§ 2.6 电磁场的边值关系

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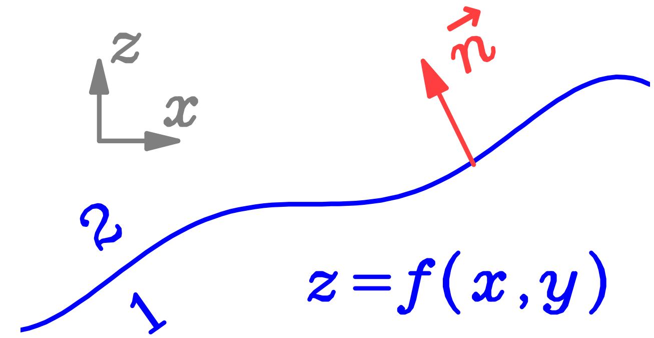
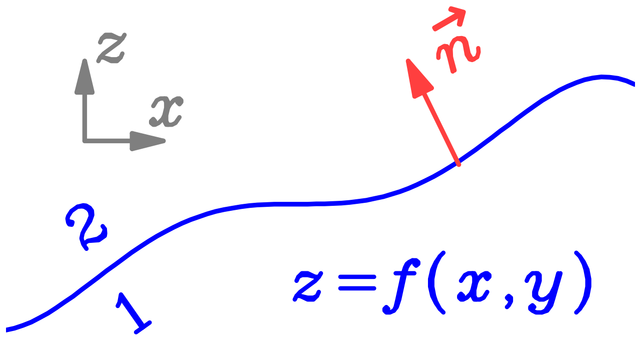
—— 这些推导均基于积分方程

基于微分方程的推导



基于微分方程的推导

如右图所示两种介质的界面由方程 $z = f(x, y)$ 给定，



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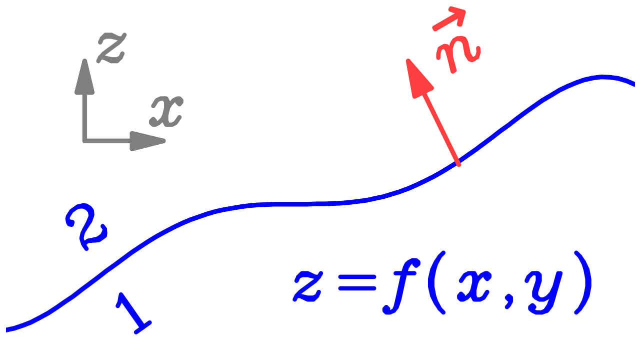

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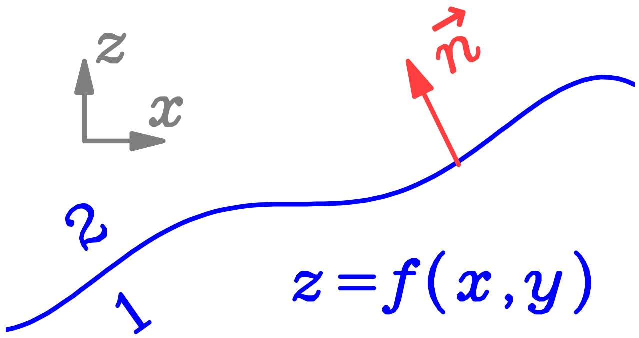

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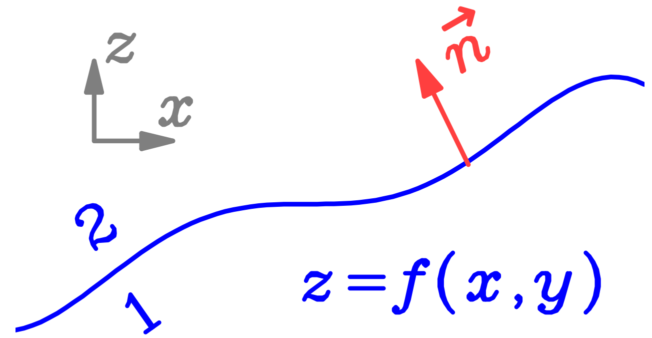

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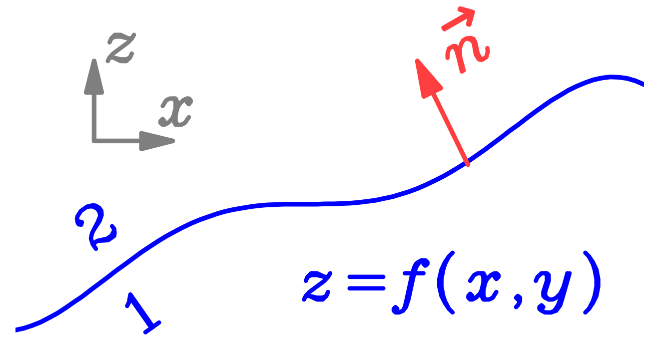

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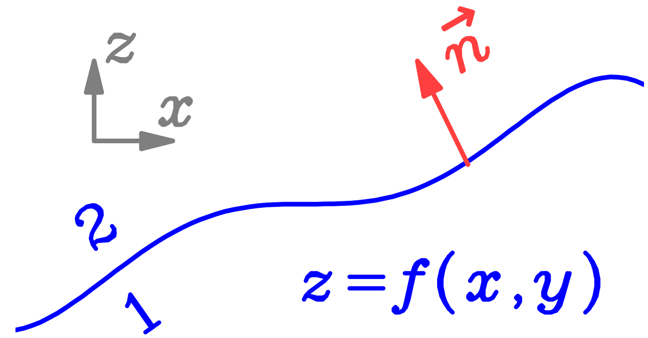

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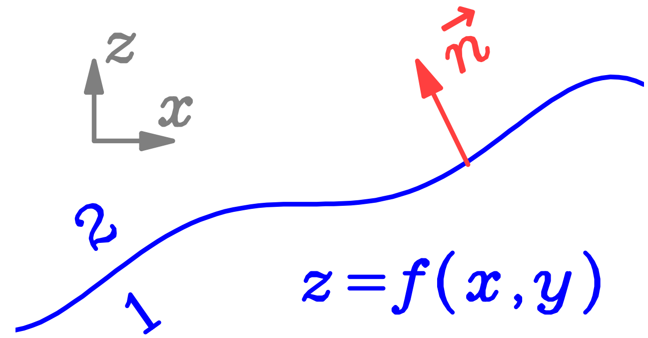
故：

$$\vec{n} = \frac{\nabla s(x,y,z)}{|\nabla s(x,y,z)|} = \left[\hat{e}_z - \frac{\partial f(x,y)}{\partial x}\ \hat{e}_x - \frac{\partial f(x,y)}{\partial y}\ \hat{e}_y\right] \Big/ \left[1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$$

基于微分方程的推导



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$\vec{n}$ 从 $z<f(x,y)$ 区（介质 1）指向 $z>f(x,y)$ 区（介质 2），故 $s(x,y,z)=z-f(x,y)$。

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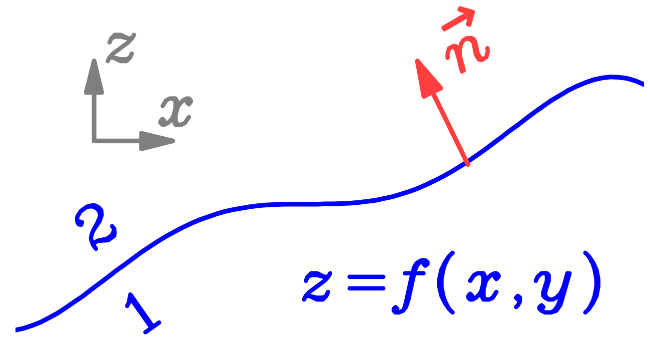

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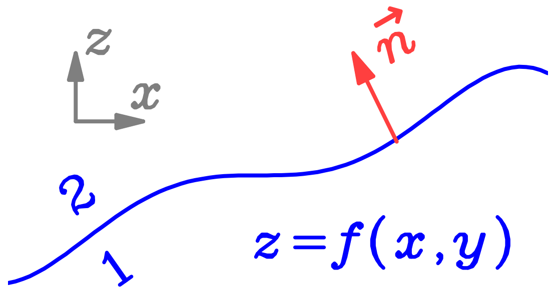
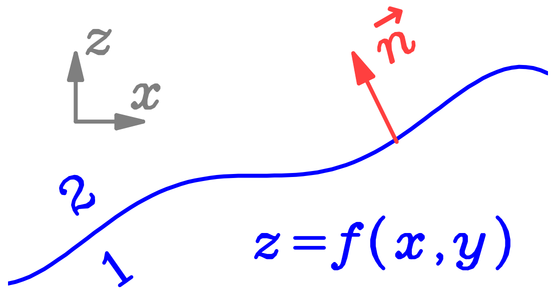
利用阶跃函数 (step function) $$u(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

及其性质： $$\frac{\mathrm{d}u(x)}{\mathrm{d}x} = \delta(x)$$

Let there be light



可将电位移矢量、电荷密度写成



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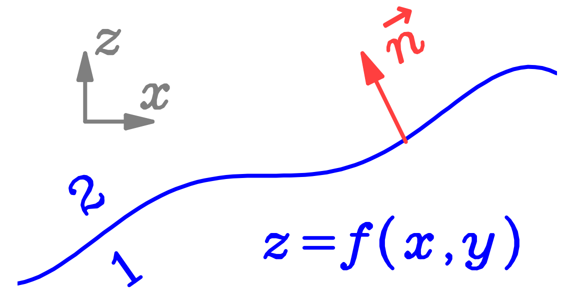

$$\vec{D} = \vec{D}_1\, u[f(x,y) - z] + \vec{D}_2\, u[z - f(x,y)]$$

$\vec{D}_1$、$\vec{D}_2$ 分别为介质 1、介质 2 的电位移矢量

$$\rho = \rho_1\, u(-s) + \rho_2\, u(s) + |\nabla s|\, \sigma_f\, \delta(s), \quad s = z - f(x,y) \qquad (1)$$

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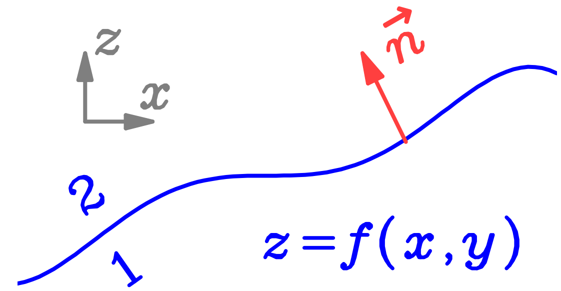

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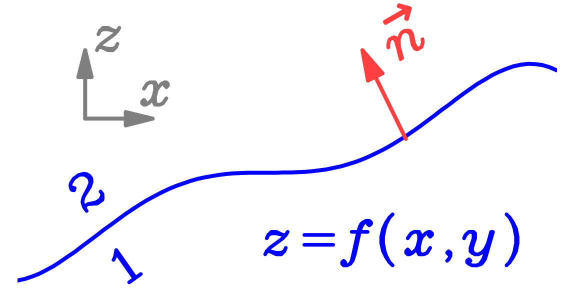

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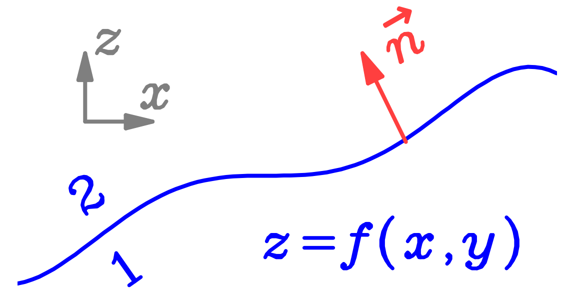

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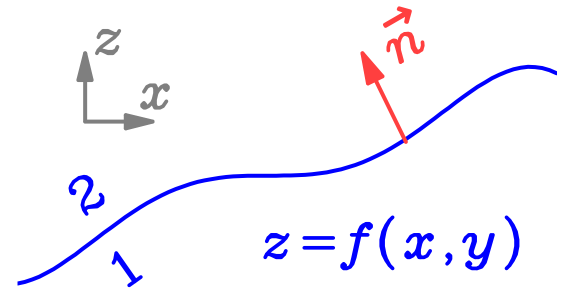

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$\nabla \cdot \vec{D}_i = \rho_i$, $i = 1, 2$，另外：$\dfrac{\mathrm{d}\, u(s)}{\mathrm{d}\, s} = \delta(s)$, $\dfrac{\mathrm{d}\, u(-s)}{\mathrm{d}\, s} = -\delta(s)$

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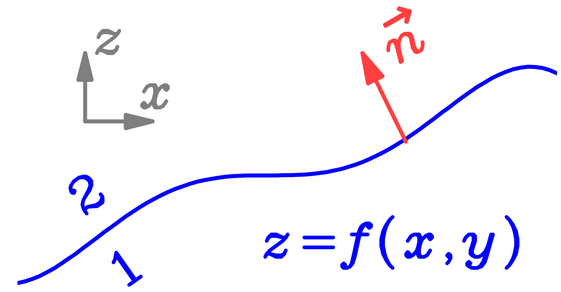

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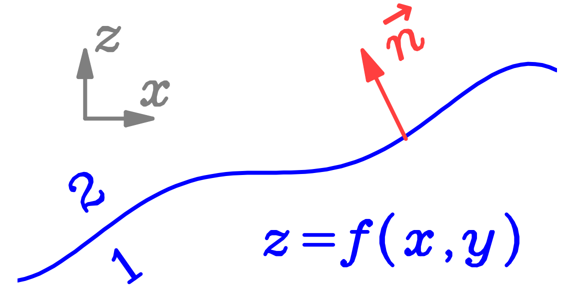
可将电位移矢量、电荷密度写成



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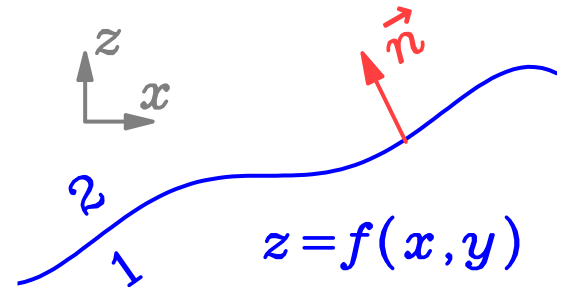

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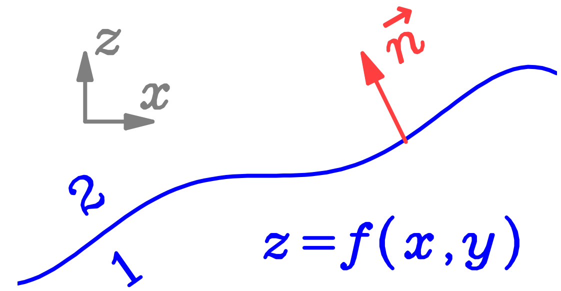

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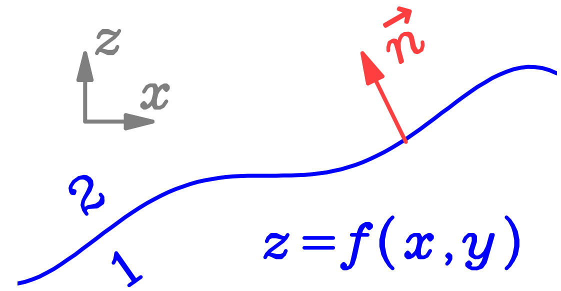

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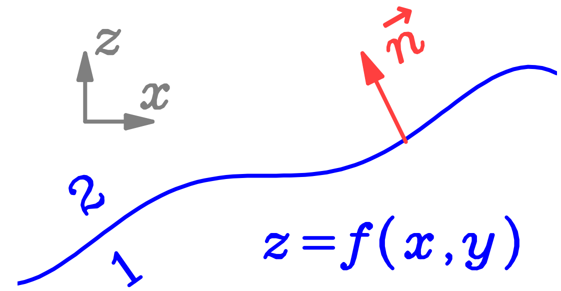

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$\xrightarrow{\text{比较}}$

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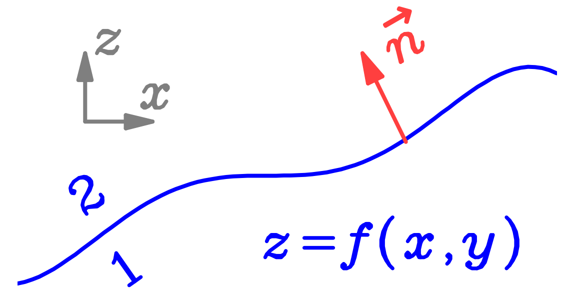

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比较 $\Longrightarrow$

$$\delta(s)\,|\nabla s|\,\vec{n}\cdot(\vec{D}_2 - \vec{D}_1) = |\nabla s|\,\sigma_f\,\delta(s)$$

$$\delta(s)\,|\nabla s|\,\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{D}}_2-\vec{\boldsymbol{D}}_1)=|\nabla s|\,\sigma_f\,\delta(s)\qquad \nabla s\neq 0$$

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$$\Longrightarrow \quad \left[\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{D}}_2-\vec{\boldsymbol{D}}_1)\right]\delta(s)=\sigma_f\,\delta(s)$$

两个包含 δ 函数的函数相等

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完全类似地，从 $\nabla\cdot\vec{\boldsymbol{B}}=0$ 可得：

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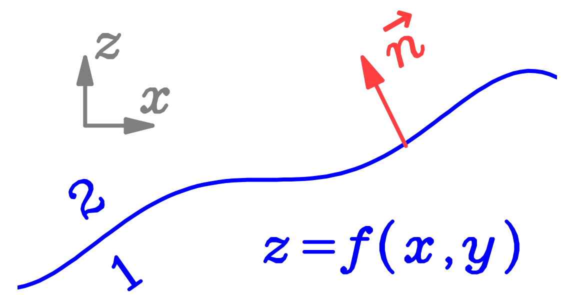

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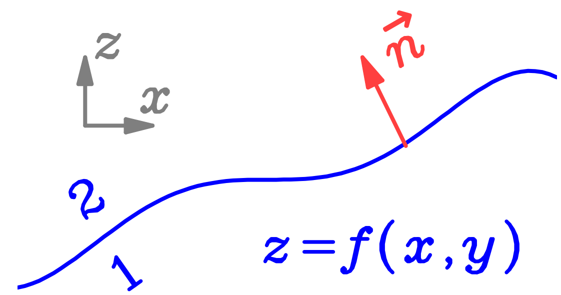
现看旋度方程：$\nabla\times\vec{\boldsymbol{H}}=\vec{j}+\dfrac{\partial\vec{\boldsymbol{D}}}{\partial t}$



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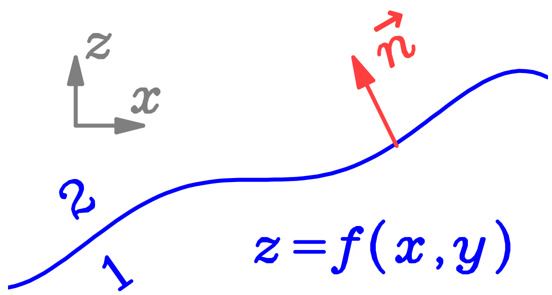

$$\vec{H} = \vec{H}_1\,u(-s) + \vec{H}_2\,u(s)$$

$\vec{H}_1$、$\vec{H}_2$ 分别为介质 1、介质 2 的磁场强度

$$\vec{D} = \vec{D}_1\,u(-s) + \vec{D}_2\,u(s) \qquad (1)$$

$\vec{D}_1$、$\vec{D}_2$ 分别为介质 1、介质 2 的电位移矢量

$$\vec{j} = \vec{j}_1\,u(-s) + \vec{j}_2\,u(s) + |\nabla s|\,\vec{\alpha}_f\,\delta(s), \qquad s = z - f(x,y) \qquad (2)$$

$\vec{j}_1$、$\vec{j}_2$ 分别为介质 1、介质 2 的自由电流密度，$\vec{\alpha}_f$ 为界面自由面电流密度

Let there be light

$$\nabla \times \vec{\boldsymbol{H}} = \nabla \times \left[\vec{\boldsymbol{H}}_1 \, u(-s) + \vec{\boldsymbol{H}}_2 \, u(s) \right]$$

Let there be light

$$\nabla \times \vec{\boldsymbol{H}} = \nabla \times \left[\vec{\boldsymbol{H}}_1 \, u(-s) + \vec{\boldsymbol{H}}_2 \, u(s)\right]$$

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利用 $\nabla\times\vec{\boldsymbol{H}}=\vec{\boldsymbol{j}}+\dfrac{\partial\vec{\boldsymbol{D}}}{\partial t}$、(1) 和 (2) 式

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利用 $\nabla\times\vec{\boldsymbol H}=\vec{\boldsymbol j}+\dfrac{\partial\vec{\boldsymbol D}}{\partial t}$、(1) 和 (2) 式

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$$\Longrightarrow$$

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$$\Longrightarrow\quad \boxed{\delta(s)\,|\nabla s|\,\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)}$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$

两个包含 δ 的函数相等且 $\nabla s\neq 0$

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$$\Rightarrow\left[\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)\right]\Big|_{s=0}=\vec{\boldsymbol{\alpha}}_f\Big|_{s=0}$$

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$$\Rightarrow\left[\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)\right]\Big|_{s=0}=\vec{\boldsymbol{\alpha}}_f\Big|_{s=0}$$ 在界面 $s=0$：

$$\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=\vec{\boldsymbol{\alpha}}_f$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

$$\Rightarrow\left[\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)\right]\Big|_{s=0}=\vec{\boldsymbol{\alpha}}_f\Big|_{s=0}$$

在界面 $s=0$： $\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=\vec{\boldsymbol{\alpha}}_f$

完全类似地，从 $\nabla\times\vec{\boldsymbol{E}}=-\dfrac{\partial\vec{\boldsymbol{B}}}{\partial t}$ 可得：

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

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微分表达式及其在交界面的形式（边值关系），$\vec{\boldsymbol{n}}$ 为界面法向，从介质 1 到介质 2

$$-\nabla\cdot\vec{\boldsymbol{P}}=\rho_P$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

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微分表达式及其在交界面的形式（边值关系），$\vec{\boldsymbol{n}}$ 为界面法向，从介质 1 到介质 2

$$-\nabla\cdot\vec{\boldsymbol{P}}=\rho_P \qquad\Longrightarrow\qquad -\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{P}}_2-\vec{\boldsymbol{P}}_1)=\sigma_P$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

$$\Rightarrow\left[\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)\right]\Big|_{s=0}=\vec{\boldsymbol{\alpha}}_f\Big|_{s=0}$$

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完全类似地，从 $\nabla\times\vec{\boldsymbol{E}}=-\dfrac{\partial\vec{\boldsymbol{B}}}{\partial t}$ 可得：

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微分表达式及其在交界面的形式（边值关系），$\vec{\boldsymbol{n}}$ 为界面法向，从介质 1 到介质 2

$$-\nabla\cdot\vec{\boldsymbol{P}}=\rho_P\qquad\Longrightarrow\qquad-\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{P}}_2-\vec{\boldsymbol{P}}_1)=\sigma_P$$

$$\nabla\cdot\vec{\boldsymbol{D}}=\rho_f$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$

两个包含 δ 的函数相等且 $\nabla s\neq 0$

$$\Rightarrow\left[\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)\right]\Big|_{s=0}=\vec{\boldsymbol{\alpha}}_f\Big|_{s=0}$$

在界面 $s=0$： $\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=\vec{\boldsymbol{\alpha}}_f$

完全类似地，从 $\nabla\times\vec{\boldsymbol{E}}=-\dfrac{\partial\vec{\boldsymbol{B}}}{\partial t}$ 可得：

在界面 $s=0$： $\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{E}}_2-\vec{\boldsymbol{E}}_1)=0$

微分表达式及其在交界面的形式（边值关系）， $\vec{\boldsymbol{n}}$ 为界面法向，从介质 1 到介质 2

$$-\nabla\cdot\vec{\boldsymbol{P}}=\rho_P\qquad\Longrightarrow\qquad-\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{P}}_2-\vec{\boldsymbol{P}}_1)=\sigma_P$$

$$\nabla\cdot\vec{\boldsymbol{D}}=\rho_f\qquad\Longrightarrow\qquad\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{D}}_2-\vec{\boldsymbol{D}}_1)=\sigma_f$$

$$\delta(s)\,|\nabla s|\vec{n}\times(\vec{H}_2-\vec{H}_1)=|\nabla s|\,\vec{\alpha}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

$$\Rightarrow\left[\vec{n}\times(\vec{H}_2-\vec{H}_1)\right]\Big|_{s=0}=\vec{\alpha}_f\Big|_{s=0}$$

在界面 $s=0$： $\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{\alpha}_f$

完全类似地，从 $\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}$ 可得：

在界面 $s=0$： $\vec{n}\times(\vec{E}_2-\vec{E}_1)=0$

微分表达式及其在交界面的形式（边值关系），$\vec{n}$ 为界面法向，从介质 1 到介质 2

$$-\nabla\cdot\vec{P}=\rho_P \quad\Longrightarrow\quad -\vec{n}\cdot(\vec{P}_2-\vec{P}_1)=\sigma_P$$

$$\nabla\cdot\vec{D}=\rho_f \quad\Longrightarrow\quad \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\sigma_f$$

$$\nabla\cdot\vec{B}=0$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

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$$\nabla\cdot\vec{\boldsymbol{D}}=\rho_f\quad\Longrightarrow\quad\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{D}}_2-\vec{\boldsymbol{D}}_1)=\sigma_f$$

$$\nabla\cdot\vec{\boldsymbol{B}}=0\quad\Longrightarrow\quad\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{B}}_2-\vec{\boldsymbol{B}}_1)=0$$

$$\delta(s)\,|\nabla s|\vec{\boldsymbol{n}}\times(\vec{\boldsymbol{H}}_2-\vec{\boldsymbol{H}}_1)=|\nabla s|\,\vec{\boldsymbol{\alpha}}_f\,\delta(s)$$ 两个包含 δ 的函数相等且 $\nabla s\neq 0$

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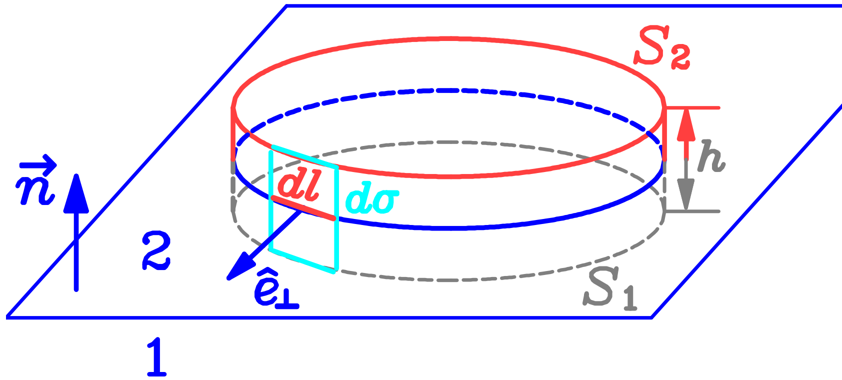

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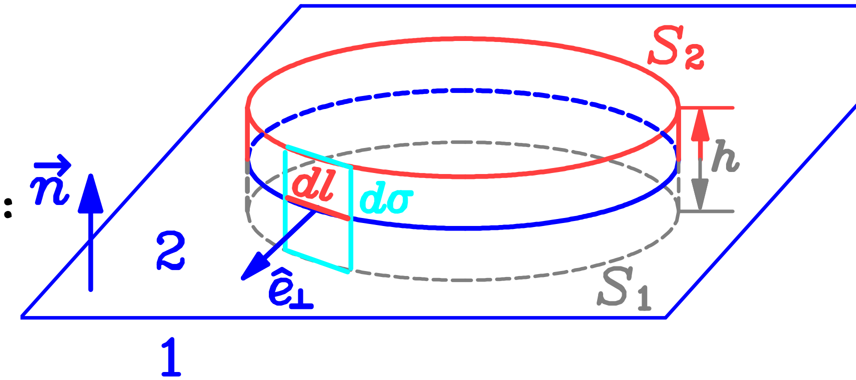

$$\oint_S \vec{j}\cdot\vec{n}_S\,\mathrm{d}\sigma+\frac{\partial}{\partial t}\int_V \rho\,\mathrm{d}\tau=\vec{j}_2\cdot\vec{n}\,S_2+\vec{j}_1\cdot(-\vec{n})\,S_1+\int_{\text{侧面}}\vec{j}\cdot\hat{e}_\perp\,\mathrm{d}\sigma+\frac{\partial}{\partial t}(S_1h\rho)$$

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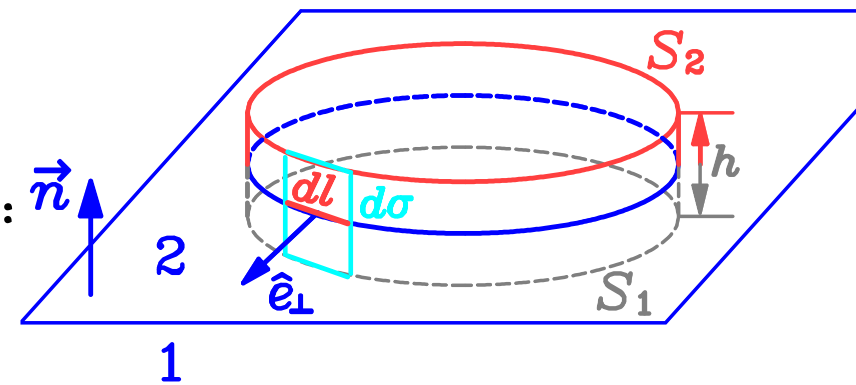

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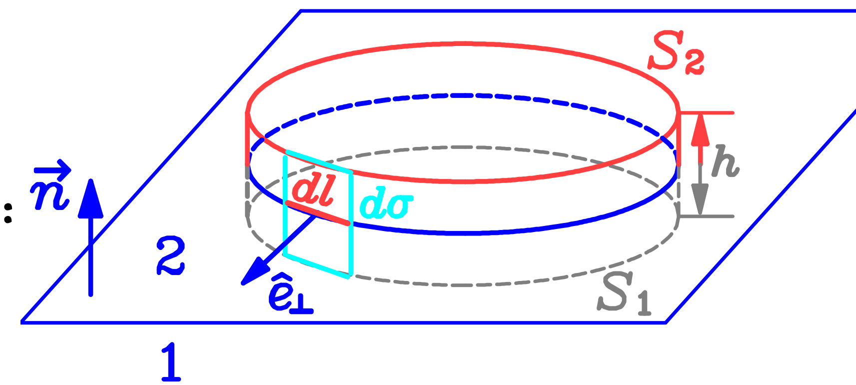

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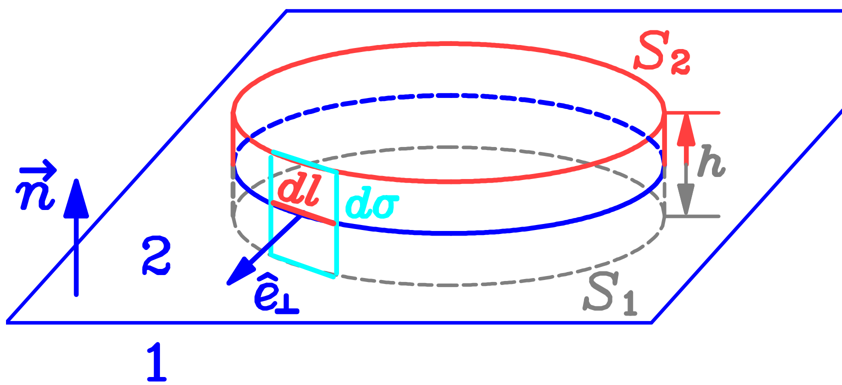

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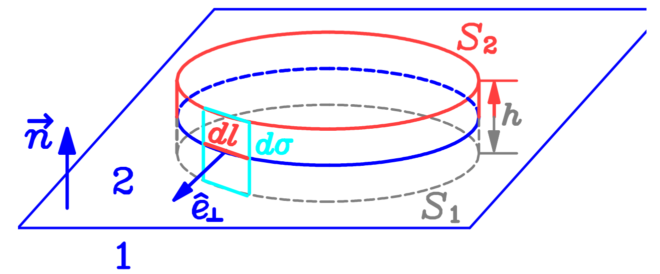
$$=S[\vec{n}\cdot(\vec{j}_2-\vec{j}_1)]+S\left[\frac{\partial}{\partial t}\lim_{\substack{h\to 0\\ \rho\to\infty}}\rho h\right]+\int_{\text{侧面}}\vec{j}\cdot\hat{e}_\perp\underbrace{h\mathrm{d}l}_{\mathrm{d}\sigma}$$

Let there be light



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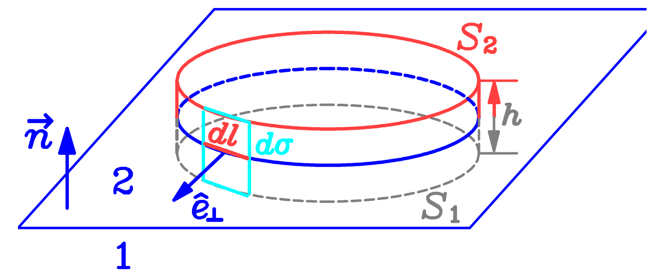
$$= S\,\vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{j}}_2-\vec{\boldsymbol{j}}_1)+S\,\frac{\partial\sigma_q}{\partial t}+\int_{\text{侧面}}\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_\perp\,h\,\mathrm{d}l$$



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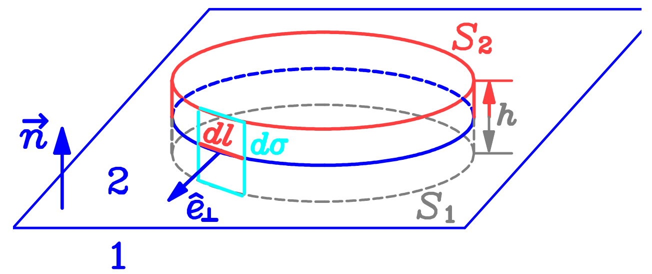

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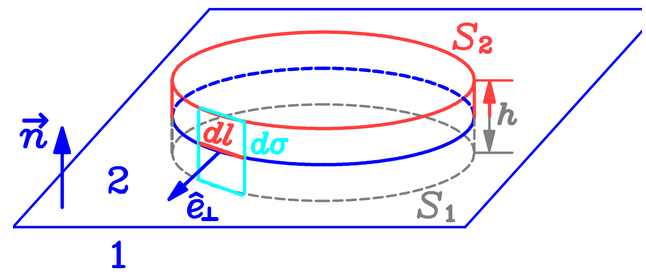

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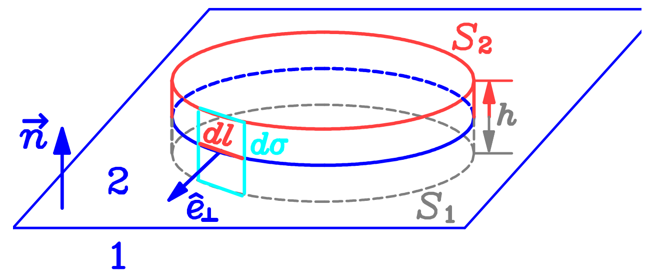

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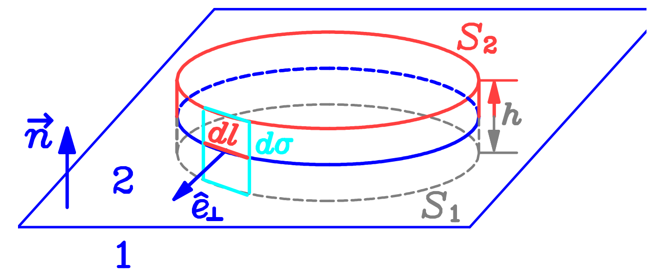

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$\mathcal{L}$ 为上图中兰色闭合线

应用二维高斯定理，化为“面积分”

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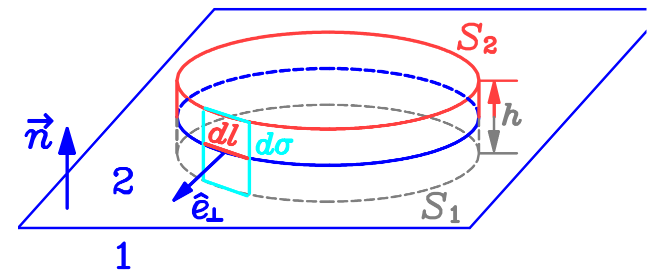

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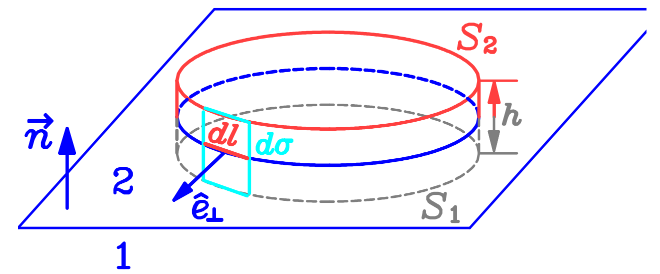
应用二维高斯定理，化为“面积分”

$$= S\vec{n}\cdot(\vec{j}_2-\vec{j}_1)+S\,\frac{\partial\sigma_q}{\partial t}+\int_S\nabla\cdot\vec{\alpha}\,\mathrm{d}\sigma$$



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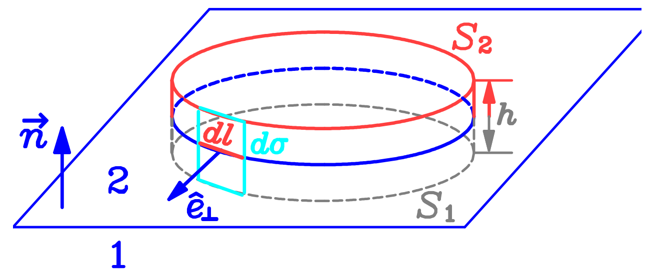

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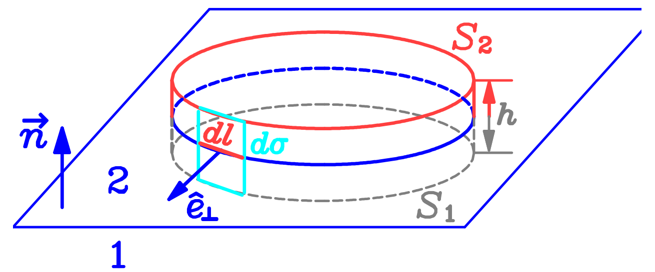

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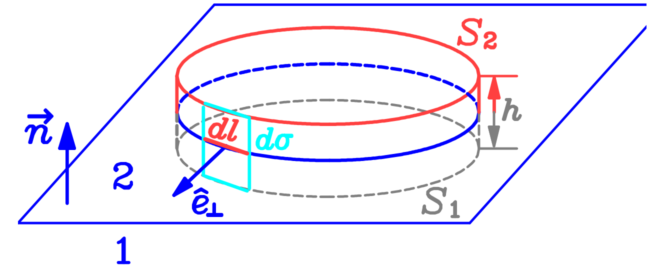
Let there be light



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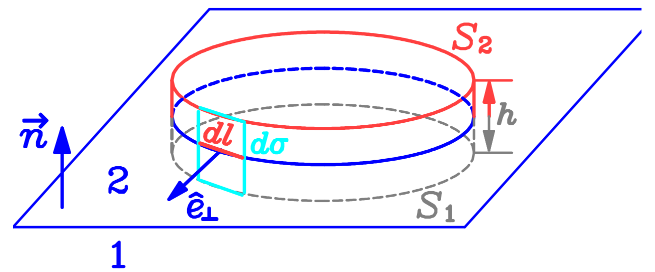

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Let there be light

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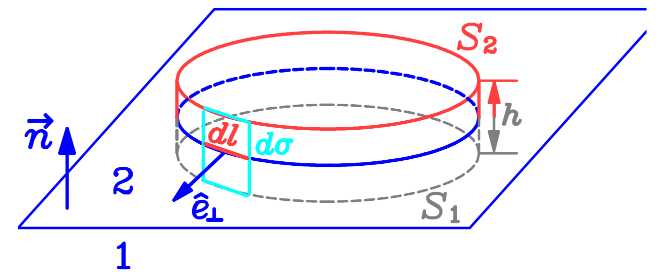

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4. 一些方程及其在交界面上的形式

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规则与物理：

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场量 $\vec{\boldsymbol{E}}$, $\vec{\boldsymbol{D}}$, $\vec{\boldsymbol{B}}$, $\vec{\boldsymbol{H}}$ 等不会趋于无穷大，在面上退化为 0，

4. 一些方程及其在交界面上的形式

$$\nabla\cdot\vec{\boldsymbol{P}}=-\rho_P \quad\Longrightarrow\quad \vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{P}}_2-\vec{\boldsymbol{P}}_1)=-\sigma_P$$

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$$\nabla\cdot\vec{\boldsymbol{B}}=0 \quad\Longrightarrow\quad \vec{\boldsymbol{n}}\cdot(\vec{\boldsymbol{B}}_2-\vec{\boldsymbol{B}}_1)=0$$

$$\nabla\times\vec{\boldsymbol{M}}=\vec{\boldsymbol{j}}_M \quad\Longrightarrow\quad \vec{\boldsymbol{n}}\times(\vec{\boldsymbol{M}}_2-\vec{\boldsymbol{M}}_1)=\vec{\boldsymbol{\alpha}}_M$$

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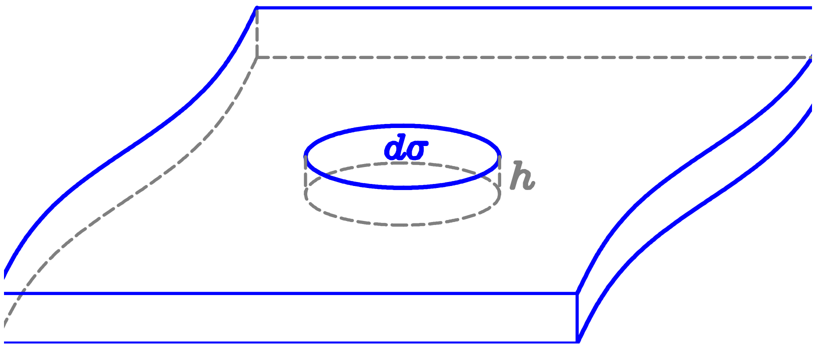

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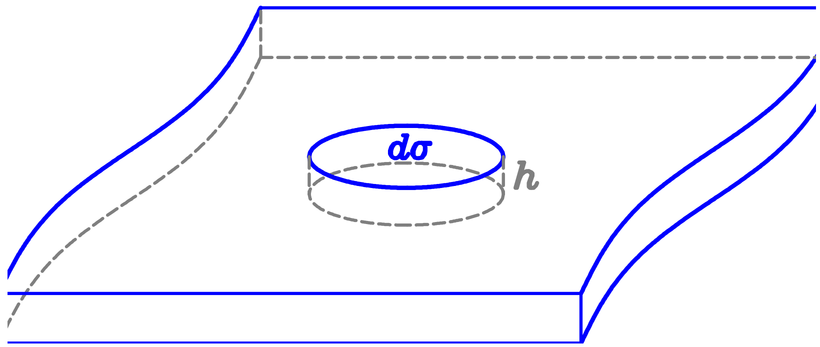

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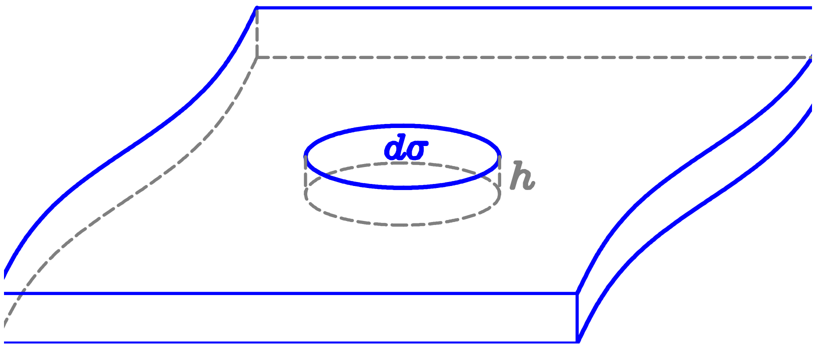

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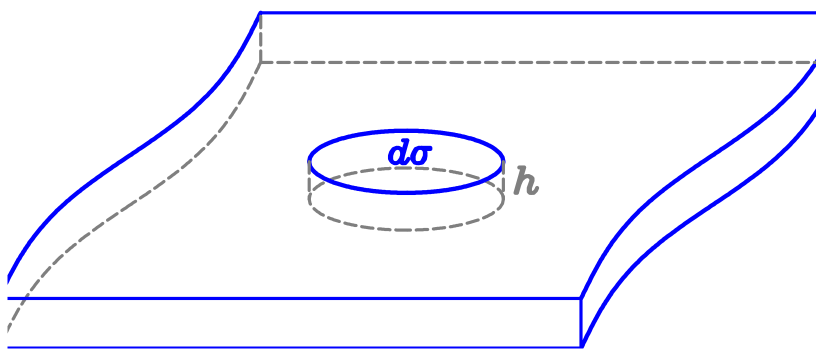

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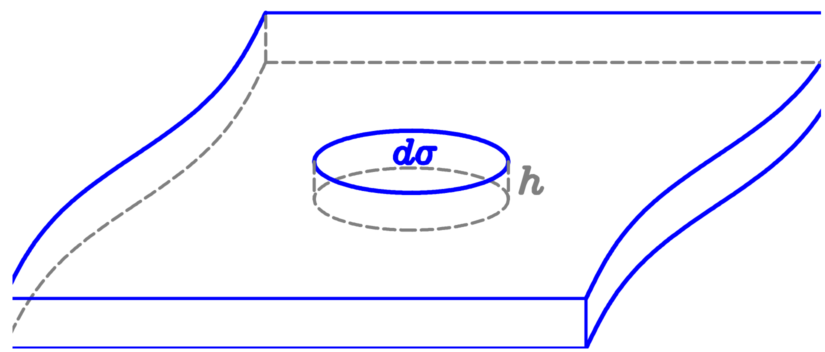

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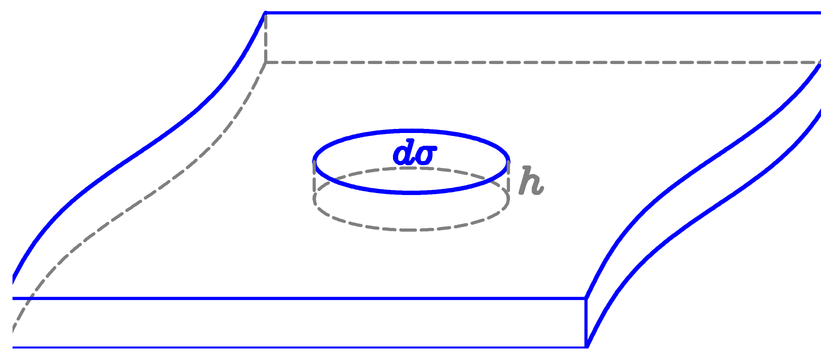

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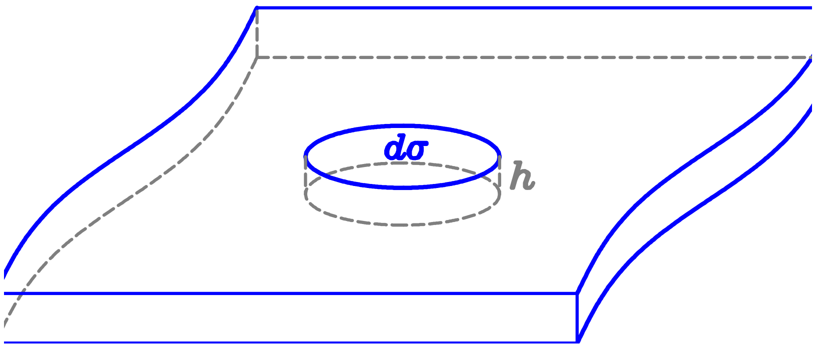

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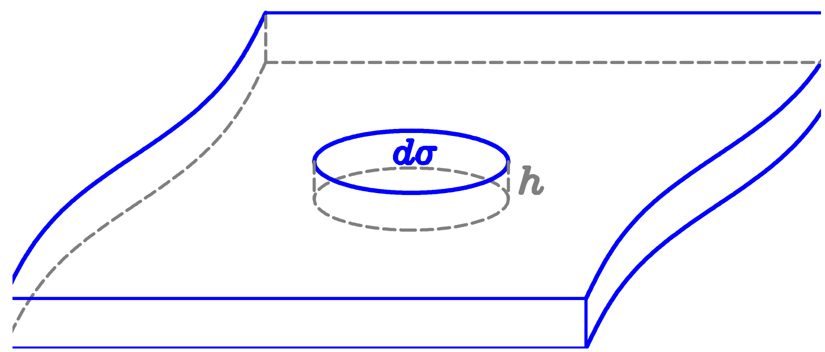

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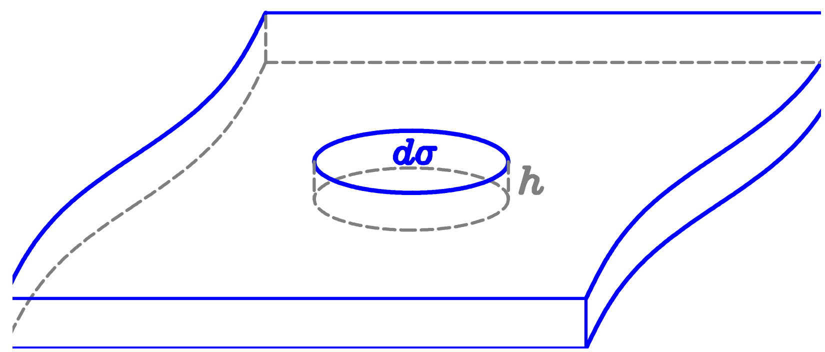

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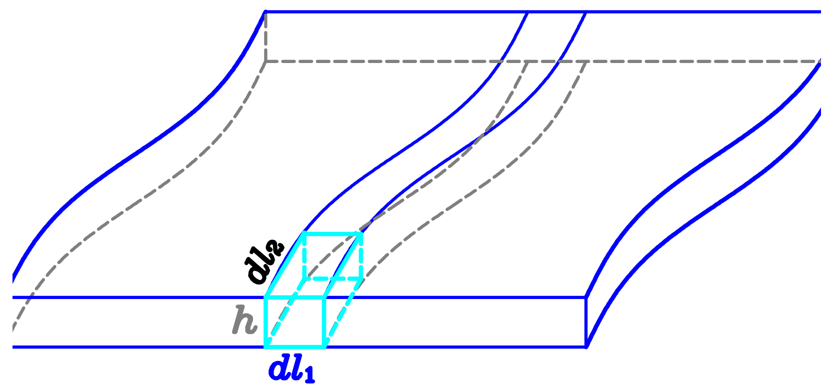

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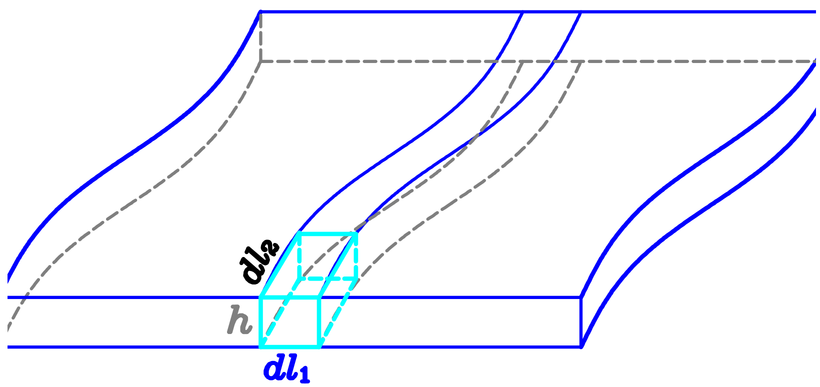

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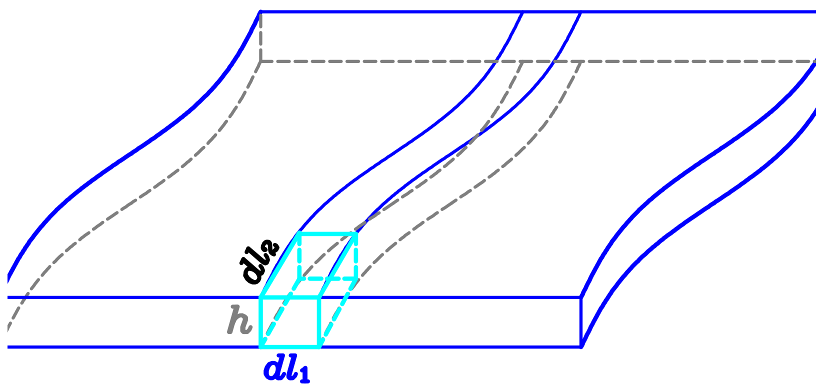

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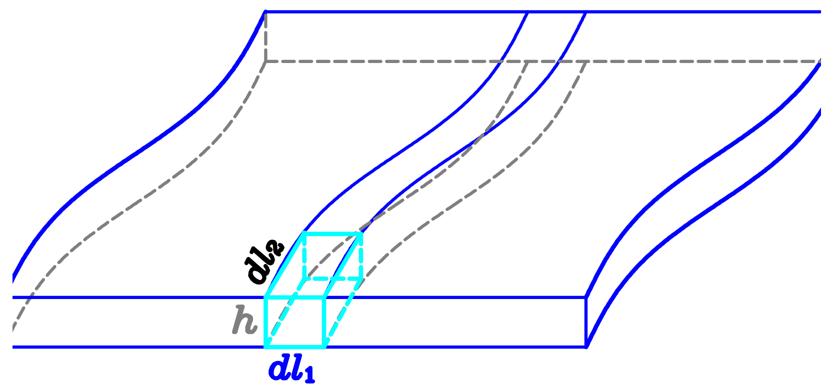

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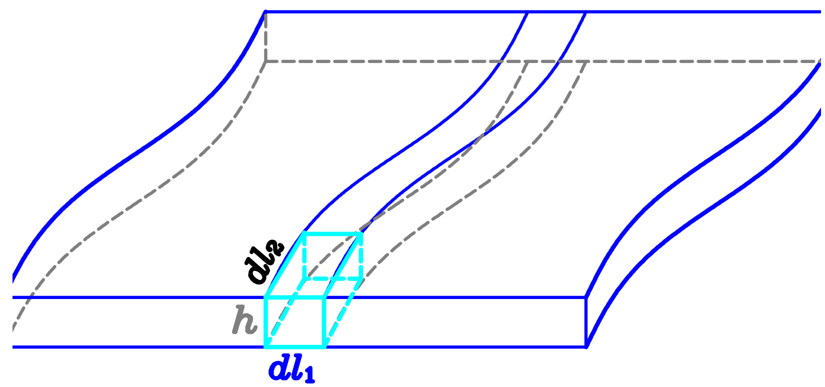

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$$= \int \vec{\alpha}\,|\nabla s|\,\delta(s)\,\mathrm{d}l_1\,\mathrm{d}l_2\,\mathrm{d}n$$

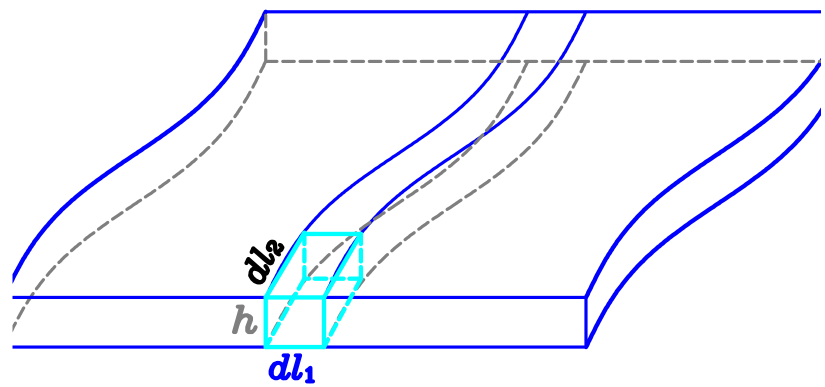
$h = \mathrm{d}n$ 沿曲面法线方向

$$\mathrm{d}n = \frac{\mathrm{d}n}{\mathrm{d}s}\mathrm{d}s = \Bigg[\underbrace{\frac{\mathrm{d}s}{\mathrm{d}n}}_{\text{方向导数}}\Bigg]^{-1}\mathrm{d}s$$

6. 用体电流密度描述面电流分布：

$$\vec{j}(\vec{r}) = \vec{\alpha}\,|\nabla s|\,\delta(s)$$

$s(x,y,z)=0$ 为面电流所在的曲面方程



必须证明：$\vec{j}\,\mathrm{d}\tau = \vec{\alpha}\,\mathrm{d}\sigma$

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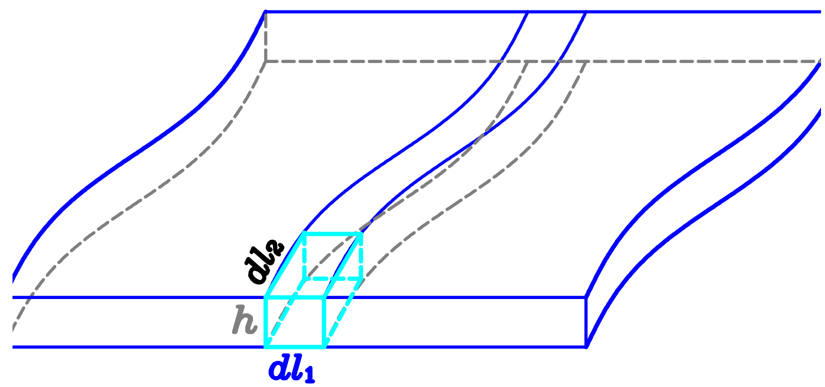

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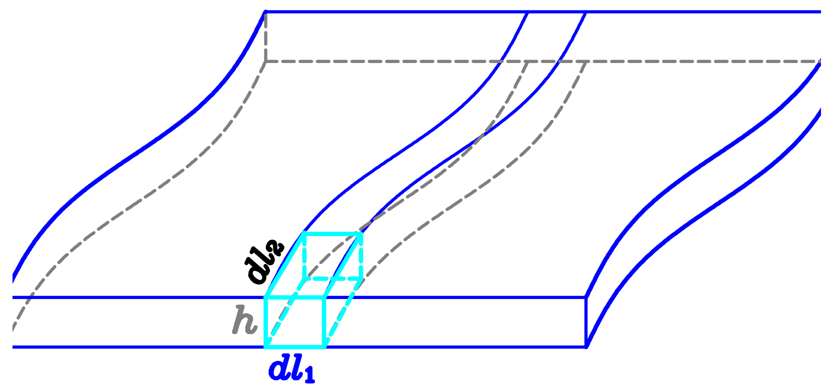

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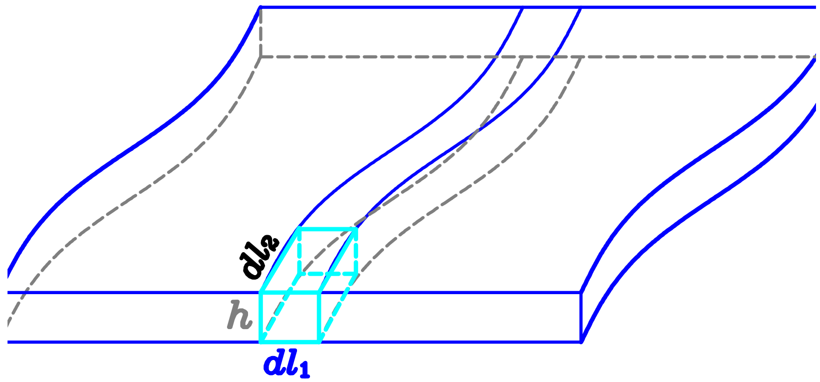

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$\mathrm{d}l_1\mathrm{d}l_2 = \mathrm{d}\sigma$ 为曲面上的小面积元

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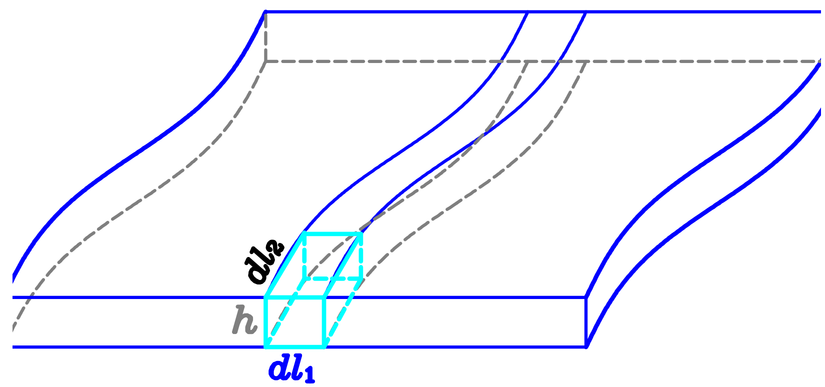

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