Solutions to problem set 4

Econ460

1. (a) Since costs are zero the profit of the firm equals its revenue.

Profit function of firm A: $\Pi^A = p_a(24 - \theta p_a + p_b)$. Similarly the profit function of firm B is given as: $\Pi^B = p_b(24 - \theta p_b + p_a)$.

The best response or the reaction function for firm A is given as: $\partial \Pi^A / \partial p_a = 0$

$$\Rightarrow 24 - 2\theta p_a + p_b = 0$$

$$p_a = \frac{24 + p_b}{2\theta} \qquad \textbf{(1)}$$

Similarly the best response for firm B is given as:

$$\Rightarrow 24 - 2\theta p_b + p_a = 0$$

$$p_b = \frac{24 + p_a}{2\theta} \qquad \textbf{(2)}$$

Solving for the Nash equilibrium prices, plugging equation (2) into equation (1) and solving for equilibrium price we have:

$$p_a^* = p_b^* = \frac{24}{2\theta - 1}$$

By plugging the equilibrium prices in the demand equations we have:

$$q_a^* = q_b^* = \frac{24\theta}{2\theta - 1}$$

Equilibrium profit: $\Pi_A^* = p_a^* q_a^* = \Pi_B^* = p_b^* q_b^* = \dfrac{576\theta}{(2\theta - 1)^2}$

(b). As obtained above the equilibrium prices are positive. If it was a standard Bertrand case i.e. if products were homogeneous then the equilibrium price would be equal to the marginal cost which in this case is zero. This is an illustration of the fact that product differentiation lends market power to firms which results in resolution of Bertrand paradox.

2. (a). Suppose the price charged by the monopolist equals p. The effective price therefore (i.e. price inclusive of the transport cost) is $p + x^2$. Gross utility of the consumer from the consumption of the ice cream equals $\bar{s}$. Therefore the net utility of the consumer who is located at a distance x is given to be $\bar{s} - p - x^2$.

   (b). As is usual in these problems the first step in solving these problems is to get the demand which we can obtain from finding the marginal consumer. Since there is only one firm in this model, the marginal consumer would be the one who is indifferent between buying from this firm and not buying. As usual lets denote the location of this consumer by $\hat{x}$. Any consumer who lies to the right of this consumer is going to have a higher transport cost and is not going to buy. Hence $\hat{x}$ must satisfy the following condition. $\bar{s} - p - \hat{x}^2 = 0$. Or $p = \bar{s} - \hat{x}^2$. $\hat{x}$ is the total quantity produced and sold. As this is the case of a monopoly he can maximize profits by choosing either price or quantity. Let him choose quantity i.e. $\hat{x}$.

   Profit of the monopoly producer is given as: $\Pi = p\hat{x} = (\bar{s} - \hat{x}^2)\hat{x}$. First order condition with respect to $\hat{x}$ is given as:

$\bar{s} - 3\hat{x}^2 = 0 \Rightarrow \hat{x} = \sqrt{\frac{\bar{s}}{3}}$. This the extent of the market served by the monopoly. Price charged by the monopolist = $\bar{s} - \hat{x}^2 = \frac{2\bar{s}}{3}$

(b). If $\bar{s} = 3$ then $\hat{x} = 1$ which implies that the entire stretch of the town is served by the monopolist.

(c). Since the costs are zero and price is positive, this firm can never break even (i.e. make zero profits). It will always make positive profits.

4. Lets denote the location of the indifferent consumer by $\hat{x}$. The location of the indifferent consumer i.e. the consumer who is indifferent between purchasing haircut from Ritz or Cheapcuts must give him same net utility purchasing from either stores.

$$50 - p_{Ritz} - 5\hat{x} = 50 - p_{cheapcuts} - 5(1-\hat{x})$$

$\Rightarrow \quad \hat{x} = \dfrac{p_C - p_R + 5}{10}$ Similarly the demand facing the Cheapcuts is

$$1 - \hat{x} = \frac{p_R - p_C + 5}{10}$$

Profit function of Ritz Salon is $\Pi^R = p_R \dfrac{p_C - p_R + 5}{10} - 20 \dfrac{p_C - p_R + 5}{10}$

Profit function of the Cheapcuts salon is given as:

$$\Pi^C = p_C \frac{p_R - p_C + 5}{10} \text{-} 10 \frac{p_R - p_C + 5}{10}$$

The best response function of the Ritz Salon is given as:

$$\partial\Pi^R / \partial p_R = 0 \Rightarrow p_R = 12.5 + \frac{p_C}{2} \tag{1}$$

The best response function of the Cheapcuts is given by

$$\partial\Pi^C / \partial p_C = 0 \Rightarrow p_c = 7.5 + \frac{p_R}{2} \tag{2}$$

Here we need to realize one important thing. A rise in cost of Ritz affects the best response function only of Ritz and not of Cheapcuts and vice versa. Of course this change in best response would in Nash equilibrium result in higher prices also for the other firm.

(b). We can obtain the Nash equilibrium prices by substituting equation 2 into equation 1: $p_R^* = 10.8$. $p_C^* = 12.9$. If both the firms have unit cost equal to 10 then the best response of Ritz is given as $p_R = 7.5 + \dfrac{p_c}{2}$ and of the cheapcuts is given as $p_c = 7.5 + \dfrac{p_R}{2}$. Solving these two equations simultaneously we have $p_R^* = p_C^* = 7.5$.

5. Just follow the class notes exactly with $\bar{s} = 50$, t = 16, c = 8 and f =1.

6. Here the demand is given in the direct form. We need to express this in the inverse form i.e. express the price of product 1 as a function of q_1 and q_2 and similarly price of good 2.

From the direct demand function of good 2 we have $p_2 = \dfrac{100 + p_1 - q_2}{1.5}$.

Plugging this into the direct demand function for good 1 we have

$$q_1 = 100 - 1.5p_1 + \frac{100 + p_1 - q_2}{1.5}$$

$$1.5q_1 = 150 - (1.5)^2 p_1 + 100 + p_1 - q_2$$

$$1.25p_1 = 250 - 1.5q_1 - q_2$$

$$p_1 = 200 - \frac{1.5}{1.25}q_1 - \frac{1}{1.25}q_2$$

Similarly the inverse demand function for good 2 is given as:

$$p_2 = 200 - \frac{1.5}{1.25}q_2 - \frac{1}{1.25}q_1$$

(b) Written in this form we can compare the coefficients to the ones used in the class

$$\alpha = 200$$
Here $\beta = 1.5/1.25$
$$\gamma = 1/1.25$$

Measure of differentiation $\delta = \dfrac{\gamma^2}{\beta^2} = 1/2.25 = 0.44$. As this measure becomes bigger we have products becoming more homogenous.

(c). Since the costs for both the firms are zero, the profit equals revenue.

$$\Pi_1 = (200 - \frac{1.5}{1.25}q_2 - \frac{1}{1.25}q_1)q_1$$

Best response of 1 is given as:

$$\partial\Pi_1 / \partial q_1 = 0$$

$$200-\frac{1.5}{1.25}q_2-\frac{2}{1.25}q_1=0$$
$$\Rightarrow q_1=\frac{250-1.5q_2}{2} \qquad (1)$$

By symmetry the best response function of 2 is given as:

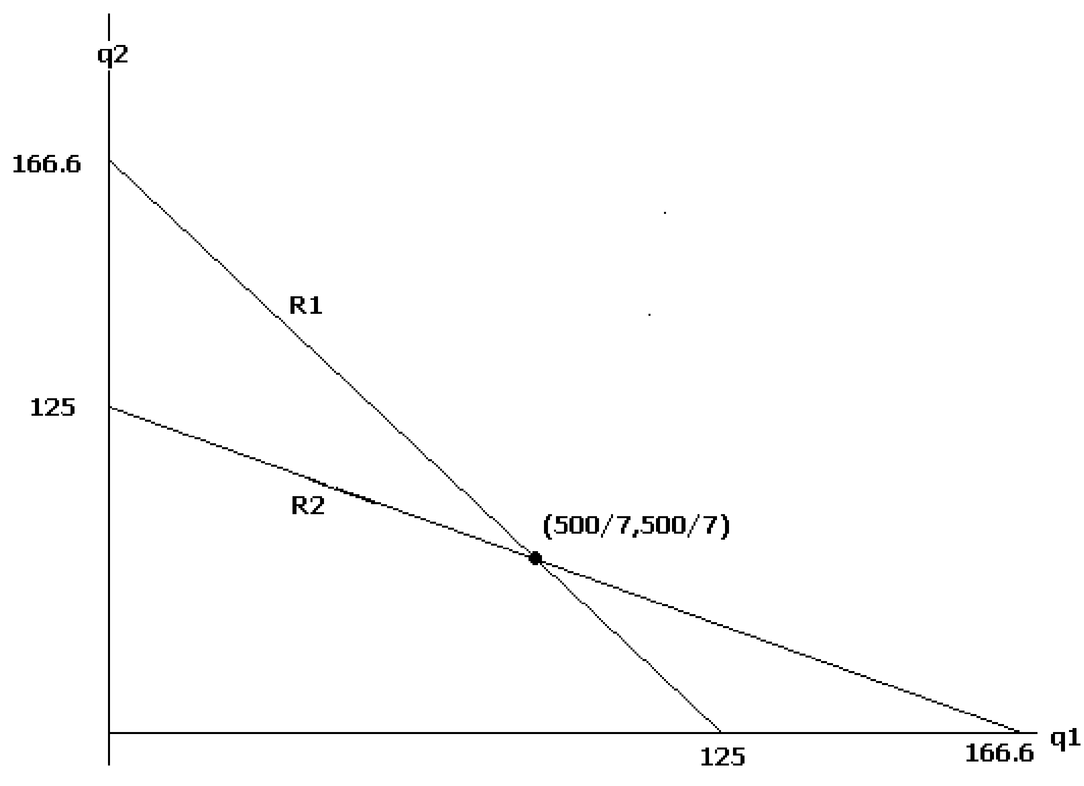
$$q_2=\frac{250-1.5q_1}{2} \qquad (2)$$

Solving for the equilibrium by plugging (2) into (1)

We get

$$q_1^*=q_2^*=500/7$$

Plot Below



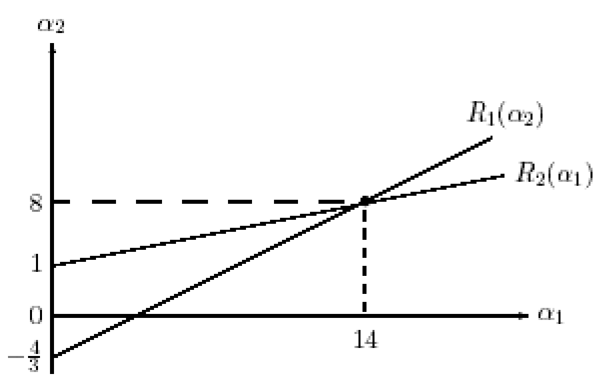
(c). We know the output produced by each firm in a homogeneous Cournot is given by $\frac{a-c}{3b}$ where a is the intercept of the inverse demand , b is the slope and c is the constant per unit cost. Remember when we have to make the products homogeneous we have to change the coefficient on the cross term to equal the coefficient on the own term i.e. with homogenous products the coefficient on both the quantities should be equal to $\frac{1.5}{1.25}$ in the inverse demand functions. This then becomes equal to b in the homogenous product case. Hence the amount of output produced by each Cournot firm in a homogenous product case would be $\frac{200}{3x\frac{1.5}{1.25}} = 250/4.5 = 55.5 < 500/7$. Differentiation raises the incentive to produce more.

Hence output produced by each firm is higher under product differentiation under Cournot competition.

Oz Shy Problems:

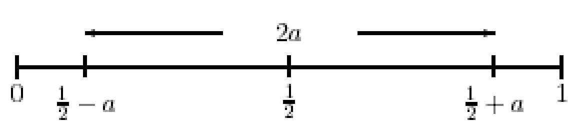
1.

$$\max_{\alpha_1} \pi_1 \quad \Rightarrow \quad R_1(\alpha_2) = 2 + \frac{3}{2}\alpha_2, \quad \text{or,} \quad \alpha_2 = -\frac{4}{3} + \frac{2}{3}R_1(\alpha_2);$$
$$\max_{\alpha_2} \pi_2 \quad \Rightarrow \quad R_2(\alpha_1) = 1 + \frac{1}{2}\alpha_1.$$



(b) The two best-response functions are upward sloping, hence strategically complements.

(c) Solving for a NE yields $\alpha_1^N = 14$ and $\alpha_2^N = 8$. The firms' profit levels in a NE are: $\pi_1 = 14^2$ and $\pi_2 = 8^2$.



2.

The indifferent consumer on each side is determined by the reservation utility, that is, $B - a - p = 0$, or, $a = B - p$. Hence, the monopoly chooses p that solves

$$\max_p \pi = p2a = 2p(B - p).$$

The solution is given by

$$p^M = \frac{B}{2}, \quad a = \frac{B}{2}, \quad \text{and} \quad \pi^M = 2pa = \frac{B^2}{2}.$$

Now, it is easy to verify that $0 < B < 1$ implies that $0 < a < 1/2$, hence, not all the market is served.

(b) When substituting $B > 1$ into the solution for 'a' found in the previous subquestion, we get that the indifferent consumers lie "outside" the city. This implies that for $B > 1$, the monopoly can increase the price and still having the entire street purchase the product. Thus, the monopoly will pick the highest possible price subject to having the consumers living at the edges of town purchase the product. Formally, set p to satisfy $B - 1/2 - p = 0$. Hence,

$$p^M = B - \frac{1}{2}, \quad a = \frac{1}{2}, \quad \text{and} \quad \pi^M = B - \frac{1}{2}.$$

3. (a) The indifferent consumer, denoted by $\hat{x}$, must satisfy

$$\hat{x} \times 1 + p_1 = (1 - \hat{x}) \times R + p_2.$$

Hence,

$$\hat{x} = \frac{R + p_2 - p_1}{1 + R}.$$

(b). For all the consumers to go to eat at restaurant 1, it must be true that even the person who is located on the right most corner of the city likes to go to the restaurant at the left end of the city. At the minimum he should be indifferent between eating at restaurant 1 or 2. In other words $\hat{x}$ at $p_1 = p_2$ must be at the minimum equal 1. Looking at $\hat{x}$ in part a at equal prices for this to be true the travel cost R must be infinite. It makes intuitive sense. If nobody is willing to dine in a restaurant on the right even if it is just one block away the reason must be that the transport costs in that direction are extremely high.