1. (a)

$$\eta_p \equiv \frac{dQ}{dp}\frac{p}{Q} = a\epsilon p^{-\epsilon-1}\frac{p}{ap^{-\epsilon}} = -\epsilon.$$

Hence, the exponential demand function has a constant price elasticity. By Proposition 3.3,

$$MR(Q) = p\left(1 + \frac{1}{-\epsilon}\right) = p\left(\frac{\epsilon - 1}{\epsilon}\right).$$

(b) Equating marginal revenue to marginal cost yields[1]

$$MR = p^M\left(\frac{\epsilon - 1}{\epsilon}\right) = c = MC; \quad \text{hence,}\ p^M = \frac{\epsilon c}{\epsilon - 1}.$$

(c) As ϵ increases, the demand becomes more elastic, hence, the monopoly price must fall. Formally,

$$\frac{dp^M}{d\epsilon} = \frac{c(\epsilon - 1) - \epsilon c}{(\epsilon - 1)^2} < 0.$$

(d).

when $\epsilon \to +1$. Clearly, $p^M \to +\infty$. The reason is, that when $\epsilon = 1$, the (entire) demand has a unit elasticity, implying that revenue is throughout constant.hence monopoly has no incentive to check the increase in price.

(e) Inverting the demand function yields $p(Q) = a^{1/\epsilon}Q^{-1/\epsilon}$. Thus,

$$TR(Q) \equiv p(Q)Q = a^{\frac{1}{\epsilon}}Q^{1-\frac{1}{\epsilon}}.$$

Hence,

$$MR(Q) \equiv \frac{dTR(Q)}{dQ} = a^{\frac{1}{\epsilon}}Q^{-\frac{1}{\epsilon}}\left(1 - \frac{1}{\epsilon}\right).$$

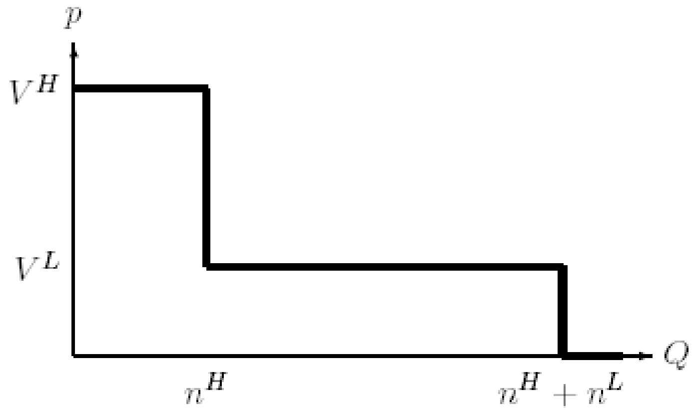
(f) Equating marginal revenue to marginal cost yields

$$a^{\frac{1}{\epsilon}}Q^{-\frac{1}{\epsilon}}\left(1 - \frac{1}{\epsilon}\right) = c,$$

yielding that the monopoly's profit maximizing output is

$$Q^M = a\left(\frac{\epsilon - 1}{\epsilon c}\right)^{\epsilon}.$$

2.
(a)



(b).

The monopoly has two options[2]: setting a high price, $p = V^H$, or a low price, $p = V^L$.

$$\pi|_{p=V^H} = n^H V^H, \quad \text{and} \quad \pi|_{p=V^L} = (n^H + n^L)V^L.$$

Comparing the two profit levels yields the monopoly's profit maximizing price. Hence,

$$p^M = \begin{cases} V^H & \text{if } V^H > (n^H + n^L)V^L/n^H \\ V^L & \text{otherwise.} \end{cases}$$

3. (a) In market 1, $p_1 = 2 - q_1$. Hence, by Proposition 3.2, $MR_1(q_1) = 2 - 2q_1$. Equating $MR_1(q_1) = c = 1$ yields $q_1 = 0.5$. Hence, $p_1 = 1.5$.

   In market 2, $p_2 = 4 - q_2$. Hence, by Proposition 3.2, $MR_2(q_2) = 4 - 2q_2$. Equating $MR_2(q_2) = c = 1$ yields $q_2 = 1.5$. Hence, $p_2 = 2.5$.

   (b) $\pi_1 = (p_1 - c)q_1 = (0.5)^2 = 0.25$, and $\pi_2 = (p_2 - c)q_2 = (1.5)^2 = 2.25$. Summing up, the monopoly's profit under price discrimination is $\pi = 2.5$.

   (c) There are two cases to be considered: (i) The monopoly sets a uniform price $p \geq 2$ thereby selling only in market 2, or (ii) setting $p < 2$, thereby selling a strictly positive amount in both markets. Let us consider these two cases:

   i. If $p \geq 2$, then $q_1 = 0$. Therefore, in this case the monopoly will set q_2 maximize its profit in market 2 only. By subquestion 3a above, $\pi = \pi_2 = 2.25$.

   ii. Here, if $p < 2$, $q_1 > 0$ and $q_2 > 0$. Therefore, aggregate demand is given by $Q(p) = q_1 + q_2 = 2 - p + 4 - p = 6 - 2p$, or $p(Q) = 3 - 0.5Q$. By Proposition 3.2, $MR(Q) = 3 - Q$. Equating $MR(Q) = c = 1$ yields $Q = 2$, hence, $p = 2$. Hence, in this case $\pi = (p - c)2 = 2 < 2.25$.

   Altogether, the monopoly will set a uniform price of $p = 2.5$ and will sell $Q = 1.5$ units in market 2 only.[4]

4. (a)

$$\pi(q_1, q_2) = (100 - q_1/2)q_1 + (100 - q_2)q_2 - (q_1 + q_2)^2.$$

   (b) The two first order conditions are given by

$$\begin{aligned} 0 = \frac{\partial \pi}{\partial q_1} &= 100 - q_1 - 2(q_1 + q_2) \\ 0 = \frac{\partial \pi}{\partial q_2} &= 100 - 2q_2 - 2(q_1 + q_2). \end{aligned}$$

   Solving for q_1^M and q_2^M yields that $q_1^M = 25$ and $q_2^M = 12.5$.

   (c) Substituting the profit maximizing sales into the market demand functions yield $p_1^M = p_2^M = 87.5$. Hence,

$$\pi(q_1^M, q_2^M) = 87.5 \times 12.5 + 87.5 \times 25 - (12.5 + 25)^2 = 1875.$$

(d) Now, that each plant sells only in one market, the two first order conditions become

$$0 = \frac{\partial \pi}{\partial q_1} = 100 - q_1 - 2q_1$$
$$0 = \frac{\partial \pi}{\partial q_2} = 100 - 2q_2 - 2q_2.$$

yielding $q_1^M = 100/3$ and $q_2^M = 25$.

(e) $p_1^M = 100 - 100/6 = 250/3$ and $p_2^M = 100 - 25 = 75$. Hence,

$$\pi = \pi_1(q_1) + \pi_2(q_2) = \frac{250}{3} \times \frac{100}{3} - \left(\frac{100}{3}\right)^2 + 75 \times 25 - 25^2 = 2917.$$

(f) This decomposition increases the monopoly's profit since the technology exhibits DRS.

5. By section 5.3, the discriminating monopoly will set quantities to satisfy $MR_1(q_1^M) = MR_2(q_2^M)$. Hence, by Proposition 3.3

$$MR_1(q_1) = p_1^M\left(1 + \frac{1}{-2}\right) = p_2^M\left(1 + \frac{1}{-4}\right) = MR_2(q_2).$$

Thus, $p_1^M = 1.5p_2^M$.

7.

(a) Equating marginal revenue to the tax inclusive unit cost yields $a - 2Q = c + t$, or $Q^M = (a - c - t)/2$. Hence, $p^M = (a + c + t)/2$.
Thus price rises by half the amount of tax imposed.

(b).

$$MR = p^M\left(1 + \frac{1}{-2}\right) = c + t, \quad \text{yielding} \quad p^M = 2(c+t).$$

Price rises by twice the amount of tax in this case.