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Cooperative behaviors in evolutionary games on complex networks

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【Abstract】 From the class study, we know defection is the rational choice in the prisoners' dilemma. However in fact, it appears cooperative behaviors among selfish individuals which seems unreasonable with the classic game theory. This article bases on the latest evolutionary game theory, using the most classic models—prisoners' dilemma to find differences of cooperative behaviors in the square lattice network and BA scale-free network. In the study, it uses theoretical analysis and Matlab simulation experiment and finds the resistance theory of cooperative nodes and the impact of network characteristic. Finally, according to the result, it gets some summaries and comes up with some improvement advice about study.

【Key Words】 cooperative behavior; evolutionary game theory; prisoners' dilemma; complex network

1. Background

Since the mathematician von Neumann and the economist Morgenstern published their book "*Theory of Games and Economic Behavior"* in 1944, game theory was used widely in various fields such as economic competition, military conflict, evolution of species and so on. Game theory provides theoretical frame to describe interacted behaviors between selfish individuals.

Although this theory which is based on rational hypothesis and individual selfish character is simple and practical, but it's not appropriate in real life. In fact, individual cognizance is limited and it cannot reach absolutely rational[1].

Evolutionary game theory is the latest research result which make improvements according to the above-mentioned problem. It roots in the Darwin's theory"Natural Selection and Survival of the Fittest". The core problem of evolutionary game theory is why it will appear widely cooperative behaviors among selfish individuals in the society. We have known a bit about it from the video shown in the class. We know defection always has higher payoff and follows the evolutionary strategy but with the evolution there will be some cooperative behaviors. So far, five mechanisms have been put forward to explain it: Kin selection, Direct reciprocity, Indirect reciprocity, Group selection and Network reciprocity.

This article will focus on cooperative behaviors in evolutionary games on complex notworks and explore the impact network makes on it.

2. Related elements

Model, network structure and evolutionary rule are three factors of networked evolutionary game.

2.1 model & parameter

2.1.1 prisoners' dilemma

Two men who cooperatively made a crime were put into prison. For better interrogation, the police have arranged them in different rooms so they cannot communicate with each other. If both of them choose to keep silent and don't admit the crime(Cooperation, C), the police will only impose a light sentence on them. In this situation, the payoff of the two is R (*Reward*). If one chooses to confess(Defection, D) but another chooses to resist, then the former will be discharged and get the payoff T (T*emptation*), the later will be given a severe judgment and have payoff S (S*uckers*). If the two both confess the crime, they will be both sentenced and get the payoff P (P*unishment*). Besides, the parameters have the relation: T>R>P>S and 2R>T+S. So we can get the payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (R,R) & (S,T) \\ (T,S) & (P,P) \end{pmatrix} \end{array}$$

To the two people, if we regard them as a whole, they will have maximal payoff 2R when they both choose to resist. But for each individual, he will always make the choice which is best for himself from the rational thinking. We can see, if the other choose to resist(C) but i choose to confess(D), so i will get T (T>R) and if the other choose to confess(D), choosing to confess(D) will still have higher payoff (P>S). So whichever choice the other makes, people should always choose to confess(D) to get the higher payoff.

From the analysis, we can know the only Nash equilibrium in the prisoners' dilemma is the two both choose confession(D,D).

2.1.2 parameters

Cooperator density ρ : the percentage of cooperators in the whole network

Temptation of defectors T : when one player chooses to defect and another chooses to cooperate, the payoff defector gets. Usually in the rule network, for easier calculation, we set R=1, P=S=0, so T is the only variate.

2.2 network structures

We use complex networks to describe game relationship among social individuals. Nodes in the network are the individuals in the game and links means the two individuals have game relationship.

Considering the variety of interaction among individuals in the real life, here we use two different network structures to describe game relationship.

2.2.1 square lattice network

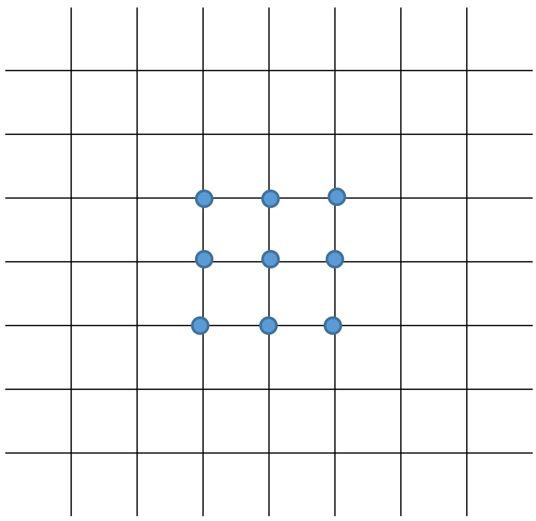


FIG. 1 square lattice network

Picture one is the square lattice network with the structure N=L*L. Square lattice network is the most common rule network. The degrees of all nodes in the network are even distribution. The characteristics of it are the small average length and big clustering coefficient.

In the picture, nodes are distributed orderly in the points of intersection and we give the corresponding coordinate (i , j) (i,j are both integers in the range of (1,L)) to each of them.

2.2.2 BA scale-free network

Barabási and Albert finds that ER random network and WS small-world network overlook two important characteristics of network in real life: growth and preferential attachment. In 1999, they set up the BA scale-free network model, solving the problems we referred.

2.2.2.1 two basic characteristics

A. Growth: the scale of network is constantly growing;

B. Preferential attachment: new nodes are more likely to connect with hub nodes which have higher degree. It's called "*rich get richer*".

2.2.2.2 procedure of setting up

A. In the beginning, there are m_0 nodes in the network. Every time adding a new node, connect the new one with the origin node in the network and generate m links(m≤ m_0)

B. When connect the new node with the origin ones, the probability follows the rule:

$$p_i = \frac{k_i}{\sum k_j}$$

C. It can't appear repeated connections in the network.

Follow the above rules to generate BA scale-free network. When the number of nodes N $\rightarrow \infty$, degree distribution follow the relation $P(k) = 2m^2 k^{-\gamma}$ and the power exponent $\gamma \to 3$, average shortest length L~ ln(N), cluster coefficient $C \sim N^{-0.75}$.

For example, when m_0 =3, m=2, the procedure of setting up BA scale-free network can be shown:

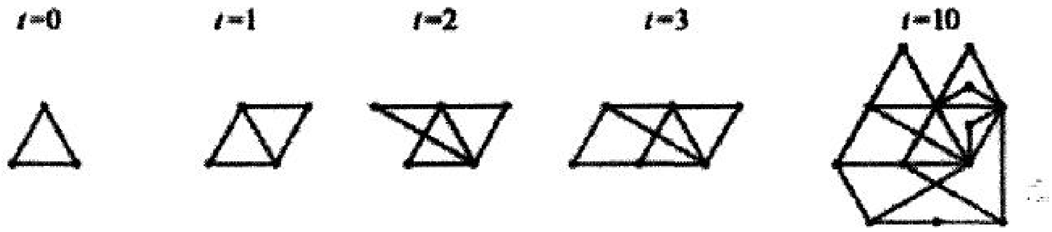


FIG 2. The evolutionary of BA scale-free network(m_0 =3, m=2)[2]

2.3 evolutionary rules

2.3.1 Optimum substitution

After every evolutionary, the node compares the payoff of all neighbor nodes and itself and chooses the strategy that the node with highest payoff used as its new game strategy.

2.3.2 Fermi rule

After every evolutionary, the node i randomly chooses a neighbor node j and according to the difference value between the two decides the possibility of the node i using the node j's strategy in the next game. The possibility can be calculated according to Fermi Function in the statistical physics:

$$w = \frac{1}{1+\exp[-(E_i - E_j)/K]}$$

E_i and E_j represents the payoff the node i and j get in the game. When the payoff of i is lower than that of j, i will tend to use j's strategy and the bigger difference value between the two nodes is, the more likely i is to use j's strategy. However, when the payoff of i is higher than that of j, it's still possible that i use j's strategy with small possibility. Parameter K represents noise figure which means the behavior can be irrational. It shows the uncertainty in the process of updating strategy. When $K \to \infty$, strategy update is totally random. When $K \to 0$, there is no uncertainty in the process, which means if the payoff of j is higher than that of i, i will be sure to use j's strategy.

3. Theoretical analysis

3.1 prisoners' dilemma in the rule network

Nowak and May made researches about the prisoner's dilemma in the two-dimensions square lattice network. As the picture one shows, in the square lattice network, every node has four neighbor nodes and connected nodes play games with each other. For easier calculation, we set up the temptation of defection T>1, the reward of cooperation R=1, both punishment of defection P and suckers S are 0, so in this situation, it's called poor prisoners' dilemma(弱囚徒困境). Nowak has proved the evolutionary results of poor prisoners' dilemma and prisoners' dilemma is the same. After each time's game, nodes will calculate the payoff of others and itself and choose the strategy that the nodes with the highest payoff used as its new strategy. So it makes a strategy update.[2]

According to Nowak's research, using this strategy update mechanism in the square lattice network, when 1≤T<2, ρ_c doesn't equal to 0 which means it exists the cooperative nodes and these nodes can gather together to form clusters in the network and show the pattern like picture two. In the picture, the dark ones are the cooperative nodes. The clusters consisting of these nodes will have changes in the process of game but will exist in the network all the time. That's because cooperative ones unite with each other and get the payoff higher than detective ones'. Clusters can prevent the invade from defection strategy.[3]

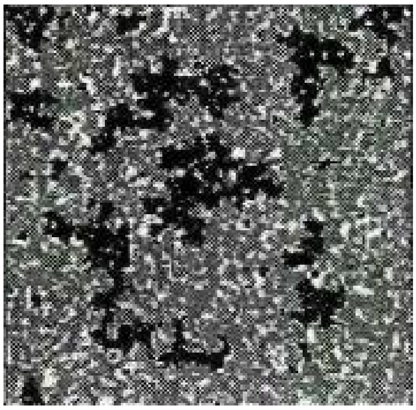


FIG. 3 clusters formed by nodes in the prisoners' dilemma evolutionary in square lattice network[4]

3.2 prisoners' dilemma in BA scale-free network

As we have learned in the class, many networks in real life are scale-free network , degree distribution of which follows the power-law distribution, such as transport network , Internet and so on. Because of the differences in topological structure between rule network and BA scale-free network, cooperative behaviors in the prisoners' dilemma evolutionary are also different. Santos has compared cooperative behaviors in different network models such as square lattice network, ER random network and scale-free network and find that scaleless property makes network easier to appear cooperative behaviors.

In the BA scale-free network, connected nodes play prisoners' dilemma game with each other and nodes will update game strategy with certain rule. Here we use the Fermi rule referred in 2.3.2 and focus on the impact hub nodes make in the network.

3.2.1 the spread of cooperative behaviors in BA scale-free network

We choose two hub nodes x,y to watch. The node x,y is directly connected and other nodes with small degree are randomly connected with one of them. In the beginning, we set x as cooperator and y as defector. Half neighbor nodes of each hub nodes are cooperators and the others are defectors. In each round of game, all nodes play prisoners' dilemma games with each of its neighbor nodes and payoff will be added up. Nodes will use Fermi rules to make evolution: each node chooses one neighbor node to compare with. If the neighbor's payoff in this round is higher than itself, the node will copy the neighbor's strategy in this round with possibility w, that means in the round, the strategy of nodes with higher payoff are more likely to be copied by their neighbors.

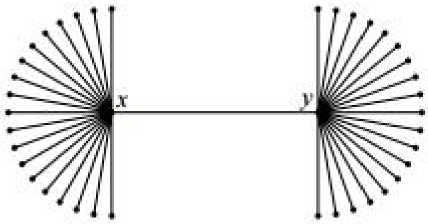


FIG. 4 A typical subgraph of a scale-free network, where two connected hub nodes are linked to many others having significantly less neighbors[5]

At first, because each hub node is surrounded by many cooperative neighbor nodes, so its accumulated payoff willl be higher than the small-degree nodes and small-degree nodes will copy the strategy of hub node it connects with. Although now cooperative x has lower payoff than defective y does, because of random choice of strategy comparing, x still has higher payoff than most of small-degree neighbor nodes, x can insist cooperative strategy for a time. As time goes by, the neighbor nodes of x tend to copy x's strategy, so there will be more and more cooperators around x, which contributes to the rise of x's payoff. On the contrary, it means there appears more and more defectors around y and y's payoff will decline and gradually be lower than its neighbor node x. At one moment, y will copy x's strategy and change into cooperator and then y's neighbor nodes will also change into cooperators. In the end, cooperation will spread in the whole network and all the hub nodes will choose to cooperate. This means if individual chooses to add up its payoff, the hub nodes in the BA scale-free network tend to choose cooperation and will affect their neighbor nodes[1].

3.2.2 the spread of defective behaviors in BA scale-free network

Besides, to find how hub nodes effectively resist the invade from defectors, we can analyze the spread of defective behaviors. In the beginning, we set only the node x with biggest degree defector and others are cooperators. Then watch the invasion hub node x make to cooperative behaviors in the network. We can find, because of high payoff x gets from its cooperative neighbor nodes, these small-degree neighbor nodes will copy its behavior and according to the research, there will be about 80% neighbor nodes changing into defectors after a time. With the fall of the percentage of cooperative neighbor nodes, the node x's payoff will be lower than some big-degree cooperative neighbor nodes and then it will realize the payoff of cooperation is higher than that of defection, so x will turn into cooperator. After that, most of its neighbor nodes will choose cooperation again.

The above analysis shows hub nodes can effectively resist the invade from defectors.

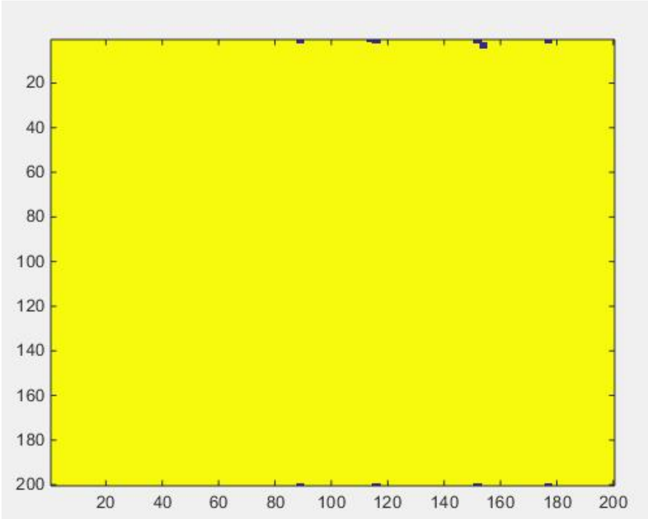
4. Simulation experiment&analysis

We use matlab to simulate the cooperative behavior evolutionary in the two different networks and set different values of T to see the change of ρ . Besides, we compare the result of two networks with different values of T and get some practical summaries.

4.1 prisoners' dilemma in the square lattice network

For easier calculation, we set R=1, S=P=0. When T>1, the game follows the prisoners' dilemma. We set the size of network is 200*200 and the evolutionary time is 50. Then we'll see the evolutionary game in the situation of T=1.3, 1.6, 1.9,

4.1.1 simulation result



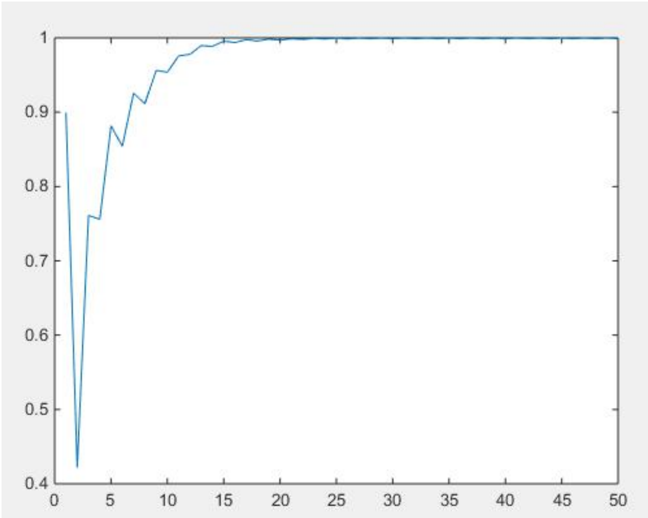
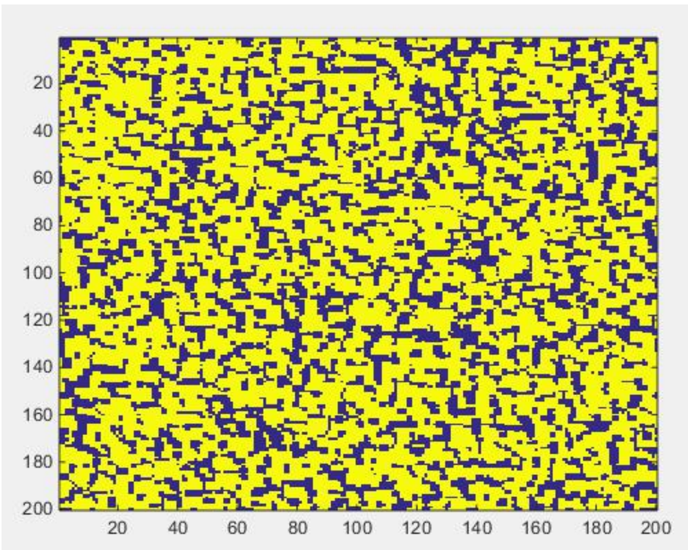


FIG. 5 The cluster graph and ρ curve when T=1.3[1](ρ_{50} =0.9977)



[1] In the cluster graph, yellow ones are cooperators and blue ones are defectors.

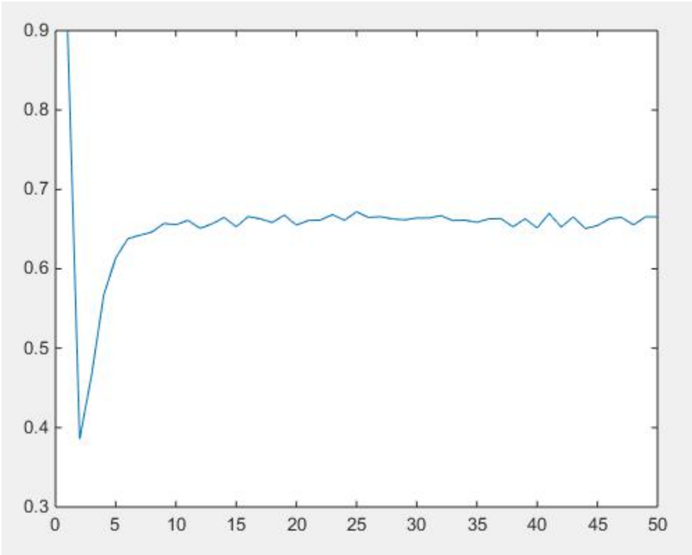
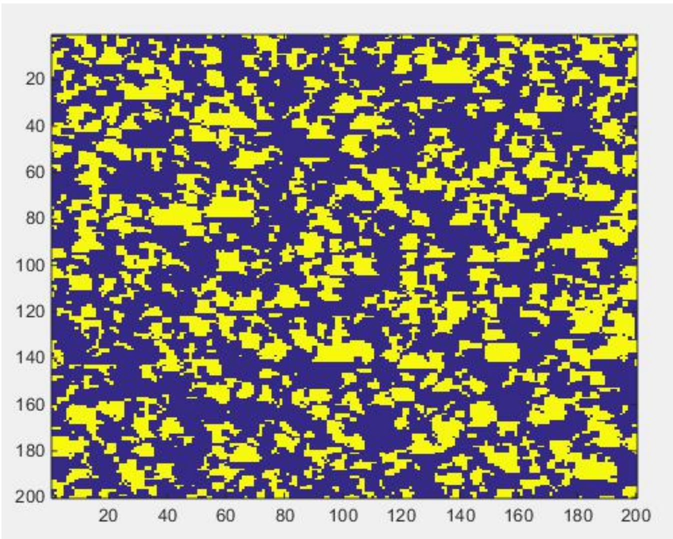


FIG. 6 The cluster graph and ρ curve when T=1.6(ρ_{50} =0.6599)



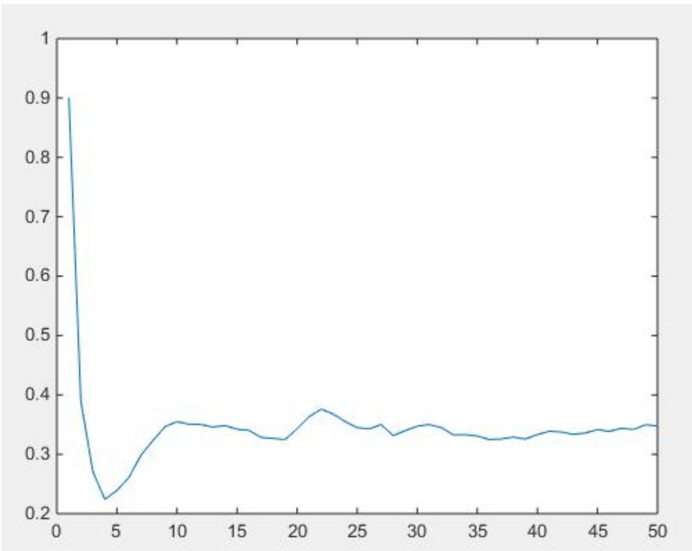


FIG. 7 The cluster graph and ρ curve when T=1.9(ρ_{50} =0.3402)

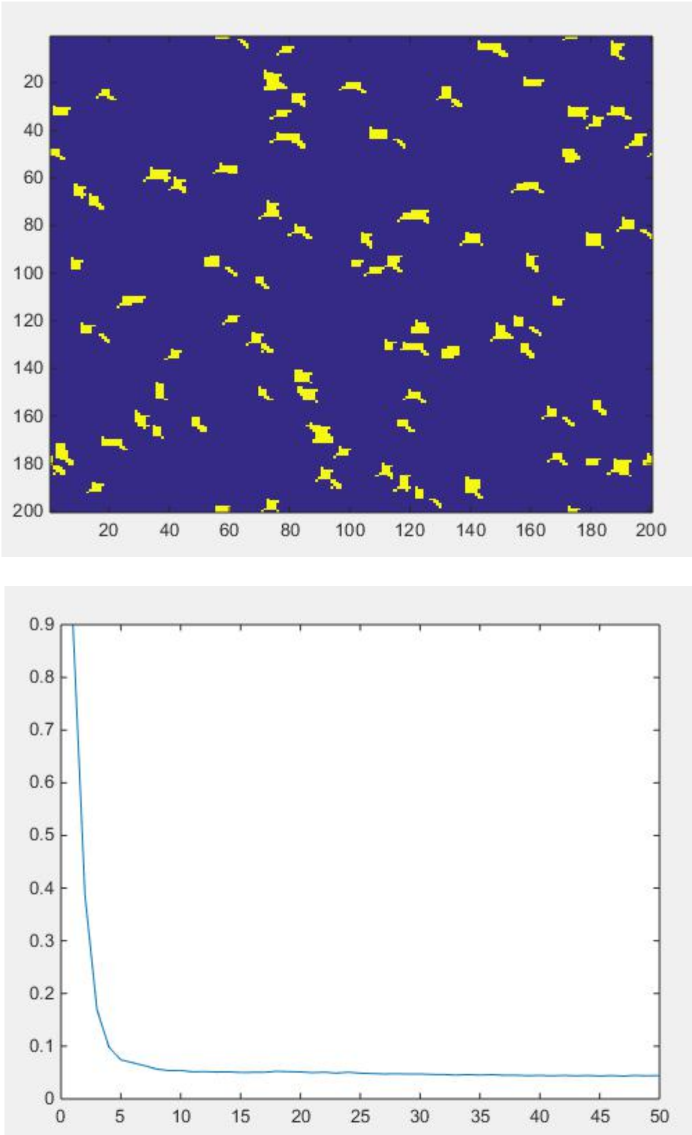


FIG. 8 The cluster graph and ρ curve when T=2(ρ_{50} =0.0441)

4.1.2 analysis

From the result we can get three conclusions:

A. When T=1.3, the network still has high cooperator density but when T=1.6 and 1.9, there is a rapid fall of cooperator density and when T=2, ρ is nearly 0 which means almost all players choose to defect. So we know T=2 is about the threshold value of disappearance of cooperators.

B. From the three plot of curve, we find within the time of 5, the network can reach a stable situation, when the cooperator density is nearly invariant.

C. From the three graphs of clusters, we can clearly see cooperators gather together to form the clusters to resist the invade from defectors. In the graphs, most clusters are small and separated and it's hard to see very big clusters. What's more, in the process of evolutionary, we find all small clusters are peripatetic but don't have constant position.

4.2 prisoners' dilemma in BA scale-free network

4.2.1 simulation result

Here we directly use the simulation result others have got and still choose the situation when T=1.3, 1.6, 1.9 and watch the change of cooperator density.

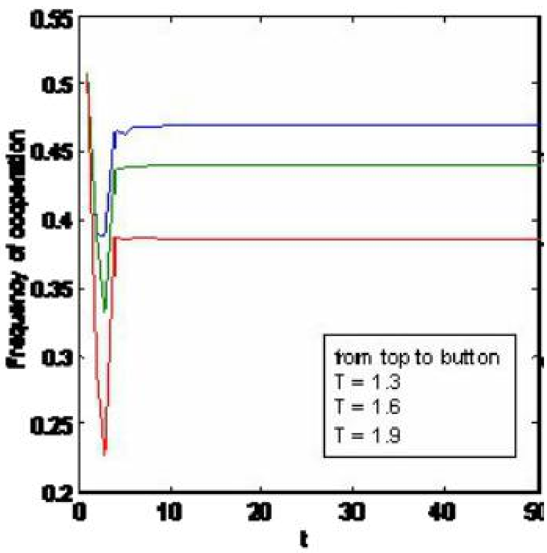


FIG. 9 ρ **curve of prisoners' dilemma in BA scale-free network**[3]

4.2.2 analysis

From the result, we can find:

A. With the rise of T, ρ is falling but the fall is relatively mean. From the picture, we can see when T=1.3, $\rho \sim 0.47$; T=1.6, $\rho \sim 0.44$;T=1.9, $\rho \sim 0.39$.

B. From the three plot of curve, we find within the time of 5, the network can reach a stable situation, when the cooperator density is nearly invariant.

4.3 comparison

A. In the prisoners' dilemma, when the value of T is small, the cooperative behavior level of square lattice network is higher than that of BA scale-free network; when the value of T is big, the cooperative behavior level of square lattice network is lower than that of BA scale-free network. So when we play the prisoners' dilemma game in the complex networks, the scale-free characteristic can raise the cooperative level of networks.

B. There is a period of transient transition in the evolutionary of cooperative strategy. When time reaches one certain value, the number of cooperators tends to be invariant.

5. Improvements

5.1 Strategy

In the above research, players usually simply choose cooperate or defection. However, compared with different kinds of players which can be divided into pure cooperators, pure defectors and even fluctuating individuals, strategy may also have other different kinds. In real world, people may take more kinds of strategy such as neutrality. For further study, we can take multi-strategy evolutionary games into consideration[6].

5.2 networks

In the above research, we just play the evolutionary game in the single network, but in fact, many complex networks in real world are mixed by different networks with different structures and functions. For example, the popular app Weibo builds a social network, but at the same time it needs Internet to connect and what's more, internet needs power network to support it. So about multi-networks mixed networks we can make some innovative researches.

5.3 incentive mechanism

In the above research, we begin with at least 50% cooperators to generate cooperation, but it doesn't correspond with the fact. In fact, cooperation always appears from a small group of people and then gradually affect others' behavior choices and it appears higher group cooperation levels. So in the research, we can try incentive mechanisms to realize more practical model simulation[7].

5.4 robustness

We only make comparison between cooperative density in different T and networks, but we have learned from *Network Science: An Introduction* that the cooperative behavior of BA scale-free network also have the characteristic of very good robustness. It's surely meaningful and practical to make research about it.

6. Summary

Why selfish individuals can have widely cooperative behaviors is the core problem of evolutionary game theory. Prisoners' dilemma is one of the most classic models and is very suitable for researches about evolutionary game, so we choose to play prisoners' dilemma game in different networks. From the above analysis,we have known cooperators in different networks will have some actions to resist the invade from defectors. Cooperators in the square lattice network will form clusters and in BA scale-free network hub nodes play an important role which have good cooperation keeping characteristic and can effectively resist the invade from defectors. Furthermore, from the simulation experiment, we find when the scale-free characteristic can raise the cooperative level in the networks.

However, there are still many drawbacks in the research and we have selected four ones to analyze and hope to improve them.

Certainly, there are many other aspects about evolutionary game challenging to be studied. As one of the most popular study fields in the international academic world, there is no doubt that evolutionary game will become one of fields which have the most development potentials.

Finally, thanks for Professor Xiang Li and Professor Cong Li who have led me into this enjoyable field and enrich my knowledge about complex networks. Thanks for Mr. Quan Yuan giving me very much help in the study. In this semester, Intro Network Science brought not only rich knowledge but also improvements of programming, team studying, self-studying and even speech making for me and it benefits me a lot. Although the class has come to an end, i believe my study about complex network just gets started.

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