Philosophy of Logics

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Chap4 Sentence Connectives

- I. Connectives
- II. Truth tables
- III. Truth Functions and Truth forms
- IV. Formal and Informal Considerations

I. Connectives

- 1. Different notations
- 2. Ordinary readings
- 3. formal definitions

1. different notations

	B. Russell (& Peano)	D. Hilbert	Lukasiewicz	Others	Mine
Negation	~A	$\bar{A}$	N_p	~A, ¬A	¬A
Conjunction	A . B	A&B	K_{pq}	AB, A∧B	A∧B
Disjunction	A∨B	A+B	A_{pq}		A∨B
Implication	A⊃B	A → B	C_{pq}		A⊃B
Equivalence	A=B	A ~ B	I_{pq}	A↔B	A≡B
Universal Quantifier	(x)F(x)	∀x F(x)	$\Pi_x f(x)$	$\forall x F(x)$, $\bigwedge_x f(x)$	∀xF(x)
Existential Quantifier	(∃x)F(x)	∃x F(x)	$\sum_x f(x)$	$\exists x F(x)$, $\bigvee_x f(x)$	∃xF(x)

2. Ordinary readings

Connectives	Reading	Ordinary Readings
¬q	not p;	p doesn't hold; p is not so; It is not the case that p; It is false that p; …
p∧q	p and q	both p and q; p but q; not only p, but also q; p, although q; p, despite q; p , yet q; p, while q; …
p∨q	p or q	p or q , or both; p, except when q; p, unless q; p and/or q; either p or q; p, if not q; …
p→q	if p, then q	provided that p, q; given that p, q; in case p, q; assuming that p, q; on the condition that p, q;
p« q	p, iff, q	p, exactly on condition that q; p, just in case q, …

3. Formal definitions

p	q	p∧q	p∨q	p⊻q	p→q	p←q	p« q
T	T	T	T	F	T	T	T
T	F	F	T	T	F	T	F
F	T	F	T	T	T	F	F
F	F	F	F	F	T	T	T

II. Truth tables

- 1. Wittgenstein's Tractatus
- 2. Truth value
- 3. Truth table

1. Wittgenstein's Tractatus

1	The world is everything that is the case.
2	What is the case, the fact, is the existence of atomic facts.
3	The logical picture of the facts is the thought.
4	The thought is the significant proposition.
5	Propositions are truth-functions of elementary propositions.
6	The general form of truth-function is:[$\bar{p}, \bar{x}, N(\bar{x})$].
7	Whereof one cannot speak, thereof one must be silent.

Bertrand Russell's Introduction

- **Wittgenstein:** Tractatus Logico-Phlosophicus
- p.7 an important event in the philosophical world.
- showing … how traditional philosophy and traditional solutions arise out of ignorance of the principles of Symbolism and out of misuse of language.
- p.23 But to have constructed a theory of logic which is not at any point obviously wrong is to have achieved a work of extraordinary difficulty and importance. This merit … makes it one which no serious philosopher can afford to neglect.

Philosophy

- Preface p27 : The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the mis-understanding of the logic of our language.
- Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.

The Vienna circle

- In 1927, Wittgenstein began to have meetings with some members of the Vienna circle. This was a group of philosophers, mathematicians and scientists led my Moritz Schlick(1882-1936),a philosopher who was assassinated by a Nazi student.
- They regard Wittgenstein and Tractatus with awesome reverence.
- Schlick: Like you, we believe philosophy should be scientific.
- Wittgenstein: If you believe that, you've completely misunderstood the TRACTATUS! It's an exploration of the ethical, the limits of language and of thought – that's what it's about.

2、Truth value

- 4.06 Propositions can be true or false only by being pictures of the reality.
- 4.05 Reality is compared with the proposition.
- 4.25 If the elementary proposition is true, the atomic fact exists; if it is false the atomic fact does not exist.
- 4.26 The specification of all true elementary propositions describes the world completely. The world is completely described by the specification of all elementary propositions plus the specification, which of them are true and which false.

truth-possibilities

- 4.27 With regard to the existence of atomic facts there are $K_n = \sum_{v=0}^{n} \binom{n}{v}$ possibilities.
  - It is possible for all combinations of atomic facts to exist, and the others not to exist.
- 4.3 The truth-possibilities of the elementary propositions mean the possibilities of the existence and non-existence of the atomic facts.

truth-conditions

- 4.4 A proposition is the expression of agreement and disagreement with the truth-possibilities of the elementary propositions.
- 4.41 The truth-possibilities of the elementary propositions are the conditions of the truth and falsehood of the propositions.
- 4.42 With regard to the agreement and disagreement of a proposition with the truth-possibilities of *n* elementary propositions there are possibilities. $\sum_{k=0}^{K_n} \binom{K_n}{k} = L_n = 2^{K_n} = 2^{2^n}$

4.431 The expression of the agreement and disagreement with the truth-possibilities of the elementary propositions expresses the truth-conditions of the proposition. The proposition is the expression of its truth-conditions.

3. Truth table

- 4.45 For *n* elementary propositions there are L_n possible groups of truth-conditions. The groups of truth-conditions which belong to the truth-possibilities of a number of elementary propositions can be ordered in a series.
- （T T F T） （p, q）
- (W W F W) (p, q)
- whar falsch

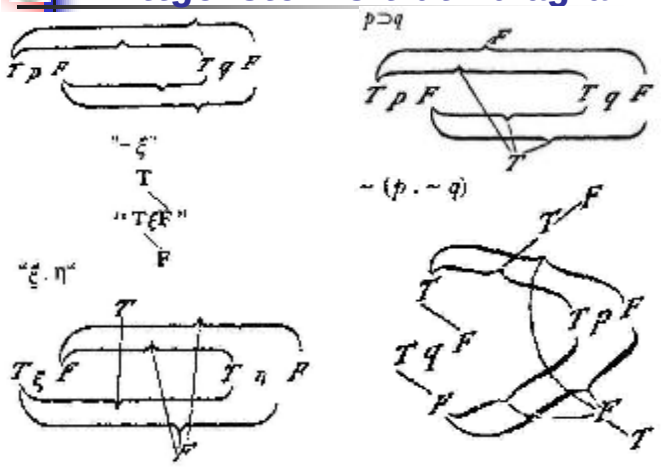
p	q	
T	T	T
F	T	T
T	F	
F	F	T

4.442 is a propositional sign.

T/F's occurrences

4.31	Line 1	Line 2	Line 3	……	Line n
composition1	T	T	T		T
composition2	F	T	T		T
composition3	T	F	T		T
composition4	F	F	T		T
……					
composition L					
T/F's occurrences	singular	Double pair	2^2	……	2^{n-1}

Wittgenstein's truth diagram



III. Truth Functions and Truth forms

- 1. Truth function
- 2. Kinds of truth functions
- 3. Kinds of truth forms
- 4. Truth operations
- 5. The function completeness of connectives

1. Truth functions

- 4.52 in some sense, one could say, that *all* propositions are **generalizations** of the elementary propositions.
- 5 Propositions are **truth-functions** of elementary propositions.
- (An elementary proposition is a **truth-function** of itself.)

2. Kinds of truth functions

- 2 arguments have $2^{2^n}=16$ kinds of truth functions, ie

A	B	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

tautology and contradiction

- 4.46 Among the possible groups of truth-conditions there are **two extreme cases**.
- In the one case the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.
- In the second case the proposition is false for all the truth-possibilities. The truth-conditions are *self-contradictory*.
- In the first case we call the proposition a **tautology**, in the second case a **contradiction**.

3. Kinds of truth forms

p	f_1	f_2	f_3	f_4
t	t	t	f	f
f	t	f	t	f
	$p\vee\neg p$ $p\supset p$ $\neg(p\wedge\neg p)$	$p\vee p$ $p\wedge p$ $\neg(\neg p)$	$\neg p$ $p\supset\neg p$ $\neg p\vee\neg p$	$p\wedge\neg p$ $\neg(p\supset p)$ $\neg(p\vee\neg p)$

Truth functions is definite, but truth forms are infinite.

f_1 tautology

f_4 contradictory

f_2、f_3 satisfiability

The truth forms of truth functions

f_1 $p\wedge q\supset p$； $p\supset p\vee q$； $(p\wedge q)\vee(p\wedge\neg q)\vee(\neg p\wedge q)\vee(\neg p\wedge\neg q)$

f_2 $p\vee q$； $\neg(\neg p\wedge\neg q)$

f_3 $\neg p\supset\neg q$； $q\supset p$； $p\vee\neg q$

f_4 $p\vee(q\wedge\neg q)$； $p\wedge(q\vee\neg q)$

f_5 $p\supset q$； $\neg p\vee q$； $\neg q\supset\neg p$

f_6 $q\vee(p\wedge\neg p)$； $q\wedge(p\vee\neg p)$

f_7 $p\neg\supset q$； $(p\supset q)\wedge(\neg p\supset\neg q)$

f_8 $p\wedge q$； $\neg(\neg p\vee\neg q)$

f_9 f_8's contradiction

f_{10} f_7's contradiction

f_{11} f_6's contradiction

f_{12} f_5's contradiction

f_{13} f_4's contradiction

f_{14} f_3's contradiction

f_{15} f_2's contradiction

f_{16} f_1's contradiction

4. truth-operations

- 5.234 The truth-functions of elementary proposition are results of operations which have the elementary propositions as bases. (I call these operations, **truth-operations**.)
- 5.2341 **Denial**, **logical addition**, **logical multiplication**, etc., etc., are operations. (Denial reverses the sense of a proposition.)
- 5.3 All propositions are results of **truth-operations** on the elementary propositions.
- The **truth-operation** is the way in which a truth-function arises from elementary propositions.

functional completeness

- Adequate set of connectives:
  functional completeness
- Classical propositional calculus operator ¬、∧、∨、→、« , are truth functional: through which compound sentences could be formed, while it's truth value only relies on that of it's constituents。
- A connective set is sufficient, if it could express all kinds of truth functions.
- {¬, →}, {¬, ∨}, {¬, ∧}, {|}, and {↓} is adequate to express them all.

Sole sufficient operator

- Both Peirce in 1880 and the American logician H. M. Sheffer in 1913 realized that the truth-functions of elementary logic could all be defined from a single function.
- Defining *p | q* to mean not both *p* and *q*, *p | p* is equivalent to not-*p*, (*p | q*) | (*p | q*) means *p* ∨ *q*, and so on.
- The sign for this truth-function is Sheffer's stroke: *p | q* is true only when *p* is false or *q* is false. A functionally complete system can also be built from a single sign for the function that is true when *p* is false and *q* is false.

Sheffer's stroke

- the Sheffer stroke, written "|" (see vertical bar, not to be confused with "||" which is often used to represent disjunction),
- "D*pq*", or "↑", denotes a logical operation that is equivalent to the negation of the conjunction operation, expressed in ordinary language as "not both".
- It is also called nand ("not and") or the alternative denial, since it says in effect that at least one of its operands is false. In Boolean algebra and digital electronics it is known as the NAND operation.
- Like its dual, the NOR operator (a.k.a. the Peirce arrow or Quine dagger), NAND can be used by itself, without any other logical operator, to constitute a logical formal system (making NAND functionally complete).

Sheffer's stroke

alternative denial Sheffer stroke Nand	joint denial Peirce's arrow Nor
¬p df= p|p	¬p df= p↓p
p∨q df= (p|p)|(q|q)	p∨q df= (p↓p)↓(q↓q)
p∧q df= (p|q)|(p|q)	p∧q df= (p↓q)↓(p↓q)
p→q df= p|(q|q)	p→q df= ((p↓p)↓q) ↓ ((p↓p)↓q))

5.1311 p|q=neither p nor q.

Thanks

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