## Industrial Organization

## Sample Final Exam

Time: 2 Hrs

## Part A: Multiple choice questions

1. Consider the following setting for n firm Stackelberg competition: Market demand curve is given as:

$$
p=1-(1 / 1000) Q
$$

Marginal cost of production is given to be equal to 28 cents: Which one of the following gives the right combination of the output produced by the leader and follower firms (in that order)
(a). $360,360 \frac{n-1}{n}$
(b). $340,360 \frac{n-1}{n}$
(c). $540,540 \frac{n+1}{n}$
(d). $540,540 \frac{n-1}{n}$

In a n firm Stackelberg the leader firm always produces $(a-c) / 2 b$. Here $a=1, b=1 / 1000, c=0.28$. This equals 360 here which is an option only in (a).
2. Consider the Hotelling linear city model where the measure of the town is 1 mile. Consider the first stage of the game i.e. the choice of where to locate. Which one of the following gives the equilibrium in locations for 3 firms case
(a). Does not exist
(b). All 3 firms in the middle
(c). One firm at the left end, one in the middle, one at the right end;
(d). Does not exist; One firm at $1 / 4$ mile, one at $1 / 2$ mile, one at $3 / 4$ mile

In case of an odd number of firms we know that for given prices an equilibrium does not exist in a Hotelling model. Hence the correct option is (a).
3. Consider an industry composed of 3 firms and facing demand: $\mathrm{P}(\mathrm{Q})=1-\mathrm{Q}$, where $\mathrm{Q}=\mathrm{q} 1+\mathrm{q} 2+$ q3.The three firms are identical, each producing using a plant that exhibits quadratic $\operatorname{cost} C\left(q_{i}\right)=1 / 2 q_{i}{ }^{2}$. What are the Cournot equilibrium quantity and prices?
(a). $3 / 5,2 / 5$
(b). $4 / 5,7 / 5$
(c). $8 / 5,6 / 5$
(d). $2 / 5,2 / 5$

The only combination of price and quantity that satisfies the demand equation is (a). Others do not.
4. Consider the Salop's circular city model. What happens to the prices charged by the firms in a symmetric equilibrium and the equilibrium number of firms as the transport costs rise
(a). Rises, falls
(B). Rises, rises
(c). Falls, falls
(d). Falls, rises

The equilibrium price in a Salop's circular city model is given to be $p^{*}=c+\frac{t}{n^{*}}$ where $n^{*}=\sqrt{t / F}$.
Plugging back we have $p^{*}=c+\sqrt{t / F}$. Hence as the transport cost rises both the number of firms as well as the prices rise. The correct answer is (b)
5. In the model with adverse selection problem (lemons model, Akerlof) which of the following will not solve the problem
(a). Price fixing by the government
(b). Warranties
(c). Repeated interaction between buyer and sellers
(d). Mechanical inspection by the buyer from an expert who cares about reputation.

In this case government has no more information than the private party which is uninformed. So unless the price is fixed by the government at the value of a lemon there would be no transaction as before. But if that is the case then the lemons problem persists. The other three are examples of screening and signaling so they can solve adverse selection.

Part B: TRUE or FALSE or UNCERTAIN and EXPLAIN: For each of the following statements, decide whether each is true or false or whether it is not possible to determine given the information. Most importantly, give the reasoning behind your answer. (Answer any four questions in this part). 1.Under second degree price discrimination with a group of low demanders and another of high demanders, a profit-maximizing monopolist will extract all the surplus available from the high demanders.

False. The Monopolist extracts all the surplus from the low type and not the high type. The high type is allowed a surplus which is just enough to prevent him from masquerading.
2.A law requiring multinational companies that have their headquarters in the U.S. to charge the same prices in home and foreign markets for their products will receive strong support from American customers.

Uncertain. With price discrimination allowed the high types loose and the low types gain so it depends which types are the American consumers.
3.The optimal number of firms undertaking research is higher under patents than under prizes (we are not going to cover this topic in spring 2003).

Lucky batch yet again: We did not reach this far.
4. Unless all consumers in a market are fully informed the full information competitive price equilibrium can never be attained.
This is False. The Tourists and the Natives model shows that we require a substantially high proportion of the population to be informed and not all.
6. In the Hotelling model with two firms where their locations are fixed at the two ends of the city there does not exist an equilibrium in the pricing game if the transport costs are linear.
This is False. Given location it does not matter whether the transport costs are linear or are quadratic. Its only when we consider both location and pricing game that it matters.

Part C: Long Questions: Answer any three from this part:

1. Consider a market characterized by competition around a unit circle, where consumers are uniformly distributed on the circle (their density is equal to 1). Consumers wish to buy one unit of the good and have a transport cost $t$ of $\$ 16$ per unit distance. Each consumer will purchase exactly one unit from the lowest-effective-price firm provided that their effective price, is less than their reservation price $s$ of $\$ 50$, and zero otherwise.

Also, assume marginal cost of production $c$ is $\$ 8$ and that there is a fixed cost $f$ of $\$ 1$ for a firm to locate on the circle. In the first stage, potential entrants simultaneously choose whether or not to enter. Let $n$ denote the number of entering firms. Firms are automatically located equidistant from one another on the circle.
a. Given $n$ firms, so that the first stage of the game has already occurred, what is the equilibrium price $p$ ?
b. What is the equilibrium number of firms in the market? What is the equilibrium price now?
(You need to show the derivation of equilibrium prices and number of firms in order to get credit)
See Problem set 4 for the solutions.
2. Consider the tourist trap model that we discussed in class. Under the assumption of search cost $c$, all firms selling identical products, all consumers being uninformed about the price charged by each firm and all consumers having identical demand functions, show how can you break the full information competitive equilibrium. Does reducing the search cost change your result. Also explain briefly without proving: does it make a difference if some consumers are informed. Do we need all the consumers to be informed?

Suppose each firm is charging the full information price $p^{c}$ then any firm can deviate to charge a higher price to the consumer by raising to the price such that it is higher by an amount less than c . This result is independent of the magnitude of the search cost because reducing the search cost does not prevent the firm from raising the price in this manner and consequently the level of information in the economy remains unchanged as consumers search no more.

It does make a difference if some consumers are informed provided their numbers are reasonably large. In such a situation by deviating to a higher price the deviant firm gets to sell only to the
uninformed consumers whose numbers being relatively few results in making it unprofitable to deviate.
3. Let the demand for products 1 and 2 be $q_{1}=10-2 p_{1}+p_{2}$ and $q_{2}=10+p_{1}-2 p_{2}$, where $q_{i}$ is the quantity of good $i$ and $p_{i}$ is the price of good $i$. Assume production costs are zero. Calculate the prices that two separate monopolies would charge when each regards the other's price as beyond its control. Calculate the prices that a single monopoly of both goods will charge. Which arrangement would the consumers prefer (you have to show it, not only argue). Using the measure of product differentiation that we have discussed in class, obtain a measure of product differentiation for the set of demand functions given here. (Remember you have to first obtain the demands in the inverse form)
Firm 1's maximization problem is $\max \Pi_{1}=p_{1}\left(10-2 p_{1}+p_{2}\right)$
Best response of the firm is given as:

$$
\begin{align*}
& \partial \Pi_{1} / \partial p_{1}=0 \\
& \Rightarrow 10-4 p_{1}+p_{2}=0  \tag{1}\\
& p_{1}=\frac{10+p_{2}}{4}
\end{align*}
$$

By symmetry the best response of the firm 2 is given as:

$$
\begin{equation*}
p_{2}=\frac{10+p_{1}}{4} \tag{2}
\end{equation*}
$$

Plugging (2) into (1) we get $p_{1}^{*}=10 / 3$ and $p_{2}^{*}=10 / 3$
If there is a single monopoly then he would maximize the joint profit $\Pi=\Pi_{1}+\Pi_{2}=p_{1}\left(10-2 p_{1}+p_{2}\right)+p_{2}\left(10-2 p_{2}+p_{1}\right)$
First order condition for the choice of the price 1

$$
\begin{align*}
& \partial \Pi / \partial p_{1}=0 \\
& \Rightarrow 10-4 p_{1}+p_{2}+p_{2}=0 \tag{3}
\end{align*}
$$

First order condition for the choice of the price 2

$$
\partial \Pi / \partial p_{2}=0
$$

$$
\begin{equation*}
\Rightarrow 10-4 p_{2}+p_{1}+p_{1}=0 \tag{4}
\end{equation*}
$$

We can use equations 3 and 4 to solve for two prices

They give $p_{1}^{*}=p_{2}^{*}=5$
Since prices are higher under monopoly the consumers are going to prefer earlier arrangement.
4. A monopoly produces and delivers goods to consumers who are located at varying distances from the factory. It costs $m$ per unit to produce the good and \$ 1 per mile to transport a unit of the good. Resales are impossible. Calculate the price that a monopoly charges consumers at location t miles if demand is $q_{t}=a-b p_{t}$, where $q_{t}$ and $p_{t}$ are the quantity and price at location t . How does $p_{t}$ change as t increases? Who bears the freight cost? If the monopolist charges different prices to different consumers, is it price discrimination? If yes which degree of price discrimination is this?

The price that the monopolist must charge the consumer at location $t$ is given as solution to the profit maximization problem with respect to the consumer at location $t$.
$\operatorname{Max} \Pi_{t}=p_{t}\left(a-b p_{t}\right)-m\left(a-b p_{t}\right)-t\left(a-b p_{t}\right)$
First order condition for the choice of $p_{t}$ gives:
$a-2 b p_{t}+m b+t b=0$
$\Rightarrow p_{t}^{*}=\frac{a+(m+t) b}{2 b}$
Here in the expression as $t$ i.e. the distance rises the monopolist charges higher price. The monopolist transfers the freight cost to the consumers but not totally. (The price rises by only $t / 2$ )

Since higher prices at more distant places just reflect the cost of serving the consumers based on the definition of price discrimination this is not price discrimination.
5. Suppose there are 100 residents (uniformly-spaced) living on a ten-mile long street. Each resident has a willingness to pay for pizza of $\$ 30$ and would buy 1 pizza per week. It costs a resident $\$ 1$ to travel one mile. Two pizza shops are considering opening shops along the street. (Hint: In the Hotelling model a ten mile long city with $\$ 1 /$ mile transportation cost is equivalent to a 1 mile long city with $\$ 10$ mile transportation costs. You can use this equivalence to make the
problem exactly identical to the Hotelling model we had in class) After opening, each pizza shop would have a marginal cost of $\$ 5$ per pizza and no fixed costs.
a) Originally, both stores have plans to open on opposite ends of the street. What prices would be charged by each firm with such a location? How much profit do the firms make per week with price competition in such a situation?
b) b) What do you think would happen to the firms' prices if they located right next to each other? Would they want to do this?

Part a is the standard Hotelling problem with $\mathrm{t}=10$.
For part $b$ realize that only if firms locate away from each other do the firms get a local monopoly which allows them market power and they charge higher prices than marginal cost. Thus they would not want to locate next to each other.

