example 'x', which represents the result of 'electing' the x's from the universe; it is the symbol for the resulting class, xy is the result of electing y's from the class x, hence the intersection of the two classes.

It holds that xy = yx and also that xx = x. x + y is the union of the two classes, x-y elects the x's that are not y. 0 is the empty class and 1 'the universe,' hence 1-x is the class of non-x's.

Augustus De Morgan

- Augustus De Morgan (1806–71) took a different path, retaining a closer connection with traditional syllogistic logic but moving the subject far beyond its traditional limits.
- When stripped of unnecessary restrictions, the syllogism would constitute an adequate basis for the representation of all modes of deductive reasoning.
- In his *Formal Logic* (1847), and in a later series of articles, he pushed the syllogistic structure so far that he called the status of the standard copula–'is'–into question. If that term could be replaced by any term relating the other components in the statement, the reach of the syllogism would be broadened: categorical statements would become relational statements.

Charles Sanders Peirce

- Charles Sanders Peirce's (1839–1914) theory of logic was once characterized as wider than anyone's. He was the first to consider himself not primarily a mathematician or philosopher, but a logician, filtering through the sieve of logic every topic he dealt with.

- On the formal level, he developed the logical lineage of Boole and De Morgan by refining the logic of relations, and devising more abstract systems of algebraic logic.

division of arguments

Deduction:	*Rule*:	All the beans in this bag are white.
	Case:	These beans are from this bag.
	∴ *Result*:	These beans are white.
Induction:	*Case*:	These beans are from this bag.
	Result:	These beans are white.
	∴ *Rule*:	All the beans in this bag are white.
Hypothesis:	*Rule*:	All the beans in this bag are white.
	… *Result*:	These beans are white.
	∴ *Case*:	These beans are from this bag.

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Gottlob Frege

- Frege (1848–1925) was a German mathematician and philosopher who set logic on a new path. He sought to connect logic and mathematics not by reducing logic to a form of algebra, but by deriving mathematics, specifically arithmetic, from the laws of logic.
- He saw that a philosophy of language was a prerequisite for this and developed much of it in his *Conceptual Notation* (*Begriffsschrift*) of 1879. Like Bolzano, but more polemically, Frege opposed any attempt to import psychology into logic, repeatedly attacking Mill for this confusion. The meaning of sentences, for instance, is not explained by the mental states of speakers, but by investigating the language itself.

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Bertrand Russell

- In 1905 Russell published "On Denoting," his finest philosophical essay, as he thought. It became a milestone in the development of analytic philosophy. A distinction is here made between proper names and expressions like 'the so and so,' which he titled *definite descriptions*.
- In English grammar, 'The present king of France is bald' has the subject 'the present King of France' and the predicate 'bald.' But this is misleading.

Principia Mathematica

- Despite his discovery of the paradox, Russell held that logicism could be made to work, if the comprehension axiom were restricted.
- He proposed several solutions, eventually the *theory of types*, fully articulated in the monumental *Principia Mathematica* authored by Russell and A. N. Whitehead (1910–13, three volumes, 1,000 pages), through which Frege's contributions entered the mainstream of logic.
- The preface states that "in all questions of logical analysis our chief debt is to Frege."

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Syntax and semantics

- There was the view that logic investigates cognitive performance, or else scientific methodology and strategy of discovery, or that it is a branch of rhetoric.
- The most important development of logic after *Principia* was to bring these two strands together.
- In propositional logic, for instance, truth tables (introduced by Wittgenstein in 1922) allow a *semantic* test for the validity of formulas and proofs, a continuation of Bolzano's project. It was then proved that the *Principia* version of propositional logic is complete, that is to say that every semantically valid formula can be derived in it and that it is consistent, that is, that only such formulas (and hence no contradiction) can be derived.
- Later Kurt Gödel proved that first order predicate logic is complete as well, but that higher order logic is not. Since the latter is needed to define arithmetic concepts, this spelled the end of the logicist project.

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Thanks

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