§4.4 静磁场矢势的多极矩展开

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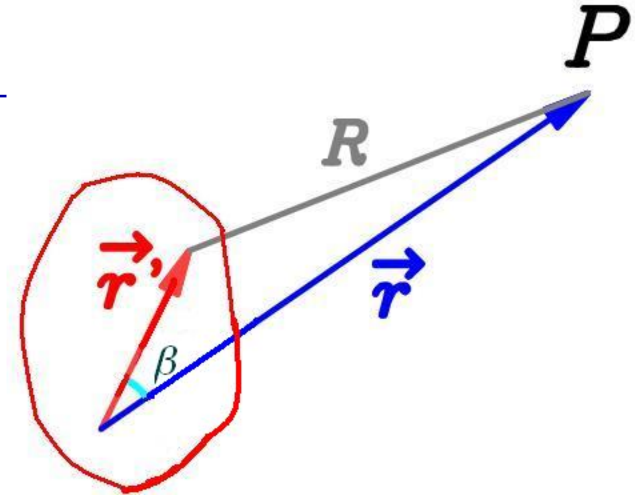
矢势的多极展开:



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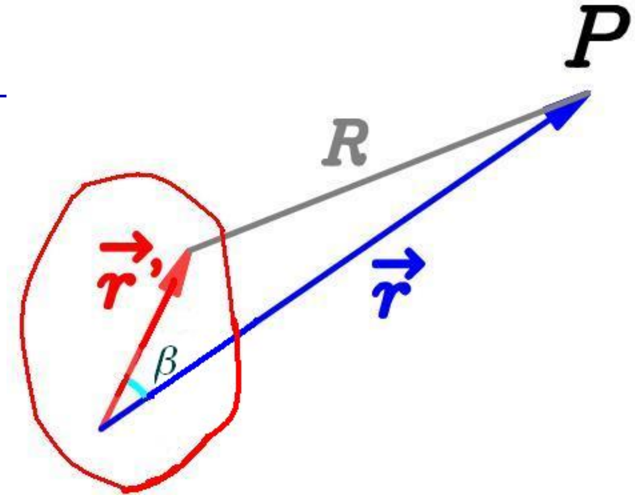

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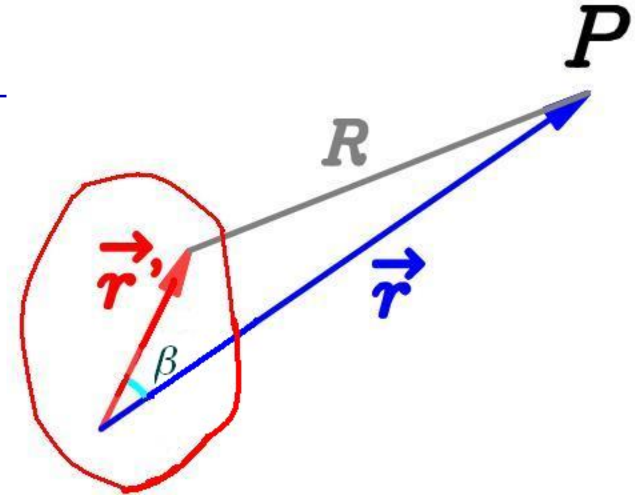

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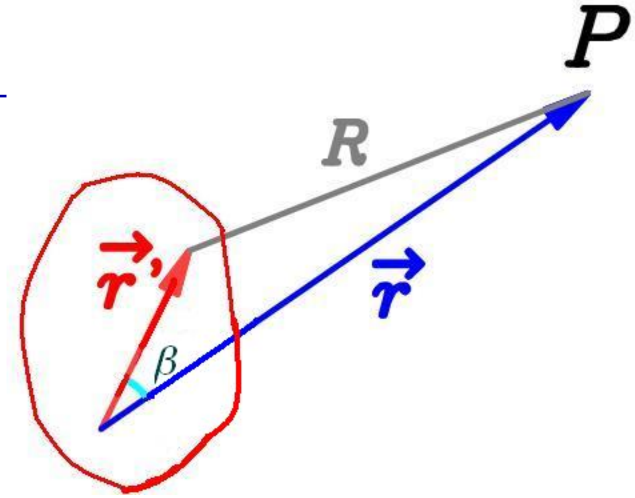

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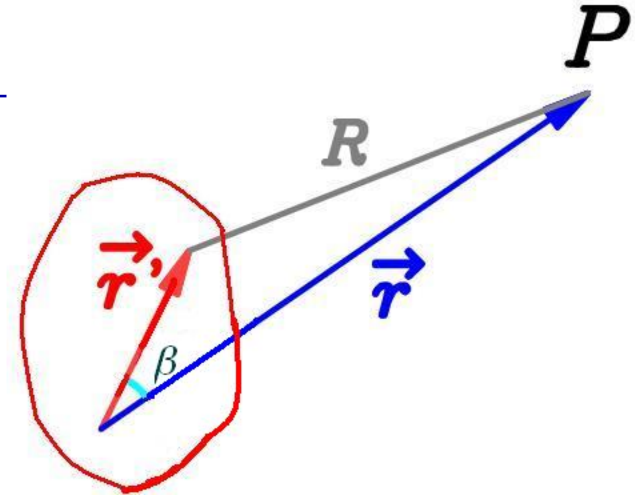

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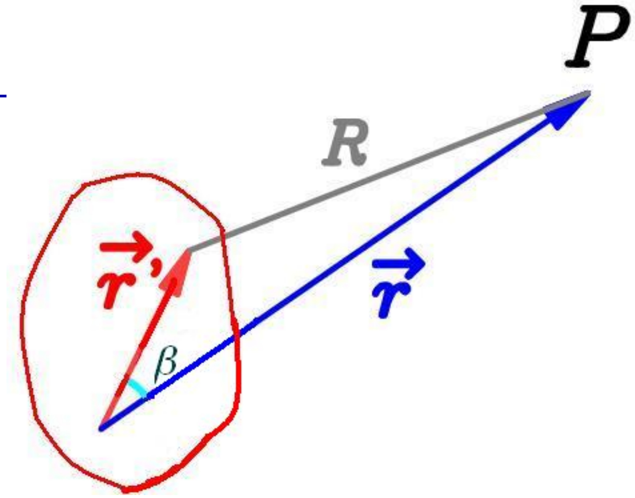

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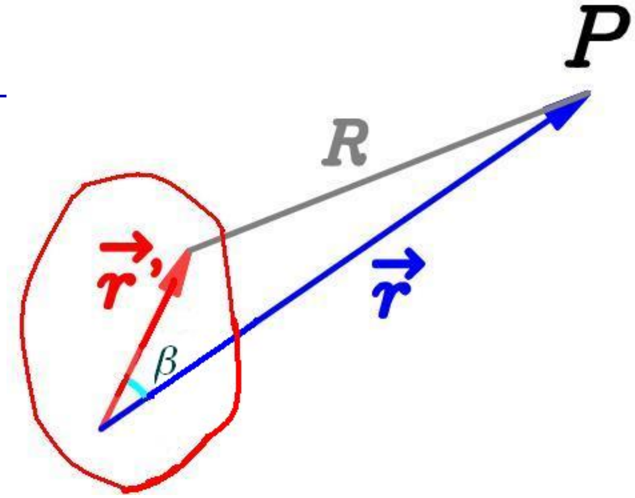

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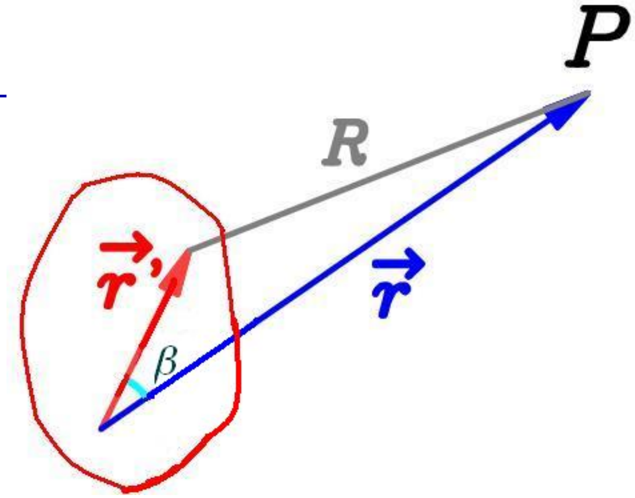

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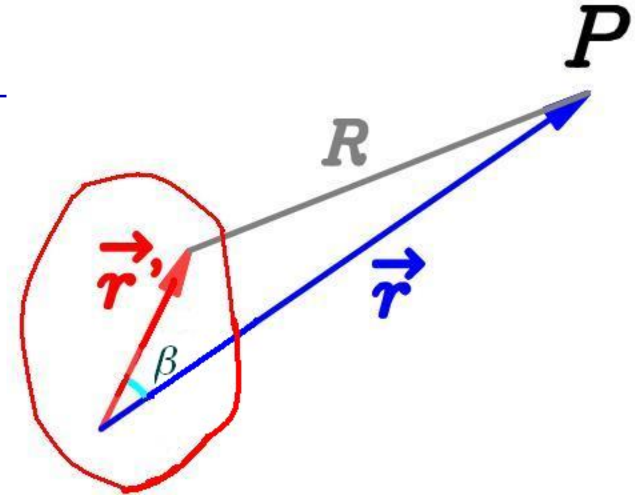
单极项 (monopole term) $n = 0$ 项， 利用 $P_0(x) = 1$



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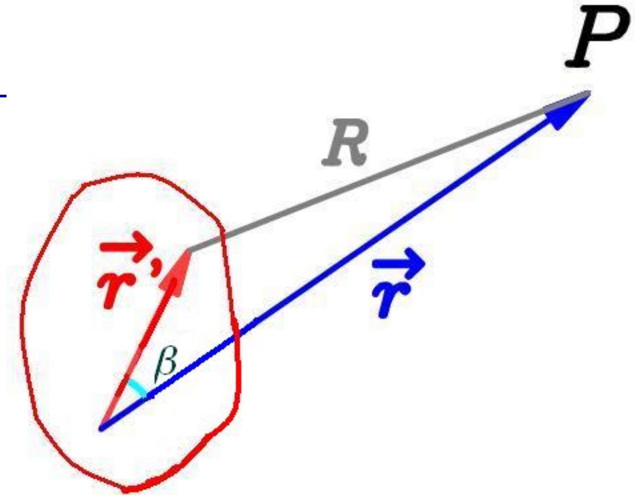

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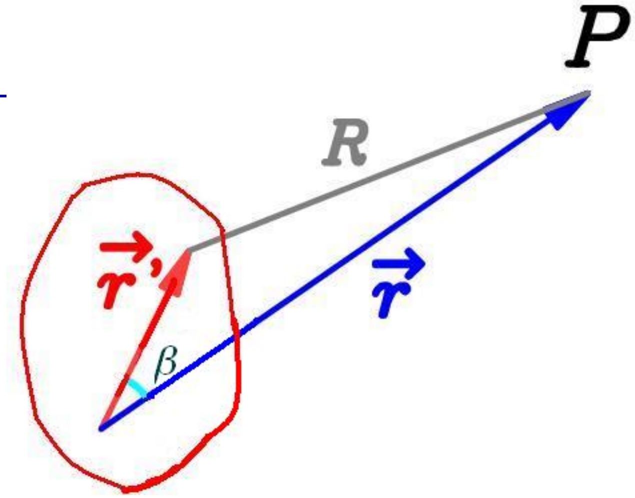

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对稳定电流 $\int \vec{j}(\vec{r}\,')\,\mathrm{d}\tau' = 0$ 稳定电流满足的第一个数学公式 —— 教材 p30 习题 1.4

Let there be light

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\mathrm{d}\tau = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}) \cdot \overbrace{(\nabla \vec{\boldsymbol{r}})}^{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau = \int \left\{ \nabla \cdot [\,\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,] - \overbrace{[\,\nabla \cdot \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,]}^{\nabla \cdot \vec{\boldsymbol{j}} = 0}\,\vec{\boldsymbol{r}} \right\} \mathrm{d}\tau$$

Let there be light

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$$= \oint_{\mathcal{S}_\infty} \vec{\boldsymbol{n}} \cdot [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}]\,\mathrm{d}\sigma = \oint_{\mathcal{S}_\infty} j_n(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,\mathrm{d}\sigma = 0$$

对有限电流分布，边界上 $j_n = 0$

Let there be light

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\mathrm{d}\tau = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\cdot\overbrace{(\nabla\vec{\boldsymbol{r}})}^{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau = \int\Big\{\,\nabla\cdot[\,\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,] - \overbrace{[\,\nabla\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,]}^{\nabla\cdot\vec{\boldsymbol{j}}=0}\,\vec{\boldsymbol{r}}\,\Big\}\,\mathrm{d}\tau$$

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Let there be light

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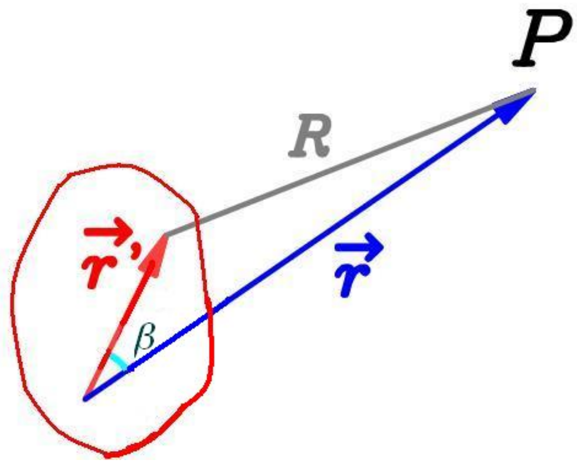


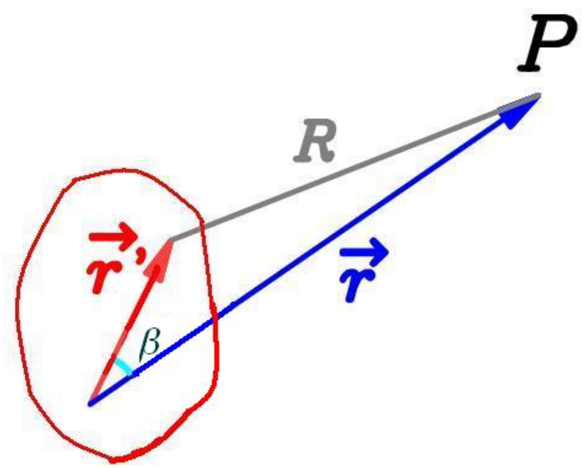
Let there be light

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\mathrm{d}\tau = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\cdot\overbrace{(\nabla\vec{\boldsymbol{r}})}^{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau = \int\{\,\nabla\cdot[\,\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,] - \overbrace{[\,\nabla\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,]}^{\nabla\cdot\vec{\boldsymbol{j}}=0}\,\vec{\boldsymbol{r}}\,\}\,\mathrm{d}\tau$$

$$= \oint_{\mathcal{S}_\infty} \vec{\boldsymbol{n}}\cdot[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}]\,\mathrm{d}\sigma = \oint_{\mathcal{S}_\infty} j_n(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,\mathrm{d}\sigma = 0$$ 对有限电流分布，边界上 $j_n = 0$

也可利用：$\nabla\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}) = (\nabla\cdot\vec{\boldsymbol{j}})\vec{\boldsymbol{r}} + (\vec{\boldsymbol{j}}\cdot\nabla)\vec{\boldsymbol{r}} = \vec{\boldsymbol{j}}$ 再两边积分

多极矩展开：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{1}{r}\sum_{n=0}^{\infty}\int\left(\dfrac{r'}{r}\right)^n P_n(\cos\beta)\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$



$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\mathrm{d}\tau = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\cdot\overbrace{(\nabla\vec{\boldsymbol{r}})}^{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau = \int \{\ \nabla\cdot[\,\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,\vec{\boldsymbol{r}}\,] - \overbrace{[\,\nabla\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})\,]}^{\nabla\cdot\vec{\boldsymbol{j}}=0}\,\vec{\boldsymbol{r}}\ \}\,\mathrm{d}\tau$$

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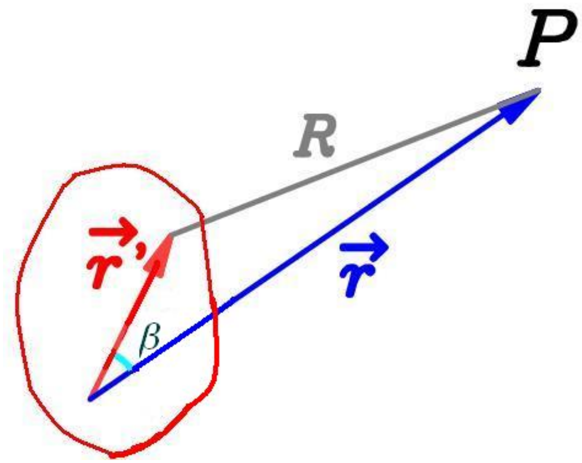

偶极项 (dipole term) $n = 1$ 项， 利用 $P_1(x) = x$

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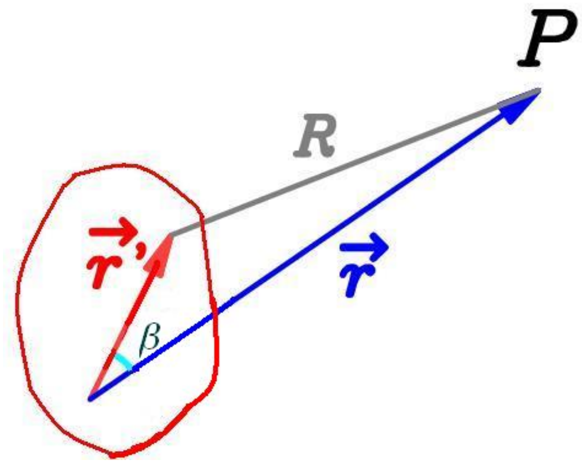

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$$\vec{\boldsymbol{A}}_1(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{1}{r}\int\frac{r'}{r}\cos\beta\,\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

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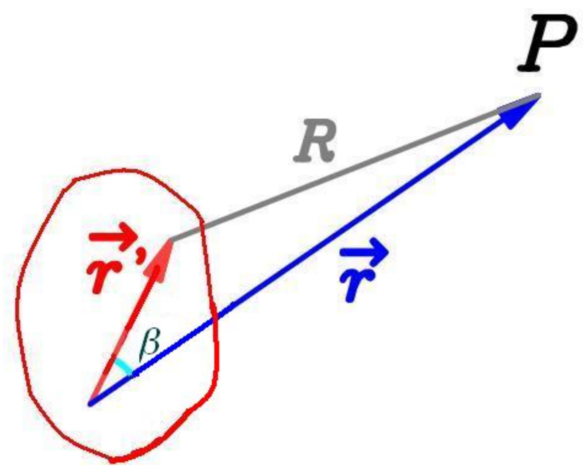

偶极项 (dipole term) $n = 1$ 项， 利用 $P_1(x) = x$

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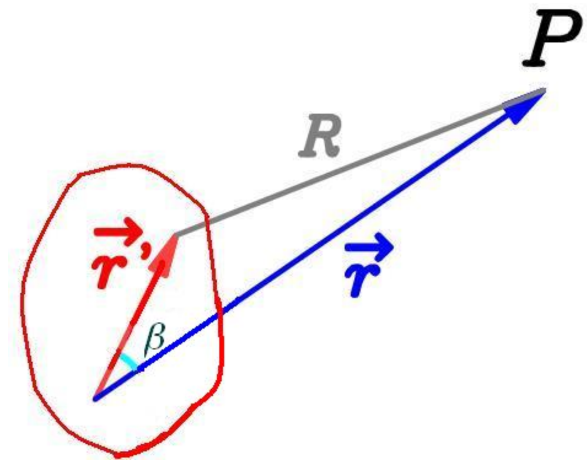

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$$\int \vec{j}(\vec{r})\,\mathrm{d}\tau = \int \vec{j}(\vec{r}) \cdot \overbrace{(\nabla \vec{r})}^{\overleftrightarrow{I}} \,\mathrm{d}\tau = \int \{\, \nabla \cdot [\,\vec{j}(\vec{r})\,\vec{r}\,] - \overbrace{[\,\nabla \cdot \vec{j}(\vec{r})\,]}^{\nabla\cdot\vec{j}=0}\,\vec{r}\,\}\,\mathrm{d}\tau$$

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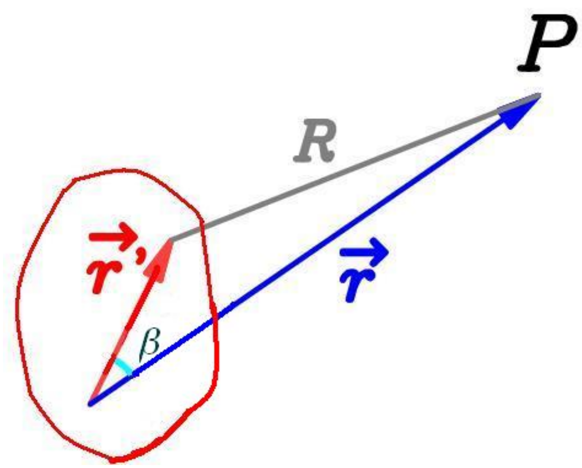

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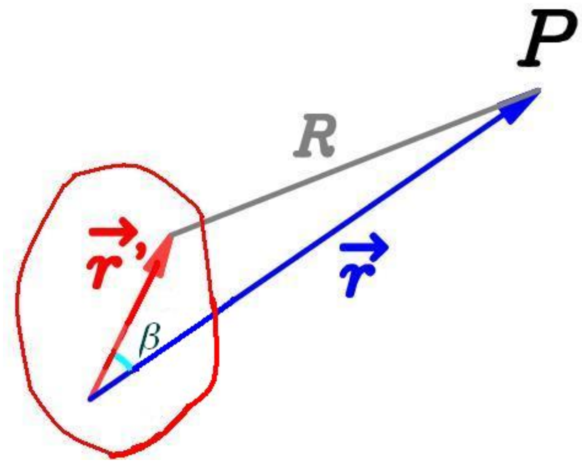

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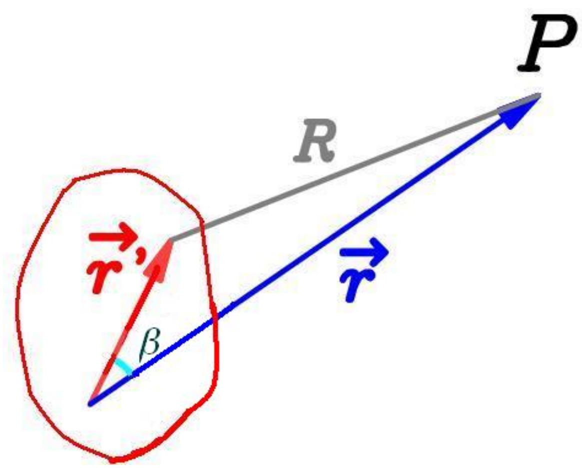

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$$\vec{\boldsymbol{A}}_1(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r - \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\mathsf{d}\tau'$$

$$
\begin{aligned}
\vec{\boldsymbol{A}}_1(\vec{\boldsymbol{r}}) &= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r-\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\mathrm{d}\tau' \\
&= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,(\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{j}}-(\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{r}}\,'\,\right]\mathrm{d}\tau'
\end{aligned}
$$

$$
\begin{aligned}
\vec{\boldsymbol{A}}_1(\vec{\boldsymbol{r}}) &= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r - \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\mathrm{d}\tau' \\
&= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\Big[\,(\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{j}} - (\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{r}}\,'\,\Big]\,\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\Big[\,\hat{\boldsymbol{e}}_r\times(\vec{\boldsymbol{j}}\times\vec{\boldsymbol{r}}\,')\,\Big]\,\mathrm{d}\tau'
\end{aligned}
$$

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$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,(\vec{\boldsymbol{r}}\,'\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{j}} - (\vec{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_r)\,\vec{\boldsymbol{r}}\,'\,\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,\hat{\boldsymbol{e}}_r\times(\vec{\boldsymbol{j}}\times\vec{\boldsymbol{r}}\,')\,\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{\boldsymbol{r}}\,'\times\vec{\boldsymbol{j}})\,\mathrm{d}\tau'\right]}_{\vec{\boldsymbol{m}}}\times\hat{\boldsymbol{e}}_r$$

$$\begin{aligned}
\vec{A}_1(\vec{r}) &= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau' \\
&= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\,\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,\hat{e}_r\times(\vec{j}\times\vec{r}\,')\,\right]\mathrm{d}\tau' \\
&= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j}\,)\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}
\end{aligned}$$

$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\,\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,\hat{e}_r\times(\vec{j}\times\vec{r}\,')\,\right]\mathrm{d}\tau'$$

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偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩：

$$\boxed{\vec{m} = \frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$$

$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int[\,(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\,]\,\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int[\,\hat{e}_r\times(\vec{j}\times\vec{r}\,')\,]\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩： $\boxed{\vec{m} = \dfrac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流：

$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,d\tau'$$

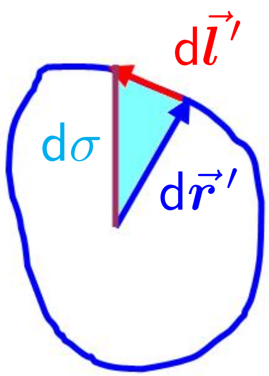
$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]d\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]d\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,d\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项：$\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$， 一般电流分布的磁偶极矩：

$$\vec{m} = \frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,d\tau'$$

对线电流：



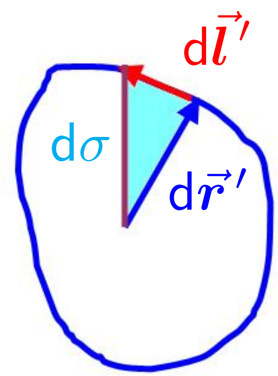
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}'\cdot\hat{e}_r - \vec{r}'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$， 一般电流分布的磁偶极矩： $\boxed{\vec{m} = \dfrac{1}{2}\int(\vec{r}'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流：$\vec{m} =$



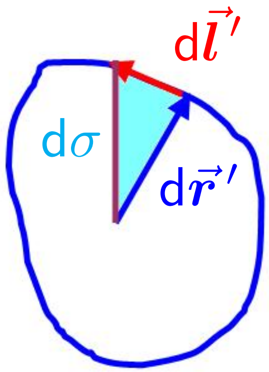
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$， 一般电流分布的磁偶极矩： $\boxed{\vec{m} = \dfrac{1}{2}\displaystyle\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流： $\vec{m} = \dfrac{1}{2}\displaystyle\oint\vec{r}\,'\times I\mathrm{d}\vec{l}\,'$



$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int (\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

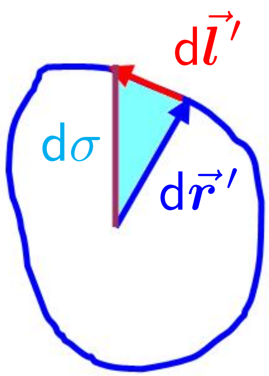
$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项：$\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩：

$$\boxed{\vec{m} = \frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$$

对线电流：$\vec{m} = \dfrac{1}{2}\oint \vec{r}\,'\times I\mathrm{d}\vec{l}\,' = I\oint\dfrac{1}{2}\vec{r}\,'\times\mathrm{d}\vec{l}\,'$



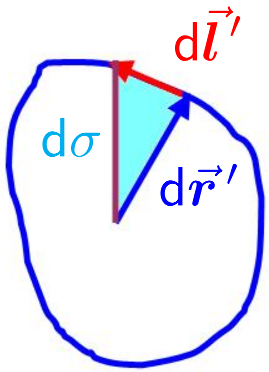
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩：$\boxed{\vec{m} = \dfrac{1}{2}\displaystyle\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流：$\vec{m} = \dfrac{1}{2}\displaystyle\oint\vec{r}\,'\times I\mathrm{d}\vec{l}\,' = I\oint\frac{1}{2}\vec{r}\,'\times\mathrm{d}\vec{l}\,' = I\oint\vec{n}\,\mathrm{d}\sigma$



$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

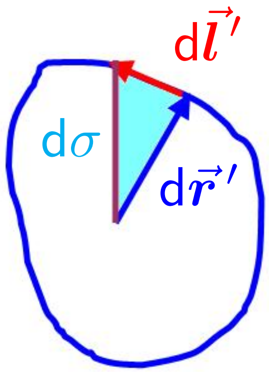
$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩：

$$\boxed{\vec{m} = \frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$$

对线电流：$\vec{m} = \dfrac{1}{2}\oint\vec{r}\,'\times I\mathrm{d}\vec{l}\,' = I\oint\dfrac{1}{2}\vec{r}\,'\times\mathrm{d}\vec{l}\,' = I\oint\vec{n}\,\mathrm{d}\sigma$



任意电流分布，远处矢势：$\vec{A}(\vec{r}) = \underbrace{\dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}}_{\sim\frac{1}{r}\frac{d}{r}} + o\left[\dfrac{1}{r}\left(\dfrac{d}{r}\right)^2\right]$

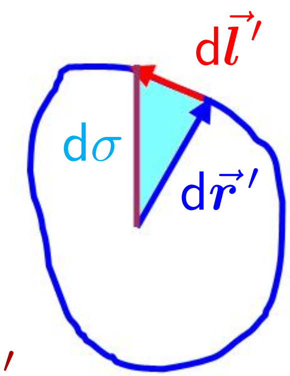
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项：$\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，一般电流分布的磁偶极矩：$\boxed{\vec{m} = \dfrac{1}{2}\displaystyle\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流：$\vec{m} = \dfrac{1}{2}\displaystyle\oint\vec{r}\,'\times I\mathrm{d}\vec{l}\,' = I\oint\frac{1}{2}\vec{r}\,'\times\mathrm{d}\vec{l}\,' = I\oint\vec{n}\,\mathrm{d}\sigma$



任意电流分布，远处矢势：$\vec{A}(\vec{r}) = \underbrace{\dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}}_{\sim\frac{1}{r}\frac{d}{r}} + o\left[\dfrac{1}{r}\left(\dfrac{d}{r}\right)^2\right]$

$$\vec{m} = \tfrac{1}{2}\int\vec{r}\,'\times\vec{j}\,\mathrm{d}\tau'$$

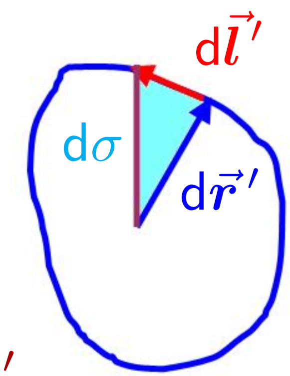
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}\,'\cdot\hat{e}_r - \vec{r}\,'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}\,'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}\,'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\,\hat{e}_r\times(\vec{j}\times\vec{r}\,')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项：$\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$，　一般电流分布的磁偶极矩：$\boxed{\vec{m} = \dfrac{1}{2}\displaystyle\int(\vec{r}\,'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流：$\vec{m} = \dfrac{1}{2}\displaystyle\oint\vec{r}\,'\times I\mathrm{d}\vec{l}\,' = I\oint\frac{1}{2}\vec{r}\,'\times\mathrm{d}\vec{l}\,' = I\oint\vec{n}\,\mathrm{d}\sigma$



任意电流分布，远处矢势：$\vec{A}(\vec{r}) = \underbrace{\dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}}_{\sim\frac{1}{r}\frac{d}{r}} + o\left[\dfrac{1}{r}\left(\dfrac{d}{r}\right)^2\right]$

$$\vec{m} = \tfrac{1}{2}\int\vec{r}\,'\times\vec{j}\,\mathrm{d}\tau'$$

远处静场：$\vec{E}\sim\dfrac{1}{r^2}$，　$\vec{B}\sim\dfrac{1}{r^3}$

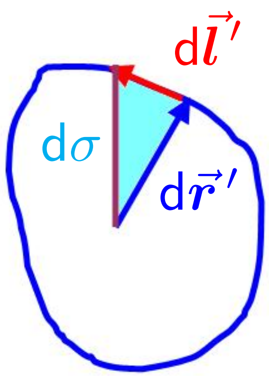
$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int(\vec{j}\,\vec{r}'\cdot\hat{e}_r - \vec{r}'\vec{j}\cdot\hat{e}_r)\,\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[(\vec{r}'\cdot\hat{e}_r)\,\vec{j} - (\vec{j}\cdot\hat{e}_r)\,\vec{r}'\right]\mathrm{d}\tau' = \frac{\mu_0}{4\pi}\frac{1}{r^2}\frac{1}{2}\int\left[\hat{e}_r\times(\vec{j}\times\vec{r}')\right]\mathrm{d}\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^2}\underbrace{\left[\frac{1}{2}\int(\vec{r}'\times\vec{j})\,\mathrm{d}\tau'\right]}_{\vec{m}}\times\hat{e}_r = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

偶极项： $\vec{A}_1(\vec{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}$， 一般电流分布的磁偶极矩： $\boxed{\vec{m} = \dfrac{1}{2}\displaystyle\int(\vec{r}'\times\vec{j})\,\mathrm{d}\tau'}$

对线电流： $\vec{m} = \dfrac{1}{2}\displaystyle\oint\vec{r}'\times I\mathrm{d}\vec{l}' = I\oint\dfrac{1}{2}\vec{r}'\times\mathrm{d}\vec{l}' = I\oint\vec{n}\,\mathrm{d}\sigma$



任意电流分布，远处矢势： $\vec{A}(\vec{r}) = \underbrace{\dfrac{\mu_0}{4\pi}\dfrac{\vec{m}\times\vec{r}}{r^3}}_{\sim\frac{1}{r}\frac{d}{r}} + o\left[\dfrac{1}{r}\left(\dfrac{d}{r}\right)^2\right]$

$\vec{m} = \frac{1}{2}\displaystyle\int\vec{r}'\times\vec{j}\,\mathrm{d}\tau'$

远处静场： $\vec{E}\sim\dfrac{1}{r^2}$， $\vec{B}\sim\dfrac{1}{r^3}$ 　物理原因： 无磁单极

下证明 $$\int (\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}})\,\mathrm{d}\tau' = 0$$ 稳定电流满足的第二个数学公式

$\vec{\boldsymbol{a}}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,'\right] = \underbrace{\left[\nabla'(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}\,')\right]}_{\vec{\boldsymbol{a}}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}\,') + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}\,')\underbrace{(\nabla'\cdot\vec{\boldsymbol{j}})}_{0}\,\vec{\boldsymbol{r}}\,' + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}\,')(\vec{\boldsymbol{j}}\cdot\nabla')\,\vec{\boldsymbol{r}}\,'$$

下证明 $$\int (\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0$$ 稳定电流满足的第二个数学公式

$\vec{a}$ 为任意常矢量

$$\nabla' \cdot \left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \underbrace{[\nabla'(\vec{a}\cdot\vec{r}\,')]}_{\vec{a}}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\underbrace{(\nabla'\cdot\vec{j})}_{0}\,\vec{r}\,' + (\vec{a}\cdot\vec{r}\,')(\vec{j}\cdot\nabla')\,\vec{r}\,'$$

$$\nabla' \cdot \left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\,\vec{j}\cdot\underbrace{(\nabla'\vec{r}\,')}_{\overleftrightarrow{I}}$$

下证明 $\int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0$ 稳定电流满足的第二个数学公式

$\vec{a}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \underbrace{[\nabla'(\vec{a}\cdot\vec{r}\,')]}_{\vec{a}}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\underbrace{(\nabla'\cdot\vec{j})}_{0}\,\vec{r}\,' + (\vec{a}\cdot\vec{r}\,')(\vec{j}\cdot\nabla')\,\vec{r}\,'$$

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\,\vec{j}\cdot\underbrace{(\nabla'\vec{r}\,')}_{\overleftrightarrow{I}} = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + \vec{a}\cdot(\vec{r}\,'\vec{j})$$

下证明 $$\int (\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j}\,)\,\mathrm{d}\tau' = 0$$ 稳定电流满足的第二个数学公式

$\vec{a}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \underbrace{[\nabla'(\vec{a}\cdot\vec{r}\,')]}_{\vec{a}}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\underbrace{(\nabla'\cdot\vec{j}\,)}_{0}\,\vec{r}\,' + (\vec{a}\cdot\vec{r}\,')(\vec{j}\cdot\nabla')\,\vec{r}\,'$$

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\,\vec{j}\cdot\underbrace{(\nabla'\vec{r}\,')}_{\overleftrightarrow{I}} = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + \vec{a}\cdot(\vec{r}\,'\vec{j}\,)$$

两边同时积分

下证明 $\displaystyle\int (\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}})\,\mathrm{d}\tau' = 0$ 稳定电流满足的第二个数学公式

$\vec{\boldsymbol{a}}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right] = \underbrace{\left[\nabla'(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\right]}_{\vec{\boldsymbol{a}}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\underbrace{(\nabla'\cdot\vec{\boldsymbol{j}})}_{0}\,\vec{\boldsymbol{r}}' + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')(\vec{\boldsymbol{j}}\cdot\nabla')\,\vec{\boldsymbol{r}}'$$

$$\nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right] = \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}} = \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}})$$

两边同时积分

$$\text{左边} = \int \nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \oint \vec{\boldsymbol{n}}\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma = \underbrace{\oint(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,j_n\,\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma}_{0}$$

下证明 $\displaystyle\int (\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}})\,\mathrm{d}\tau' = 0$ 稳定电流满足的第二个数学公式

$\vec{\boldsymbol{a}}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right] = \underbrace{\left[\nabla'(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\right]}_{\vec{\boldsymbol{a}}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\underbrace{(\nabla'\cdot\vec{\boldsymbol{j}})}_{0}\,\vec{\boldsymbol{r}}' + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')(\vec{\boldsymbol{j}}\cdot\nabla')\,\vec{\boldsymbol{r}}'$$

$$\nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right] = \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}} = \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}') + \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}})$$

两边同时积分

$$\text{左边} = \int \nabla'\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \oint \vec{\boldsymbol{n}}\cdot\left[(\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma = \underbrace{\oint (\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{r}}')\,j_n\,\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma}_{0}$$

$$\text{右边} = \int \vec{\boldsymbol{a}}\cdot(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\,\vec{\boldsymbol{j}})\,\mathrm{d}\tau = \vec{\boldsymbol{a}}\cdot\int(\vec{\boldsymbol{j}}\,\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\,\vec{\boldsymbol{j}})\,\mathrm{d}\tau$$

下证明 $\displaystyle\int (\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0$ 稳定电流满足的第二个数学公式

$\vec{a}$ 为任意常矢量

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \underbrace{\left[\nabla'(\vec{a}\cdot\vec{r}\,')\right]}_{\vec{a}}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\underbrace{(\nabla'\cdot\vec{j})}_{0}\,\vec{r}\,' + (\vec{a}\cdot\vec{r}\,')(\vec{j}\cdot\nabla')\,\vec{r}\,'$$

$$\nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\vec{j}\,\vec{r}\,'\right] = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + (\vec{a}\cdot\vec{r}\,')\,\vec{j}\cdot\underbrace{(\nabla'\vec{r}\,')}_{\overleftrightarrow{I}} = \vec{a}\cdot(\vec{j}\,\vec{r}\,') + \vec{a}\cdot(\vec{r}\,'\vec{j})$$

两边同时积分

$$\text{左边} = \int \nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\,\vec{j}\,\vec{r}\,'\right]\mathrm{d}\tau' = \oint \vec{n}\cdot\left[(\vec{a}\cdot\vec{r}\,')\,\vec{j}\,\vec{r}\,'\right]\mathrm{d}\sigma = \underbrace{\oint(\vec{a}\cdot\vec{r}\,')\,j_n\,\vec{r}\,'\,\mathrm{d}\sigma}_{0}$$

$$\text{右边} = \int \vec{a}\cdot(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau = \vec{a}\cdot\int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau$$

$$\text{左边} = \text{右边} \quad\Longrightarrow\quad \int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0 \qquad \text{qed}$$

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两边同时积分

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$$\text{右边} = \int \vec{a}\cdot(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau = \vec{a}\cdot\int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau$$

$$\text{左边} = \text{右边} \quad\Longrightarrow\quad \int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0$$ qed

如了解三阶张量，则：$\nabla\cdot(\vec{j}\,\vec{r}\,\vec{r}) = (\nabla\cdot\vec{j})\,\vec{r}\,\vec{r} + [(\vec{j}\cdot\nabla)\vec{r}]\vec{r} + \vec{r}[(\vec{j}\cdot\nabla)\vec{r}] = \vec{j}\,\vec{r} + \vec{r}\,\vec{j}$

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两边同时积分

左边 $$= \int \nabla'\cdot\left[(\vec{a}\cdot\vec{r}\,')\,\vec{j}\,\vec{r}\,'\right]\mathrm{d}\tau' = \oint \vec{n}\cdot\left[(\vec{a}\cdot\vec{r}\,')\,\vec{j}\,\vec{r}\,'\right]\mathrm{d}\sigma = \underbrace{\oint (\vec{a}\cdot\vec{r}\,')\,j_n\,\vec{r}\,'\,\mathrm{d}\sigma}_{0}$$

右边 $$= \int \vec{a}\cdot(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau = \vec{a}\cdot\int(\vec{j}\,\vec{r}\,' + \vec{r}\,'\,\vec{j})\,\mathrm{d}\tau$$

左边 $=$ 右边 $$\Longrightarrow \int (\vec{j}\,\vec{r}\,' + \vec{r}\,'\vec{j})\,\mathrm{d}\tau' = 0$$ qed

如了解三阶张量，则：$\nabla\cdot(\vec{j}\,\vec{r}\,\vec{r}) = (\nabla\cdot\vec{j})\,\vec{r}\,\vec{r} + [(\vec{j}\cdot\nabla)\vec{r}]\vec{r} + \vec{r}[(\vec{j}\cdot\nabla)\vec{r}] = \vec{j}\,\vec{r} + \vec{r}\,\vec{j}$

即：$\nabla\cdot(\vec{j}\,\vec{r}\,\vec{r}) = \vec{r}\,\vec{j} + \vec{j}\,\vec{r}$，两边积分即得：$\int(\vec{j}\,\vec{r} + \vec{r}\vec{j})\,\mathrm{d}\tau = 0$