INTRODUCTION TO COMPLEX NETWORKS

IN-CLASS PRESENTATION

GROUP1

On the speech

'How social networks predict epidemics' by Nicholas Christakis

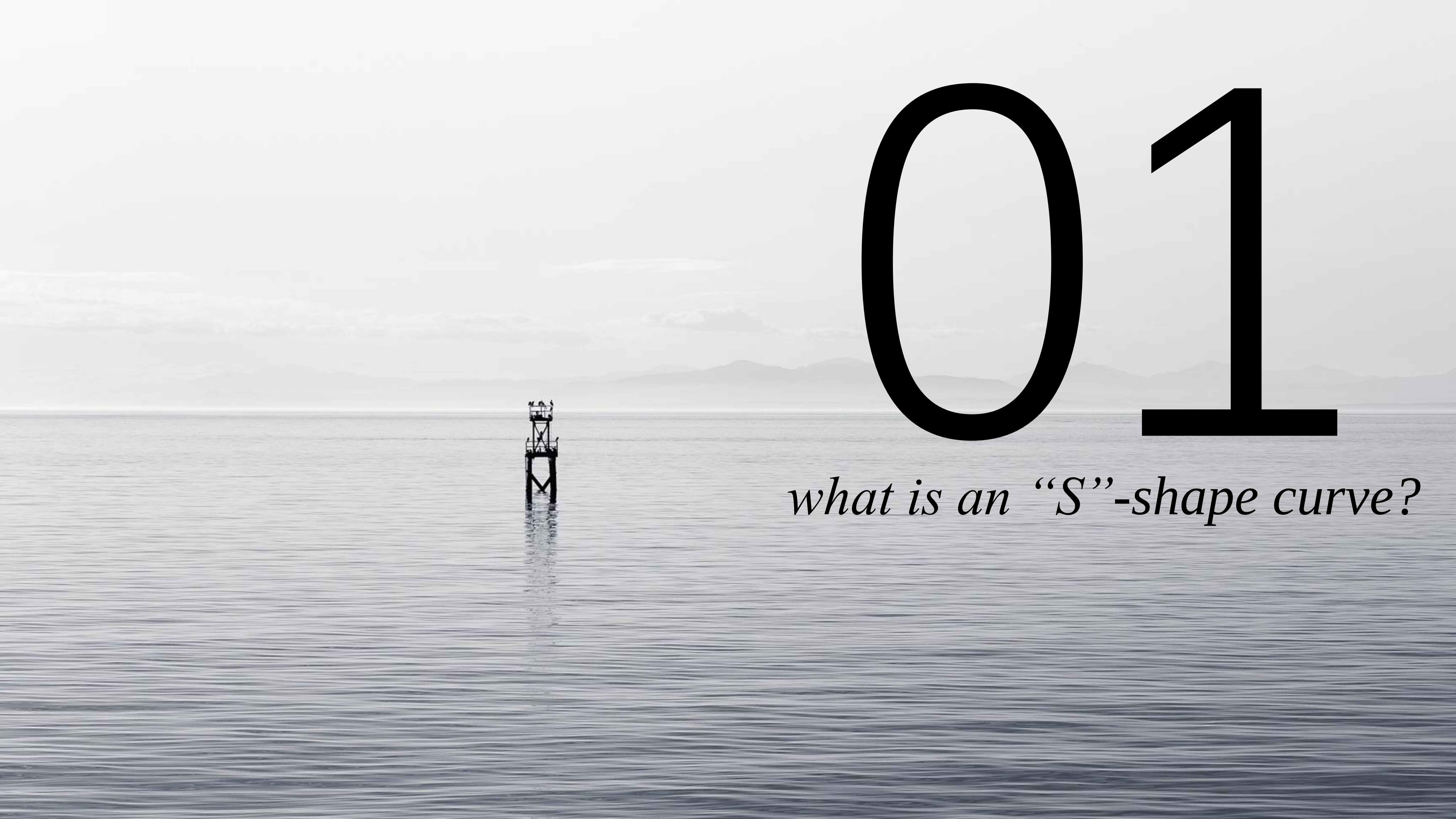
Our mission

3'08'' & 6'30'' the "S"-shape curve;

Explain the difference between the two curves (yellow line and red dot) ;

why the yellow curve shifts to left side?

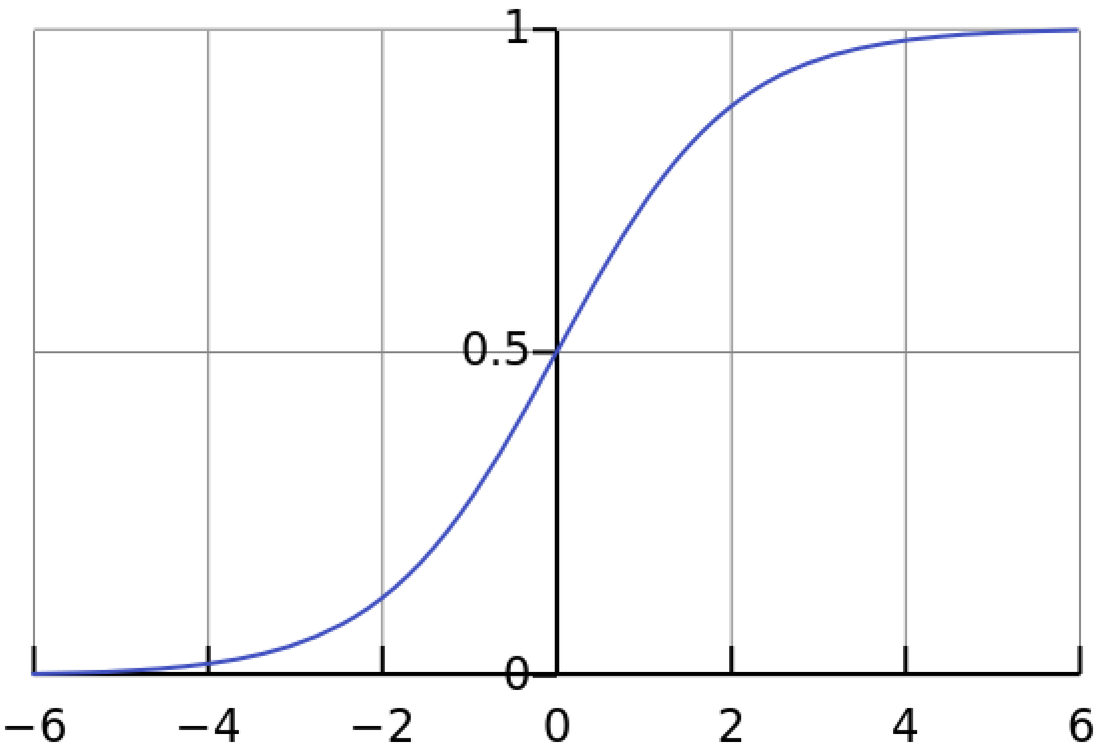
CONTENTS



01

what is an “S”-shape curve?

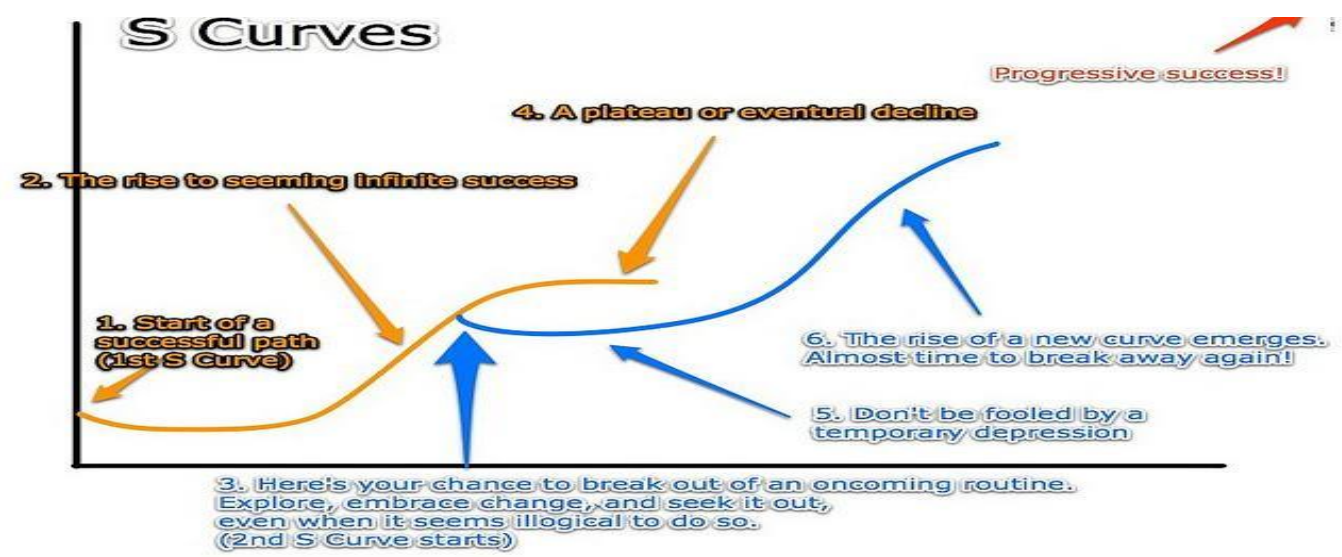
what is an “S-shape”curve?

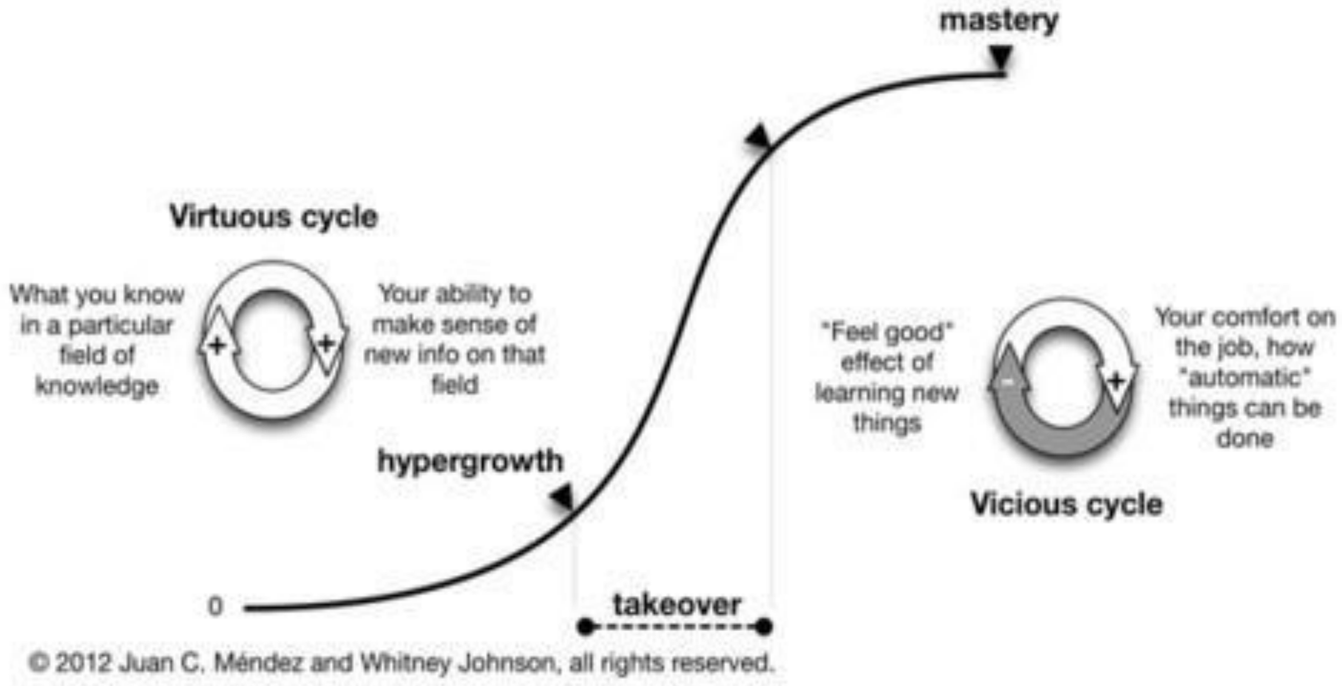


A sigmoid function is a mathematical function having a characteristic "S"-shaped curve or sigmoid curve.

“S-shape”curve

A wide variety of sigmoid functions have been used as the activation function of artificial neurons. Sigmoid curves are also common in statistics as cumulative distribution functions (which go from 0 to 1), such as the integrals of the logistic distribution, the normal distribution, and Student's t-distribution probability density functions.





“S-shape”curve

- Logistic function

$$f(x) = \frac{1}{1+e^{-x}}$$

- hyperbolic tangent

$$f(x) = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- arctangent function

$$f(x) = \arctan x$$

- Gudermannian function

$$f(x) = \operatorname{gd}(x) = \int_0^x \frac{1}{\cosh t}\, dt$$

- Error function

$$f(x) = \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

- Error function

$$f(x) = \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

- Generalised logistic function

$$f(x) = (1+e^{-x})^{-\alpha}, \quad \alpha > 0$$

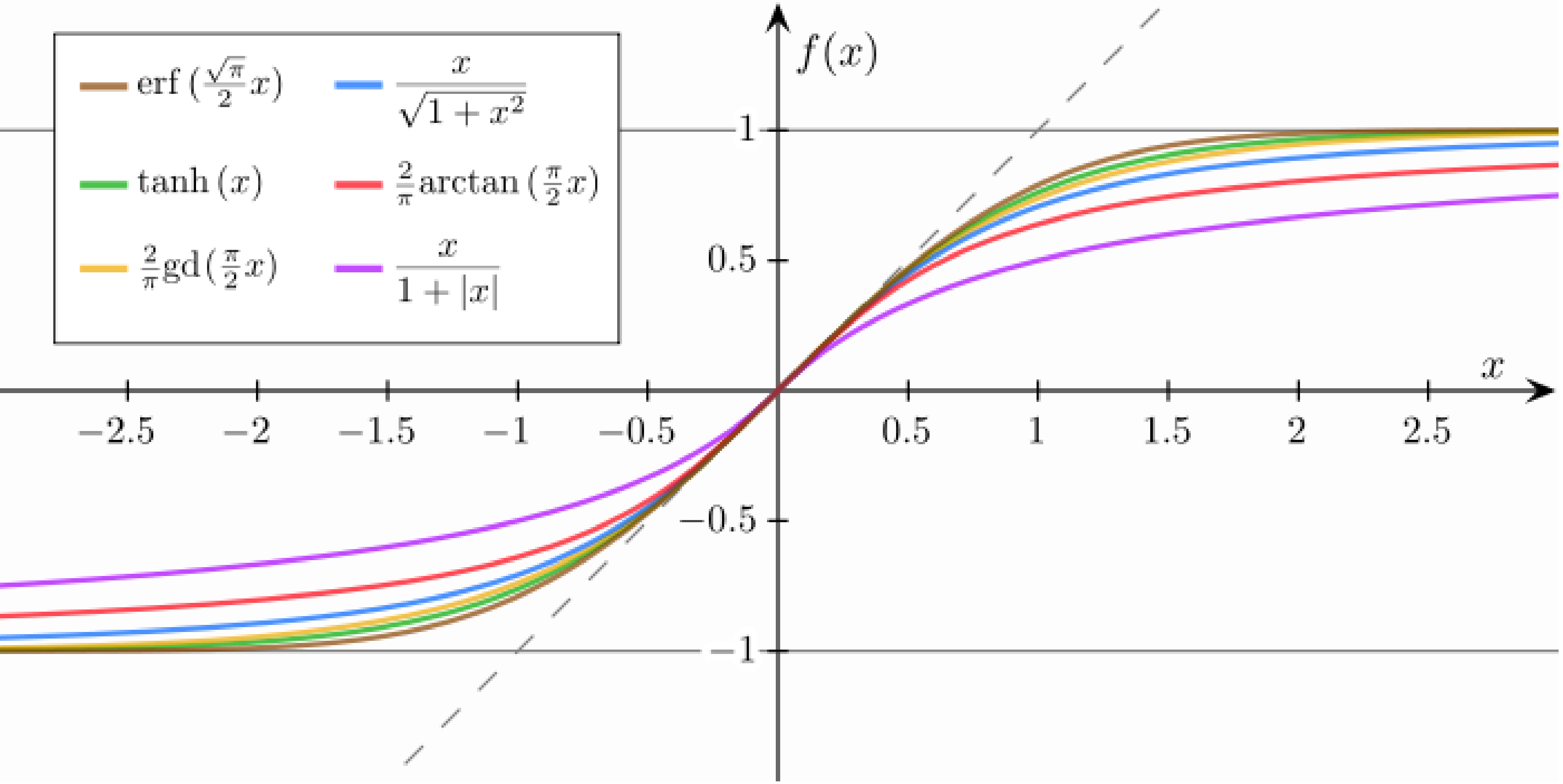
- Smoothstep function

$$f(x) = \begin{cases} \left(\int_0^1 (1-u^2)^N\, du\right)^{-1} \int_0^x (1-u^2)^N\, du & |x| \le 1 \\ \operatorname{sgn}(x) & |x| \ge 1 \end{cases} \quad N \ge 1$$

- Specific algebraic functions

$$f(x) = \frac{x}{\sqrt{1+x^2}}.$$

“S-shape”curve



Often, sigmoid function refers to the special case of the logistic function shown in the first figure and defined by the formula

$$S(x) = \frac{1}{1+e^{-x}}$$

Other examples of similar shapes include the **Gompertz curve** (used in modeling systems that saturate at large values of x) and **the ogee curve** (used in the spillway of some dams). Sigmoid functions have have domain of all real numbers, with return value monotonically increasing most often from 0 to 1 or alternatively from −1 to 1.

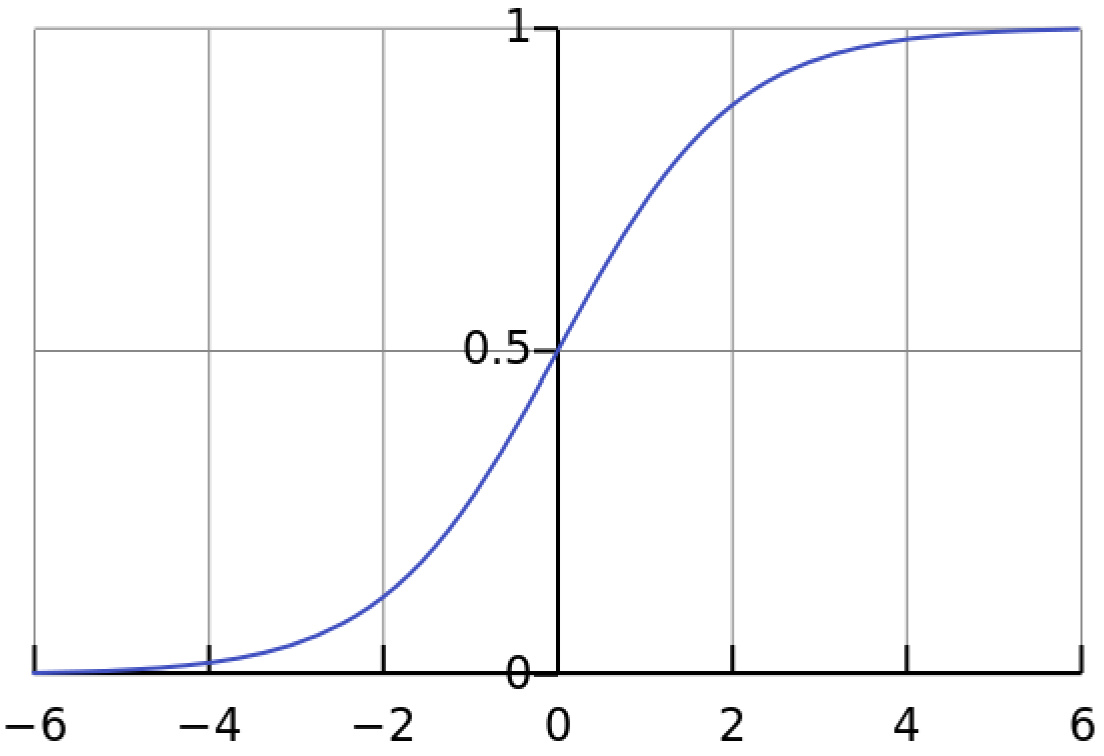
“S-shape”curve

A logistic function or logistic curve is a common "S" shape (sigmoid curve), with equation:

$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}}$$

where

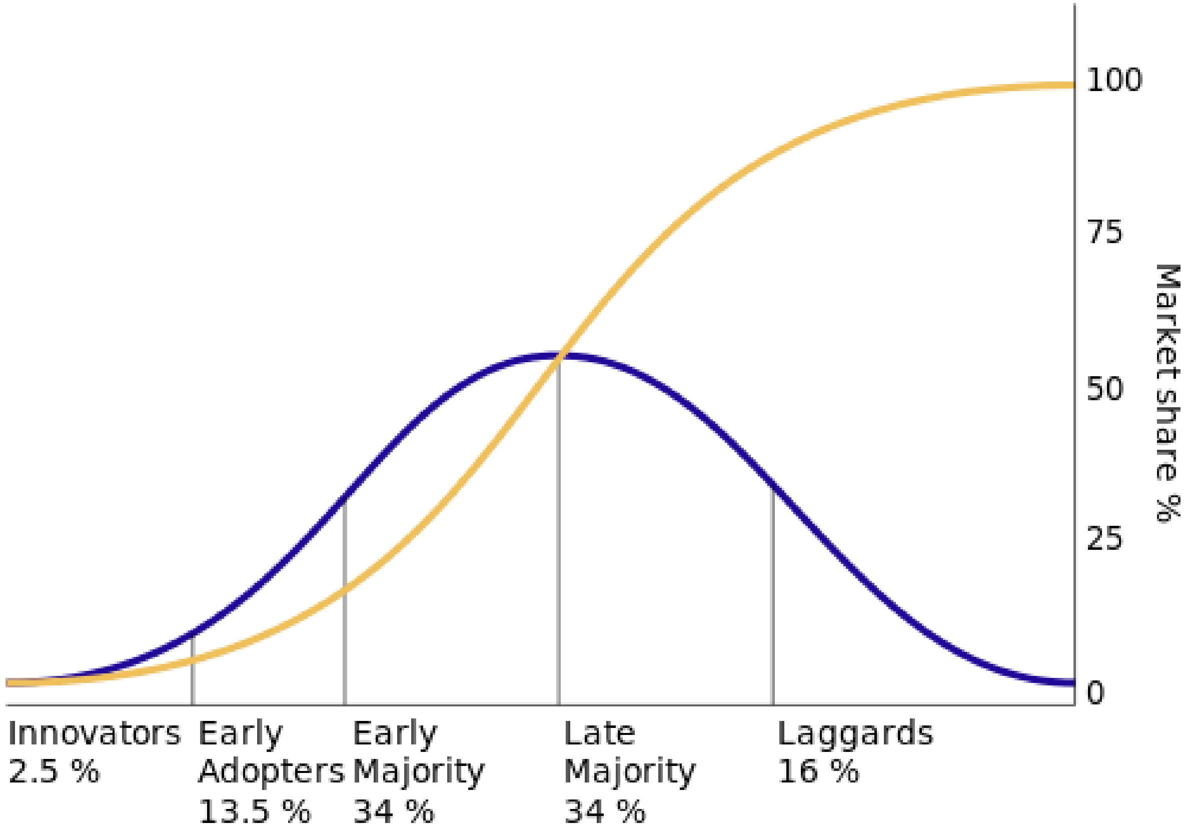
e = the natural logarithm base (also known as Euler's number),
x0 = the x-value of the sigmoid's midpoint,
L = the curve's maximum value, and
k = the steepness of the curve.



The logistic function finds applications in a range of fields, including artificial neural networks, biology (especially ecology), biomathematics, chemistry, demography, economics, geoscience, mathematical psychology, probability, sociology, political science, linguistics, and statistics.

“S-shape”curve

Diffusion of innovations is a theory that seeks to explain how, why, and at what rate new ideas and technology spread. Rogers proposes that four main elements influence the spread of a new idea: the innovation itself, communication channels, time, and a social system. This process relies heavily on human capital. The innovation must be widely adopted in order to self-sustain. Within the rate of adoption, there is a point at which an innovation reaches critical mass.



Epidemiology Fundamentals

Susceptible-Infected (SI) model

Any infected individual may infect each of its neighbors independently with probability β.
Also, the SI model assumes every infected individual stays infected forever.
the dynamic process of the SI model can be characterized by the following differential equation.

$$\frac{dI}{dt} = \beta \times (N - I) \times I$$

the disease propagation network is a clique of N nodes
where I is the number of infected nodes at time t

“S-shape”curve

Another popular disease model that we use in this paper is the so-called SEIR model

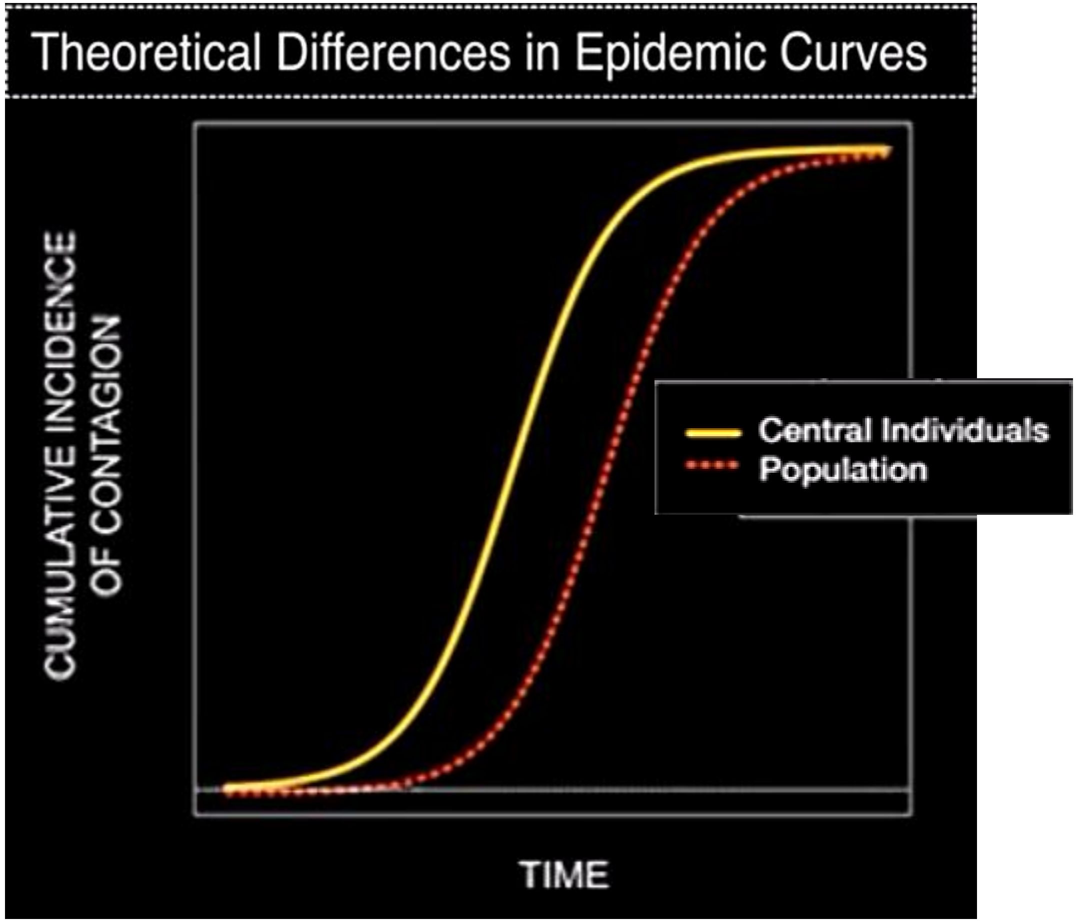
$$\frac{dS}{dt} = -\beta SI \qquad \frac{dI}{dt} = \alpha E - \gamma I$$
$$\frac{dE}{dt} = \beta SI - \alpha E \qquad \frac{dR}{dt} = \gamma I \ ,$$

where S, E, I and R denote the number of individuals in the corresponding states at time t, and S+E+I+R = N.

Here β, α and γ represent the transition rates between the different states. Notice that since we are considering disease epidemics during a short period of time in this paper, we ignore the birth and death rates in the standard SEIR model here.


02
Differences of two curves

S-shaped curve of adoption



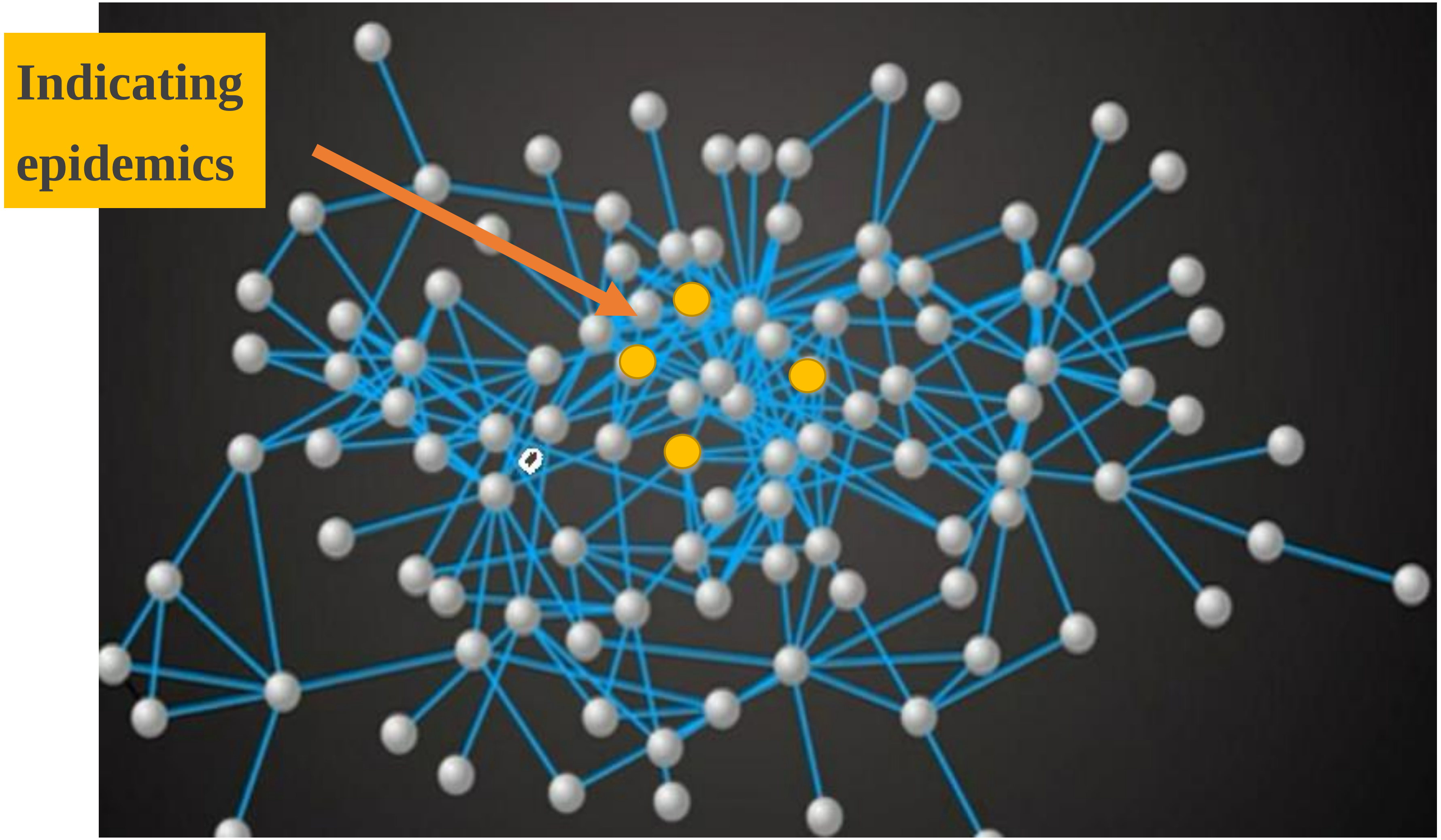
Dotted red line:

adoption in the random people

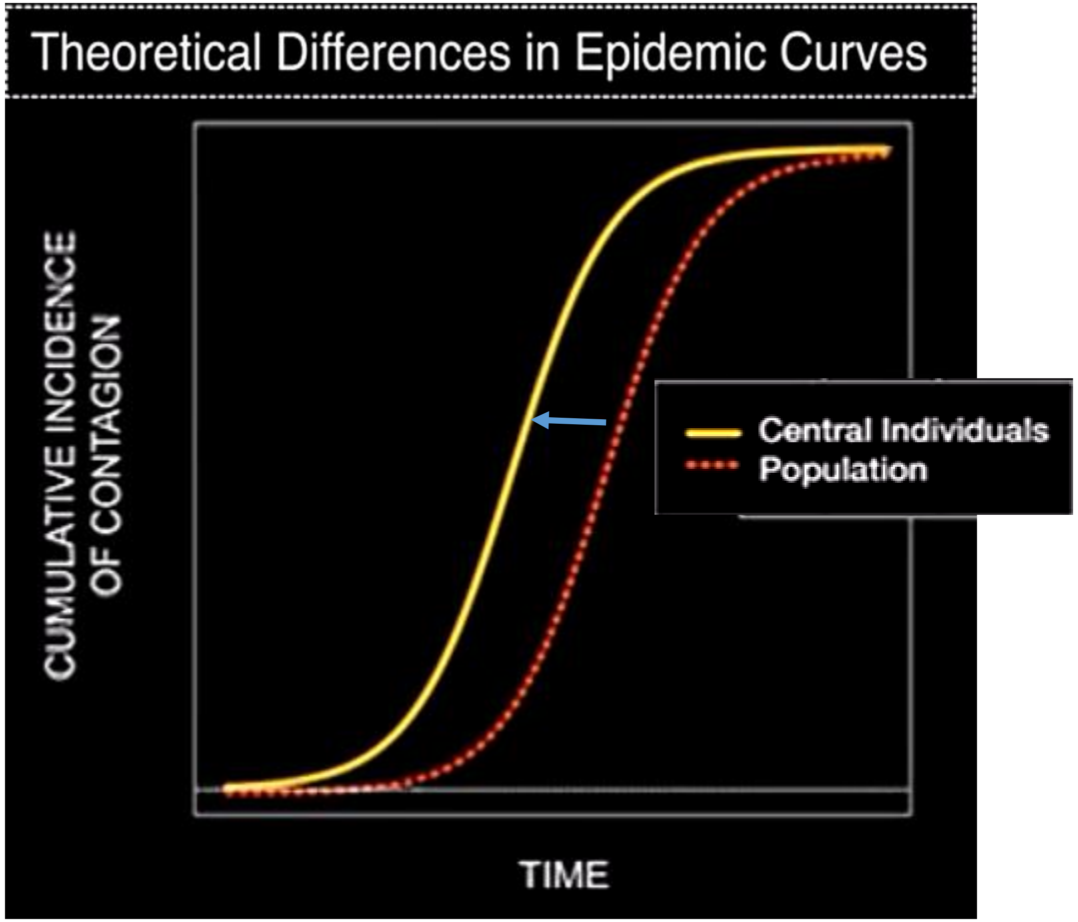
Yellow line:

adoption in the central individuals

Choose of central individuals



S-shaped curve of adoption



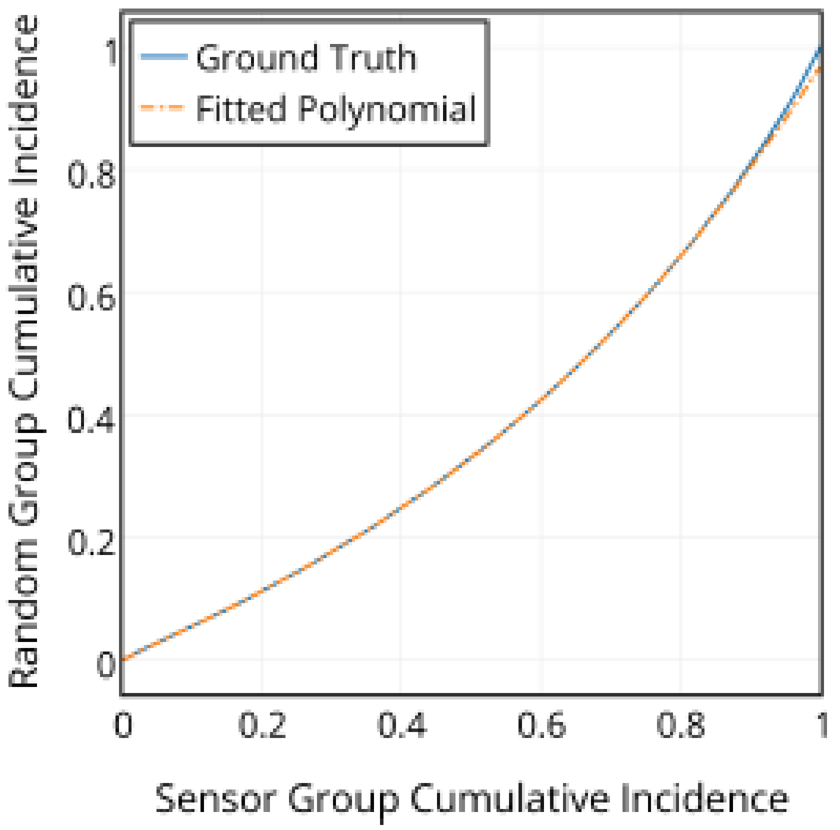
Yellow curve:

adoption in the random people

Red curve:

adoption in the central individuals

Fitted polynomial regression model



Sensor group(predictor):

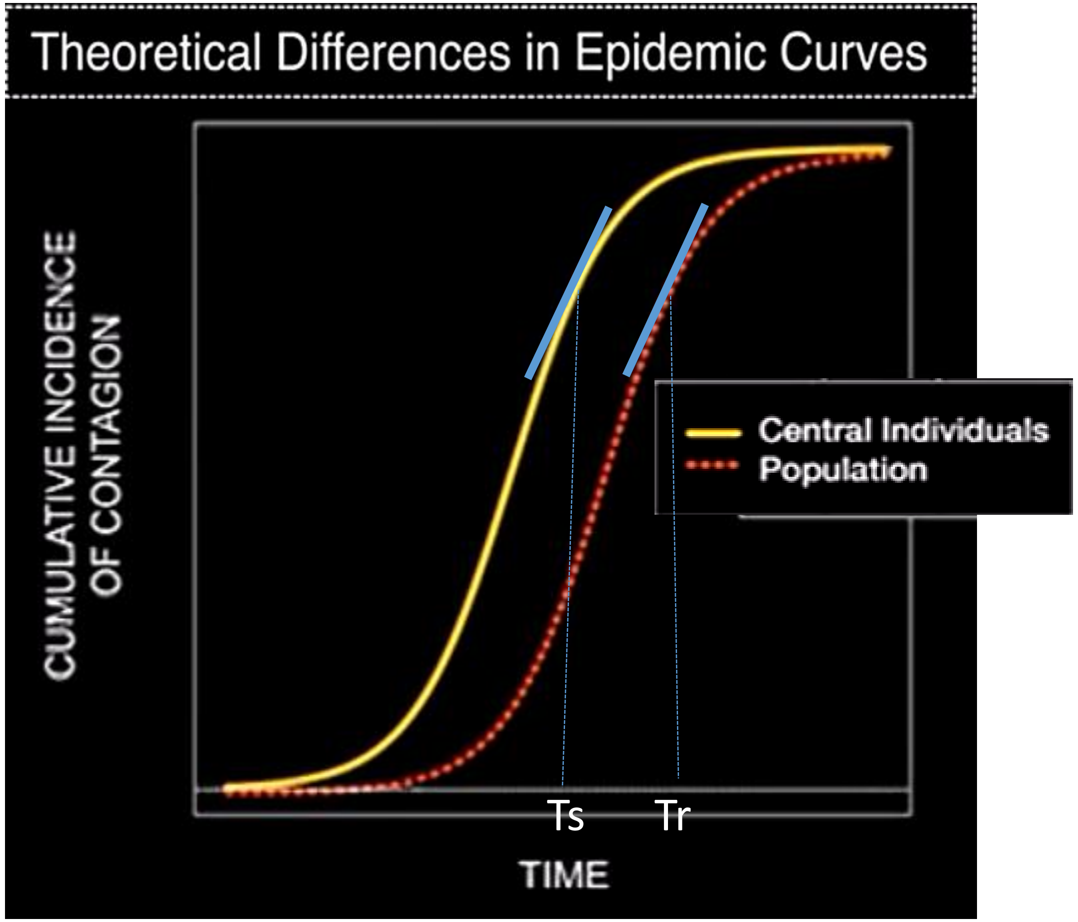
the first 150 days

Random group(responses):

truth: the rest 150 days

prediction: predict from sensor group

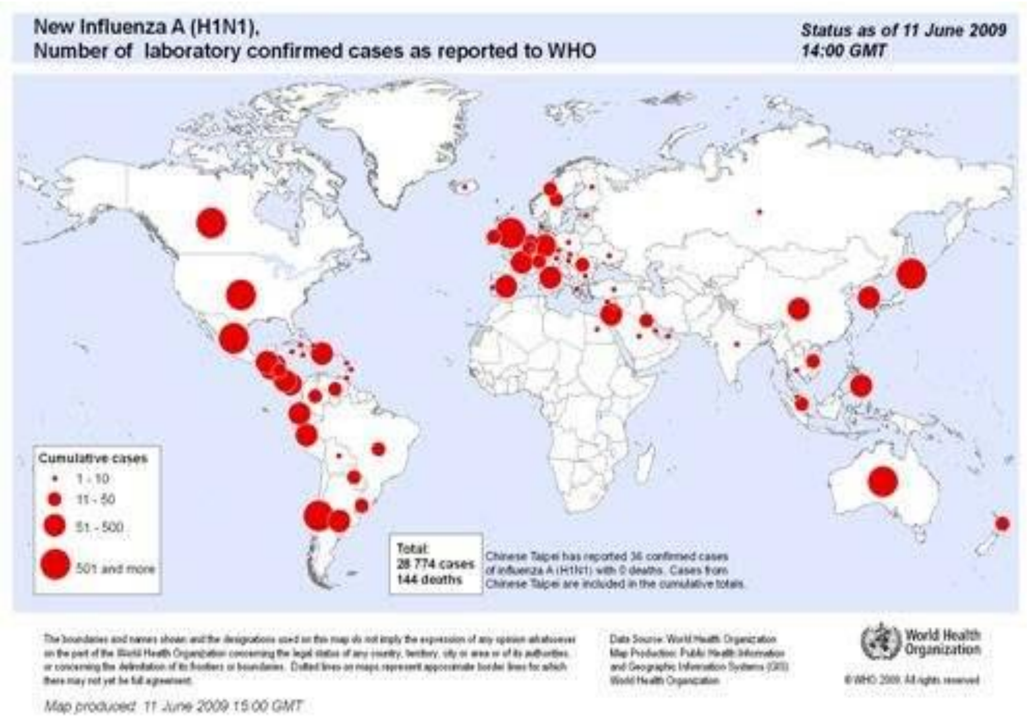
S-shaped curve of adoption



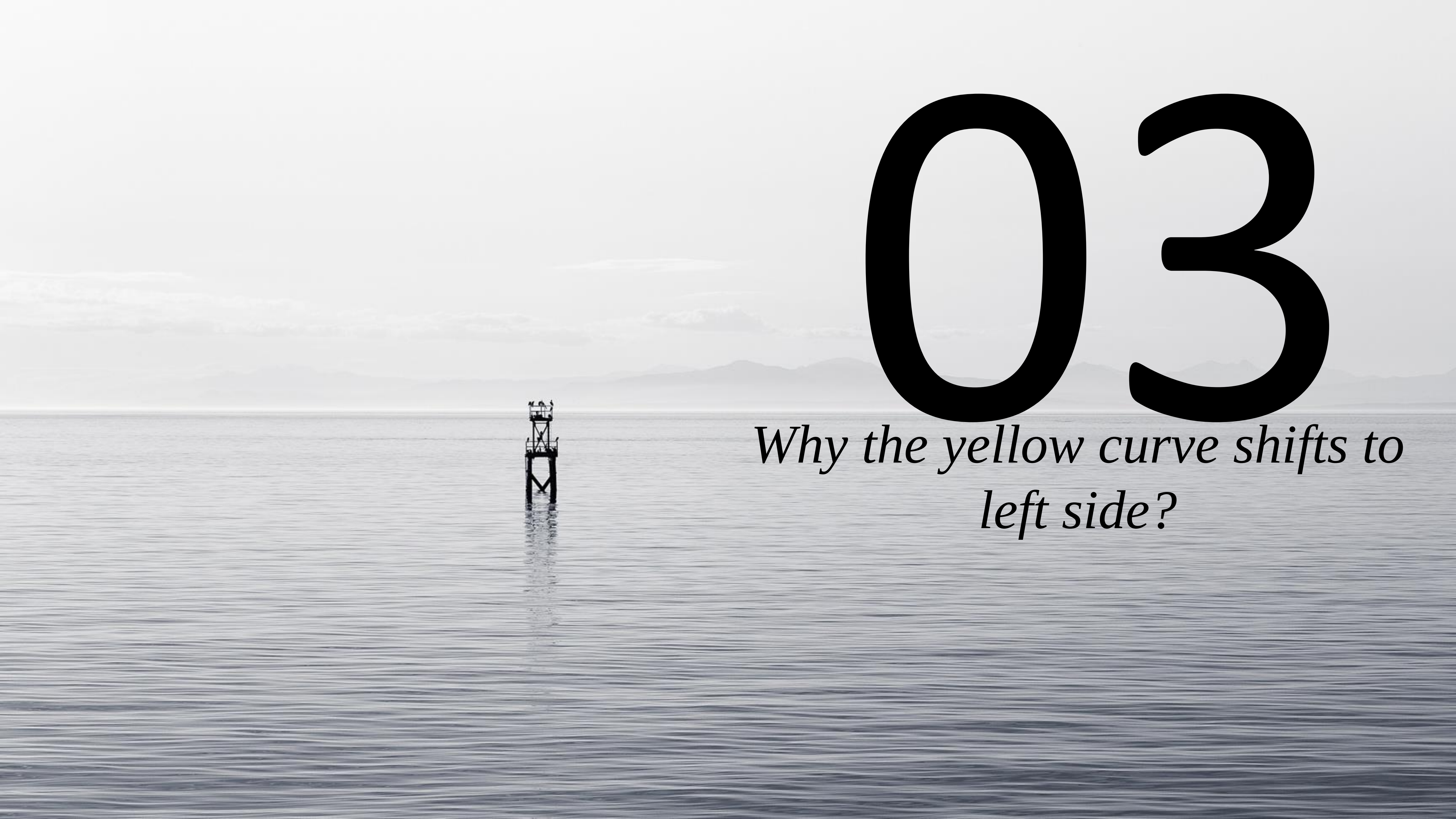
Lead time：

$$\triangle t = tr - ts$$

Practical significance



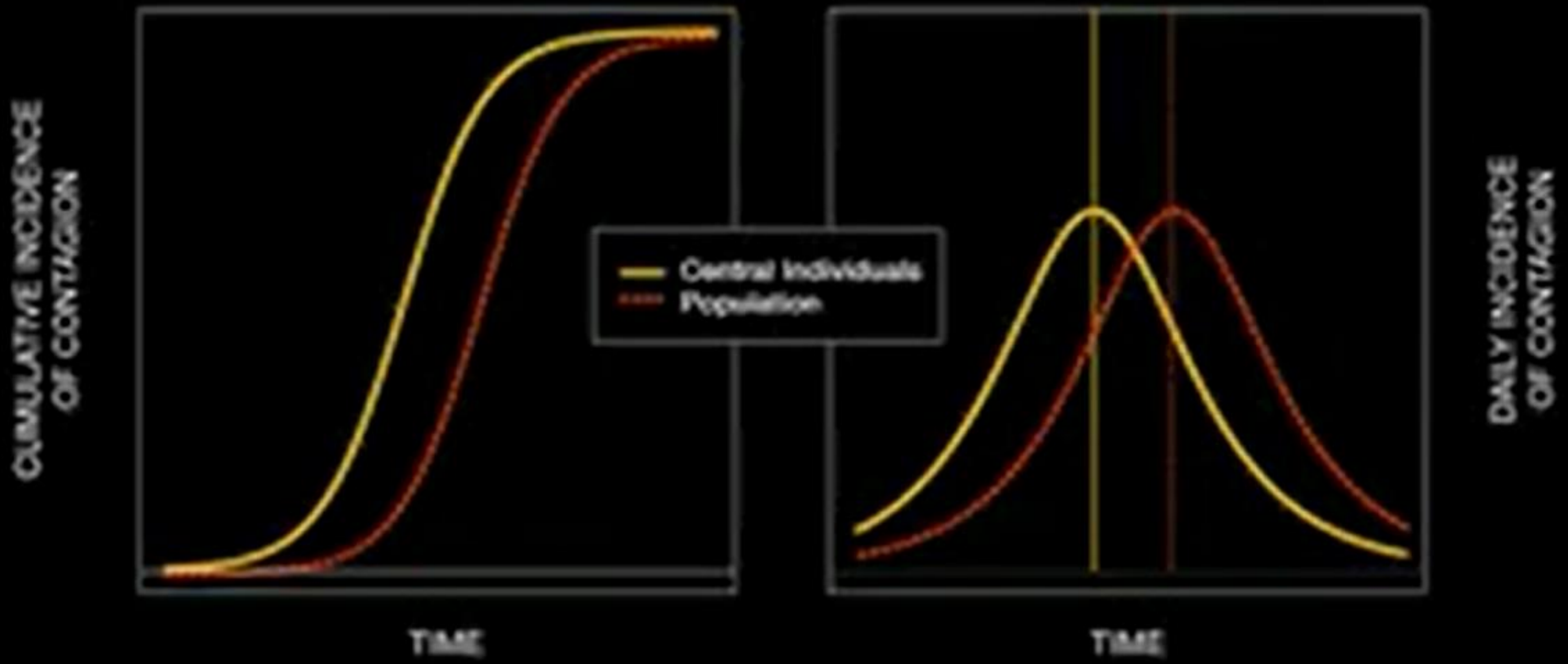
Better engineer health care responses!



03

Why the yellow curve shifts to left side?

Theoretical Differences in Epidemic Curves



NA Christakis and J H Fowler, "Social Network Sensors for Early Detection of Contagious Outbreaks" http://arxiv.org/abs/1004.4792

Why shift?

Red curve

The adoption in the random people

Yellow curve

The adoption in the central individuals

The left-hand panel

The Y-axis is the cumulative instances of contagion, and on the X-axis is the time

The right-hand panel

The Y-axis is the daily incidence , and on the X-axis is the time.

Why shift?

- At the very beginning, one or two people are infected, or affected by the thing and then they affect, or infect, two people, who in turn affect four, eight, 16 and so forth.

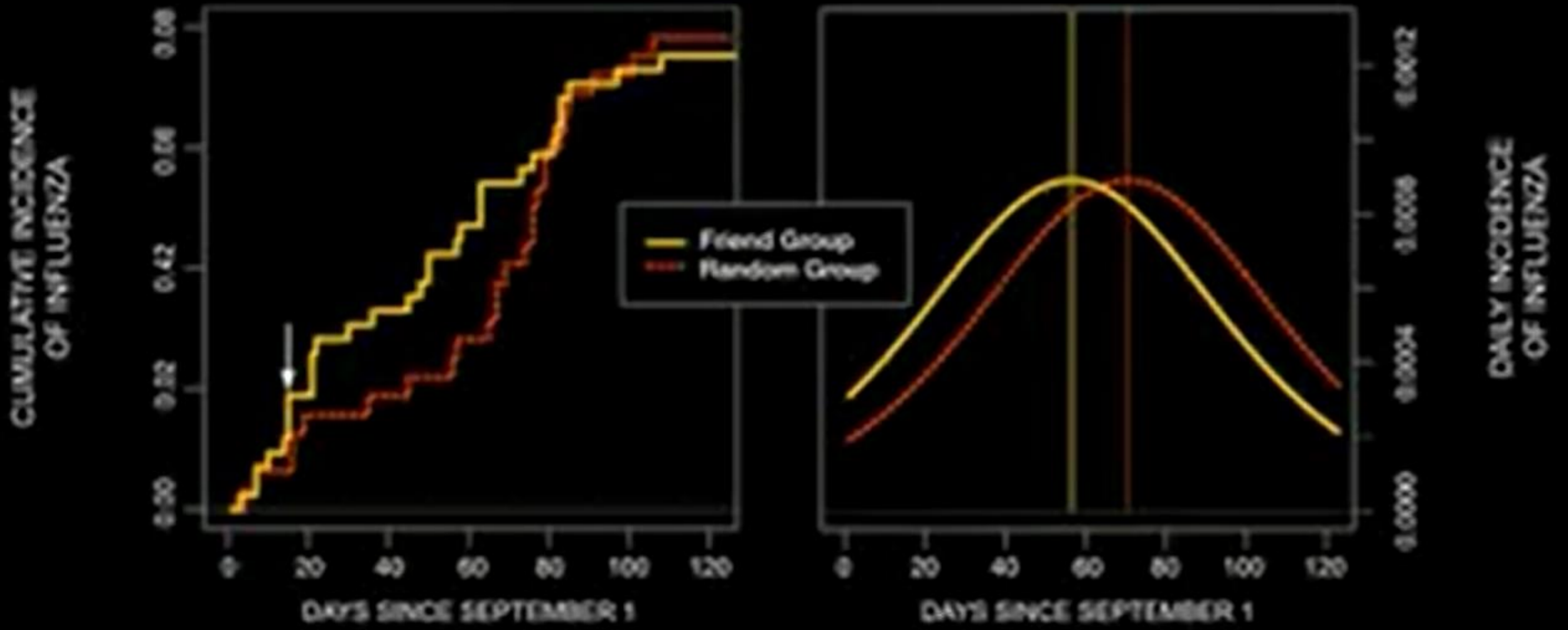
- And eventually, you saturate the population. There are fewer and fewer people who are still available that you might infect. So we get this two panels.

Why shift?

- Since the X-axis is the time and the Y-axis is the incidence of contagion ,the yellow curve shifts to the left side means what’s occurring in the central individuals is earlier than that in the random people.

- If the central individuals contract a germ , soon the whole network will contract a germ . The conclusion can be used to predict epidemics

Observed Differences in Epidemic Curves



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Application of the theory

An outbreak of H1N1 flu at Harvard College in the fall and winter of 2009.There are two groups, 1,300 randomly selected undergraduates and friends they nominated. So the random group is in the red line. The epidemic in the friends group has shifted to the left. And the difference in the two is 16 days. By monitoring the friends group, we could get 16 days advance warning of an impending epidemic in this human population.

THANK YOU