§ 6.3 谐变电荷电流分布的多极辐射

§6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

§6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

§6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{r},t)=\rho(\vec{r})e^{-i\omega t}$，$\quad \vec{j}(\vec{r},t)=\vec{j}(\vec{r})e^{-i\omega t}$ (实际电荷电流需取实部)

§6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{\boldsymbol{j}}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{\boldsymbol{r}},t)=\rho(\vec{\boldsymbol{r}})e^{-i\omega t}$，$\quad \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}},t)=\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})e^{-i\omega t}$ (实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{\boldsymbol{j}}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{\boldsymbol{j}}=i\omega\rho$

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{\boldsymbol{j}}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{\boldsymbol{r}},t)=\rho(\vec{\boldsymbol{r}})e^{-i\omega t}$， $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}},t)=\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})e^{-i\omega t}$ (实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{\boldsymbol{j}}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{\boldsymbol{j}}=i\omega\rho$

对谐变源，给定电流分布 $\vec{\boldsymbol{j}}$，电荷分布 ρ 也即给定。

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{r},t)=\rho(\vec{r})e^{-i\omega t}$，$\vec{j}(\vec{r},t)=\vec{j}(\vec{r})e^{-i\omega t}$ (实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{j}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{j}=i\omega\rho$

对谐变源，给定电流分布 $\vec{j}$，电荷分布 ρ 也即给定。

如果只研究所谓定态 (steady state) 问题，势和场也是谐变的。

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{\boldsymbol{j}}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{\boldsymbol{r}}, t) = \rho(\vec{\boldsymbol{r}})e^{-i\omega t}$，　$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}, t) = \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}})e^{-i\omega t}$　　(实际电荷电流需取实部)

电荷守恒：$\nabla \cdot \vec{\boldsymbol{j}} + \dfrac{\partial \rho}{\partial t} = 0 \quad \Longrightarrow \quad \nabla \cdot \vec{\boldsymbol{j}} = i\omega\rho$

对谐变源，给定电流分布 $\vec{\boldsymbol{j}}$，电荷分布 ρ 也即给定。

如果只研究所谓定态 (steady state) 问题，势和场也是谐变的。

由洛仑兹规范：$\nabla \cdot \vec{\boldsymbol{A}} + \dfrac{1}{c^2}\dfrac{\partial \varphi}{\partial t} = 0 \quad \Longrightarrow \quad \varphi = \dfrac{c^2}{i\omega}\nabla \cdot \vec{\boldsymbol{A}}$

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{r},t)=\rho(\vec{r})e^{-i\omega t}$，$\quad \vec{j}(\vec{r},t)=\vec{j}(\vec{r})e^{-i\omega t}$ (实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{j}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{j}=i\omega\rho$

对谐变源，给定电流分布 $\vec{j}$，电荷分布 ρ 也即给定。

如果只研究所谓定态 (steady state) 问题，势和场也是谐变的。

由洛仑兹规范：$\nabla\cdot\vec{A}+\dfrac{1}{c^2}\dfrac{\partial\varphi}{\partial t}=0 \quad\Longrightarrow\quad \varphi=\dfrac{c^2}{i\omega}\nabla\cdot\vec{A}$

对谐变源的定态解，求得矢势即可求标势

§6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{r},t)=\rho(\vec{r})e^{-i\omega t}$，　$\vec{j}(\vec{r},t)=\vec{j}(\vec{r})e^{-i\omega t}$　　(实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{j}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{j}=i\omega\rho$

对谐变源，给定电流分布 $\vec{j}$，电荷分布 ρ 也即给定。

如果只研究所谓定态 (steady state) 问题，势和场也是谐变的。

由洛仑兹规范：$\nabla\cdot\vec{A}+\dfrac{1}{c^2}\dfrac{\partial\varphi}{\partial t}=0 \quad\Longrightarrow\quad \varphi=\dfrac{c^2}{i\omega}\nabla\cdot\vec{A}$

对谐变源的定态解，求得矢势即可求标势

从而电磁场由 $\vec{B}=\nabla\times\vec{A}$，　$\vec{E}=-\nabla\varphi-\dfrac{\partial\vec{A}}{\partial t}$ 确定。

§ 6.3 谐变电荷电流分布的多极辐射

一般电荷电流分布的场归结为推迟势的计算。对任意随时间变化的 ρ，$\vec{j}$，总可以通过 Fourier 分析，分解为谐变分量的叠加，本节讨论谐变电荷电流分布的辐射场。

一、谐变电荷、电流系统辐射场的势

谐变：$\rho(\vec{r},t)=\rho(\vec{r})e^{-i\omega t}$，　$\vec{j}(\vec{r},t)=\vec{j}(\vec{r})e^{-i\omega t}$　(实际电荷电流需取实部)

电荷守恒：$\nabla\cdot\vec{j}+\dfrac{\partial\rho}{\partial t}=0 \quad\Longrightarrow\quad \nabla\cdot\vec{j}=i\omega\rho$

对谐变源，给定电流分布 $\vec{j}$，电荷分布 ρ 也即给定。

如果只研究所谓定态 (steady state) 问题，势和场也是谐变的。

由洛仑兹规范：$\nabla\cdot\vec{A}+\dfrac{1}{c^2}\dfrac{\partial\varphi}{\partial t}=0 \quad\Longrightarrow\quad \varphi=\dfrac{c^2}{i\omega}\nabla\cdot\vec{A}$

对谐变源的定态解，求得矢势即可求标势

从而电磁场由 $\vec{B}=\nabla\times\vec{A}$，　$\vec{E}=-\nabla\varphi-\dfrac{\partial\vec{A}}{\partial t}$ 确定。

求谐变源的定态解问题归结为从电流分布求矢势问题：$\vec{j} \quad\Longrightarrow\quad \vec{A}$

一般电荷电流分布的推迟势：

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}, t) \quad = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t_r)}{|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}\,'|} \, \mathrm{d}\tau',$$

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) \quad = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\,\mathrm{d}\tau',$$

$$t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau',$$

$$t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\mathrm{d}\tau',$$

$\vec{\boldsymbol{R}} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}\,'$, $\quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|$, $\quad k = \omega/c$

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\mathrm{d}\tau',$$

$\vec{\boldsymbol{R}} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}\,', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|, \quad k = \omega/c$

e^{ikR}:

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\,\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\,\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau', \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|, \quad k = \omega/c$$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\mathrm{d}\tau', \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|, \quad k = \omega/c$$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}\,'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

分区：

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\mathrm{d}\tau', \qquad t_r = t-\frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}{c} = t-\frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\mathrm{d}\tau',$$

定态：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\mathrm{d}\tau',$$

$\vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'$, $R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|$, $k = \omega/c$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

分区：

(1) $R \ll \lambda = \dfrac{2\pi}{k}$：近区，$e^{ikR}\sim 1$ 推迟因子可略，电磁场基本与稳恒场相同

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad \text{定态：} \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\mathrm{d}\tau', \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|, \quad k = \omega/c$$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}\,'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

分区：

(1) $R \ll \lambda = \dfrac{2\pi}{k}$：近区，$e^{ikR} \sim 1$ 推迟因子可略，电磁场基本与稳恒场相同

取 $e^{ikR} \sim 1$ 的近似称为准静态近似 常用于：低频、体系波长小情况

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\mathrm{d}\tau', \qquad t_r = t-\frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}{c} = t-\frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|}\mathrm{d}\tau', \qquad \text{定态：}\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int\frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\mathrm{d}\tau', \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'|, \quad k = \omega/c$$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

分区：

(1) $R \ll \lambda = \frac{2\pi}{k}$：近区，$e^{ikR} \sim 1$　　推迟因子可略，电磁场基本与稳恒场相同

取 $e^{ikR} \sim 1$ 的近似称为准静态近似　　常用于：低频、体系波长小情况

(2) $R \sim \lambda = \frac{2\pi}{k}$：感应区　　较少讨论

一般电荷电流分布的推迟势：

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t_r)}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad t_r = t - \frac{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}{c} = t - \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{-i\omega t}e^{i\omega R/c}}{|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|}\mathrm{d}\tau', \qquad \text{定态：}\ \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}},t) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{-i\omega t}$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\mathrm{d}\tau', \qquad \vec{\boldsymbol{R}} = \vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,', \quad R = |\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}\,'|, \quad k = \omega/c$$

e^{ikR}：推迟因子，表示电磁波由源点 $\vec{\boldsymbol{r}}\,'$ 到观察点 $\vec{\boldsymbol{r}}$ 相位滞后了 kR。

分区：

(1) $R \ll \lambda = \dfrac{2\pi}{k}$：近区，$e^{ikR} \sim 1$　　推迟因子可略，电磁场基本与稳恒场相同

取 $e^{ikR} \sim 1$ 的近似称为准静态近似　　常用于：低频、体系波长小情况

(2) $R \sim \lambda = \dfrac{2\pi}{k}$：感应区　　较少讨论

(3) $R \gg \lambda = \dfrac{2\pi}{k}$：远区　　实际辐射及其应用多落于此区

二、远区辐射场的势：级数展开

二、远区辐射场的势：级数展开

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

二、远区辐射场的势：级数展开

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{r}\,'$ 视为一小量

$$\frac{e^{ikR}}{R} = \frac{e^{ikr}}{r} + \vec{r}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \frac{1}{2!}\vec{r}\,'\vec{r}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \dots$$

二、远区辐射场的势：级数展开

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{\boldsymbol{r}}\,'$ 视为一小量

$$\begin{aligned}\frac{e^{ikR}}{R} &= \frac{e^{ikr}}{r} + \vec{\boldsymbol{r}}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{\boldsymbol{r}}\,'=0} + \frac{1}{2!}\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{r}}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{\boldsymbol{r}}\,'=0} + \dots\\ &= \frac{e^{ikr}}{r} - \vec{\boldsymbol{r}}\,'\cdot\left[\nabla\frac{e^{ikr}}{r}\right] + \frac{1}{2!}\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{r}}\,':\left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \dots\end{aligned}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

二、远区辐射场的势：级数展开

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{r}\,'$ 视为一小量

$$\begin{aligned}
\frac{e^{ikR}}{R} &= \frac{e^{ikr}}{r} + \vec{r}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \frac{1}{2!}\vec{r}\,'\vec{r}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \dots \\
&= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\nabla\frac{e^{ikr}}{r}\right] + \frac{1}{2!}\vec{r}\,'\vec{r}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \dots \\
&= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\frac{ik}{r} - \frac{1}{r^2}\right]\frac{\vec{r}}{r}e^{ikr} + \frac{1}{2!}\vec{r}\,'\vec{r}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \dots
\end{aligned}$$

二、远区辐射场的势：级数展开

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{\boldsymbol{r}}\,'$ 视为一小量

$$\begin{aligned}\frac{e^{ikR}}{R} &= \frac{e^{ikr}}{r} + \vec{\boldsymbol{r}}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{\boldsymbol{r}}\,'=0} + \frac{1}{2!}\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{r}}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{\boldsymbol{r}}\,'=0} + \ldots \\ &= \frac{e^{ikr}}{r} - \vec{\boldsymbol{r}}\,'\cdot\left[\nabla\frac{e^{ikr}}{r}\right] + \frac{1}{2!}\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{r}}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \ldots \\ &= \frac{e^{ikr}}{r} - \vec{\boldsymbol{r}}\,'\cdot\left[\frac{ik}{r} - \frac{1}{r^2}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr} + \frac{1}{2!}\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{r}}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \ldots\end{aligned}$$

代入 $\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\frac{e^{ikR}}{R}\,\mathrm{d}\tau'$

二、远区辐射场的势：级数展开

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{r}\,'$ 视为一小量

$$\begin{aligned}
\frac{e^{ikR}}{R} &= \frac{e^{ikr}}{r} + \vec{r}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \frac{1}{2!}\vec{r}\,'\vec{r}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \dots \\
&= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\nabla\frac{e^{ikr}}{r}\right] + \frac{1}{2!}\vec{r}\,'\vec{r}\,':\left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \dots \\
&= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\frac{ik}{r}-\frac{1}{r^2}\right]\frac{\vec{r}}{r}e^{ikr} + \frac{1}{2!}\vec{r}\,'\vec{r}\,':\left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \dots
\end{aligned}$$

代入 $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\frac{e^{ikR}}{R}\,\mathrm{d}\tau'$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\frac{e^{ikr}}{r}\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\,\vec{r}\,'\cdot\left[\frac{ik}{r}-\frac{1}{r^2}\right]\frac{\vec{r}}{r}e^{ikr}\,\mathrm{d}\tau' + \dots$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

二、远区辐射场的势：级数展开

远区：电荷电流分布在一小区域 (坐标原点取在该小区域)，$\vec{r}\,'$ 视为一小量

$$\begin{aligned}\frac{e^{ikR}}{R} &= \frac{e^{ikr}}{r} + \vec{r}\,'\cdot\left[\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \frac{1}{2!}\vec{r}\,'\vec{r}\,'\left[\nabla'\nabla'\frac{e^{ikR}}{R}\right]\Big|_{\vec{r}\,'=0} + \ldots \\ &= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\nabla\frac{e^{ikr}}{r}\right] + \frac{1}{2!}\vec{r}\,'\vec{r}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \ldots \\ &= \frac{e^{ikr}}{r} - \vec{r}\,'\cdot\left[\frac{ik}{r} - \frac{1}{r^2}\right]\frac{\vec{r}}{r}e^{ikr} + \frac{1}{2!}\vec{r}\,'\vec{r}\,' : \left[\nabla\nabla\frac{e^{ikr}}{r}\right] + \ldots\end{aligned}$$

代入 $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\frac{e^{ikR}}{R}\,\mathrm{d}\tau'$

$$\begin{aligned}\vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\frac{e^{ikr}}{r}\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int \vec{j}(\vec{r}\,')\,\vec{r}\,'\cdot\left[\frac{ik}{r} - \frac{1}{r^2}\right]\frac{\vec{r}}{r}e^{ikr}\,\mathrm{d}\tau' + \ldots \\ &= \underbrace{\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{j}(\vec{r}\,')\,\mathrm{d}\tau'}_{\vec{A}^{(0)}(\vec{r})} + \underbrace{\frac{\mu_0}{4\pi}\left[\int \vec{j}(\vec{r}\,')\vec{r}\,'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2} - \frac{ik}{r}\right]\frac{\vec{r}}{r}e^{ikr}}_{\vec{A}^{(1)}(\vec{r})} + \ldots\end{aligned}$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

$$= \frac{\mu_0}{4\pi r}\left|\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r}-ik\right]\right| \le \frac{\mu_0\, l'}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|\left|\frac{1}{r}-ik\right|$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

$$= \frac{\mu_0}{4\pi r}\left|\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r}-ik\right]\right| \le \frac{\mu_0\, l'}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|\left|\frac{1}{r}-ik\right|$$

$$\frac{\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right|}{\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|} \le l'\left|\frac{1}{r}-ik\right|,$$

l' 为电荷电流分布的尺度

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

$$= \frac{\mu_0}{4\pi r}\left|\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r}-ik\right]\right| \leq \frac{\mu_0\, l'}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|\left|\frac{1}{r}-ik\right|$$

$$\frac{\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right|}{\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|} \leq l'\left|\frac{1}{r}-ik\right|,$$

l' 为电荷电流分布的尺度

如果：$kl' = 2\pi l'/\lambda \ll 1$ 且 $l' \ll r$ $\Longrightarrow$ $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \ll \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\ \mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

$$= \frac{\mu_0}{4\pi r}\left|\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r}-ik\right]\right| \le \frac{\mu_0\, l'}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|\left|\frac{1}{r}-ik\right|$$

$$\frac{\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right|}{\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|} \le l'\left|\frac{1}{r}-ik\right|,$$

l' 为电荷电流分布的尺度

如果：$kl' = 2\pi l'/\lambda \ll 1$ 且 $l' \ll r$ $\Longrightarrow$ $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \ll \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|$

对远区 $r \gg \lambda$，如果 $\lambda \gg l'$，则 $\vec{\boldsymbol{A}}^{(1)}$ 远小于 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$ 与 $\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 的大小

$$\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right| = \frac{\mu_0}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|$$

$$\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| = \left|\frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}\right|$$

$$= \frac{\mu_0}{4\pi r}\left|\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r}-ik\right]\right| \leq \frac{\mu_0\, l'}{4\pi r}\left|\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'\right|\left|\frac{1}{r}-ik\right|$$

$$\frac{\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right|}{\left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|} \leq l'\left|\frac{1}{r}-ik\right|,$$

l' 为电荷电流分布的尺度

如果：$kl' = 2\pi l'/\lambda \ll 1$ 且 $l' \ll r$ $\Longrightarrow$ $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \ll \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right|$

对远区 $r \gg \lambda$，如果 $\lambda \gg l'$，则 $\vec{\boldsymbol{A}}^{(1)}$ 远小于 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})$

类似地可证明：如果 $\boxed{r \gg \lambda = \frac{2\pi c}{\omega} \gg l'}$，矢势级数快速收敛。

三、电偶极辐射

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau'$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}\,')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau'$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}\,')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,',t)\cdot\nabla'\right]\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau'$

对稳恒电流 $\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}\,')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\cdot\nabla'\right]\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'$$

$$= \int \left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\vec{\boldsymbol{r}}\,'\right] - \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,', t)\right]\vec{\boldsymbol{r}}\,'\right\}\mathrm{d}\tau'$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\begin{aligned}
\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' &= \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \int \left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right] - \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\right]\vec{\boldsymbol{r}}'\right\}\mathrm{d}\tau' \\
&= \oint \vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma - \int \left[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'
\end{aligned}$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\begin{aligned}
\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' &= \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overset{\leftrightarrow}{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\big]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \int \Big\{\nabla'\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big] - \big[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\big]\vec{\boldsymbol{r}}'\Big\}\,\mathrm{d}\tau' \\
&= \oint \vec{\boldsymbol{n}}\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big]\,\mathrm{d}\sigma - \int \Big[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\Big]\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},
\end{aligned}$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})=\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\int\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'=0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)$ 有：

$$\int\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\,\mathrm{d}\tau'=\int\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau'=\int\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\cdot\nabla'\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$=\int\left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\vec{\boldsymbol{r}}'\right]-\left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\right]\vec{\boldsymbol{r}}'\right\}\mathrm{d}\tau'$$

$$=\oint\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma-\int\left[-\frac{\partial\rho(\vec{\boldsymbol{r}}',t)}{\partial t}\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$=\oint j_n(\vec{\boldsymbol{r}}',t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma+\frac{\partial}{\partial t}\int\rho(\vec{\boldsymbol{r}}',t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'=\frac{\partial\vec{\boldsymbol{p}}}{\partial t},$$

$$\boxed{\int\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}',t)\mathrm{d}\tau'=\frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{A}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{j}(\vec{r}\,')\,\mathrm{d}\tau'$

对稳恒电流 $\int \vec{j}(\vec{r}\,')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{j}(\vec{r}\,', t)$ 有：

$$\int \vec{j}(\vec{r}\,', t)\,\mathrm{d}\tau' = \int \vec{j}(\vec{r}\,', t)\cdot \underbrace{(\nabla'\vec{r}\,')}_{\overleftrightarrow{I}}\,\mathrm{d}\tau' = \int \left[\vec{j}(\vec{r}\,', t)\cdot\nabla'\right]\vec{r}\,'\,\mathrm{d}\tau'$$

$$= \int \left\{\nabla'\cdot\left[\vec{j}(\vec{r}\,', t)\vec{r}\,'\right] - \left[\nabla'\cdot\vec{j}(\vec{r}\,', t)\right]\vec{r}\,'\right\}\mathrm{d}\tau'$$

$$= \oint \vec{n}\cdot\left[\vec{j}(\vec{r}\,', t)\vec{r}\,'\right]\mathrm{d}\sigma - \int \left[-\frac{\partial\rho(\vec{r}\,', t)}{\partial t}\right]\vec{r}\,'\,\mathrm{d}\tau'$$

$$= \oint j_n(\vec{r}\,', t)\vec{r}\,'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{r}\,', t)\,\vec{r}\,'\,\mathrm{d}\tau' = \frac{\partial \vec{p}}{\partial t},$$

$$\boxed{\int \vec{j}(\vec{r}\,', t)\mathrm{d}\tau' = \frac{\partial \vec{p}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{A}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}}$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\begin{aligned}
\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' &= \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \int \left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right] - \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\right]\vec{\boldsymbol{r}}'\right\}\mathrm{d}\tau' \\
&= \oint \vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma - \int\left[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},
\end{aligned}$$

$$\boxed{\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$ 为了求电磁场，需计算 $\nabla\dfrac{e^{ikr}}{r}$

三、 电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \int \left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right] - \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\right]\vec{\boldsymbol{r}}'\right\}\mathrm{d}\tau'$$

$$= \oint \vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma - \int\left[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},$$

$$\boxed{\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$　　为了求电磁场，需计算 $\nabla\dfrac{e^{ikr}}{r}$

$$\nabla\frac{e^{ikr}}{r}$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\big]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \int\Big\{\nabla'\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big] - \big[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\big]\vec{\boldsymbol{r}}'\Big\}\,\mathrm{d}\tau'$$

$$= \oint \vec{\boldsymbol{n}}\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big]\,\mathrm{d}\sigma - \int\Big[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\Big]\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},$$

$$\boxed{\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$ 为了求电磁场，需计算 $\nabla\frac{e^{ikr}}{r}$

$$\nabla\frac{e^{ikr}}{r} = \left(1 + \frac{i}{kr}\right) ik\,\frac{\vec{\boldsymbol{r}}}{r}\,\frac{e^{ikr}}{r},$$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\begin{aligned}
\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' &= \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \int \left\{\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right] - \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\right]\vec{\boldsymbol{r}}'\right\}\mathrm{d}\tau' \\
&= \oint \vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\right]\mathrm{d}\sigma - \int\left[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\right]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' \\
&= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},
\end{aligned}$$

$$\boxed{\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$ 　为了求电磁场，需计算 $\nabla\dfrac{e^{ikr}}{r}$

$$\nabla\frac{e^{ikr}}{r} = \left(1 + \frac{i}{kr}\right)ik\frac{\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r},$$

因为 $r \gg \lambda$，故 $kr \gg 1$

三、电偶极辐射

既然矢势级数快速收敛，先看第一项：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

对稳恒电流 $\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau' = 0$，对时变电流 $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)$ 有：

$$\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\,\mathrm{d}\tau' = \int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\underbrace{(\nabla'\vec{\boldsymbol{r}}')}_{\overleftrightarrow{\boldsymbol{I}}}\,\mathrm{d}\tau' = \int \big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\cdot\nabla'\big]\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \int \Big\{\nabla'\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big] - \big[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\big]\vec{\boldsymbol{r}}'\Big\}\,\mathrm{d}\tau'$$

$$= \oint \vec{\boldsymbol{n}}\cdot\big[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\big]\,\mathrm{d}\sigma - \int \Big[-\frac{\partial\rho(\vec{\boldsymbol{r}}', t)}{\partial t}\Big]\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'$$

$$= \oint j_n(\vec{\boldsymbol{r}}', t)\vec{\boldsymbol{r}}'\,\mathrm{d}\sigma + \frac{\partial}{\partial t}\int \rho(\vec{\boldsymbol{r}}', t)\,\vec{\boldsymbol{r}}'\,\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t},$$

$$\boxed{\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}', t)\mathrm{d}\tau' = \frac{\partial\vec{\boldsymbol{p}}}{\partial t}}$$

$\Longrightarrow$ 偶极辐射矢势：$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$ 为了求电磁场，需计算 $\nabla\frac{e^{ikr}}{r}$

$$\nabla\frac{e^{ikr}}{r} = \left(1+\frac{i}{kr}\right)ik\frac{\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r},$$

因为 $r \gg \lambda$，故 $kr \gg 1$ $\Longrightarrow$ $\boxed{\nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r}}$

Let there be light

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$$

Let there be light

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\dfrac{1}{r}$ 的求导必导致 $\dfrac{X}{r}$ 项，

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\dfrac{1}{r}$ 的求导必导致 $\dfrac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\dfrac{1}{r}$ 的求导必导致 $\dfrac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区}\ (r \gg \lambda)\ \text{的辐射场时：}\nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\dfrac{1}{r}$ 的求导必导致 $\dfrac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区}\ (r \gg \lambda)\ \text{的辐射场时：}\nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$$

偶极辐射场：

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$$

偶极辐射场：

$$\vec{\boldsymbol{B}} = \nabla \times \vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}} \times \vec{\boldsymbol{A}}$$

$$\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{r}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{n})\,\frac{e^{ikr}}{r}, \qquad \vec{n} = \frac{\vec{r}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{n}$$

偶极辐射场：

$$\vec{\mathcal{B}} = \nabla \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}}^{(0)}$$

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$\Longrightarrow$ 在求远区 $(r \gg \lambda)$ 的辐射场时：$\nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$

偶极辐射场：

$$\vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{p}}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{\boldsymbol{n}}\times\ddot{\vec{\boldsymbol{p}}}$$

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\dfrac{1}{r}$ 的求导必导致 $\dfrac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$$

偶极辐射场：

$$\vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{p}}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{\boldsymbol{n}}\times\ddot{\vec{\boldsymbol{p}}}$$

$$\text{由 } \nabla\times\vec{\boldsymbol{H}} = \frac{\partial\vec{\boldsymbol{D}}}{\partial t} \quad \Longrightarrow \quad \vec{\boldsymbol{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\boldsymbol{B}}\text{，故}$$

$$\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{r}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{n})\,\frac{e^{ikr}}{r}, \qquad \vec{n} = \frac{\vec{r}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$\Longrightarrow$ 在求远区 $(r \gg \lambda)$ 的辐射场时：$\nabla \Longrightarrow ik\,\vec{n}$

偶极辐射场：

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\vec{n}\times\vec{\mathcal{A}} = ik\vec{n}\times\vec{\mathcal{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{n}\times\dot{\vec{p}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}$$

由 $\nabla\times\vec{\mathcal{H}} = \frac{\partial\vec{\mathcal{D}}}{\partial t} \quad\Longrightarrow\quad \vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\mathcal{B}}$，故

$$\vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\mathcal{B}} = \frac{i}{\omega\epsilon_0\mu_0}(ik\vec{n})\times\vec{\mathcal{B}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$$

$$\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{\boldsymbol{r}}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{\boldsymbol{n}})\,\frac{e^{ikr}}{r}, \qquad \vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{r}}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$\Longrightarrow$ 在求远区 $(r \gg \lambda)$ 的辐射场时：$\nabla \quad \Longrightarrow \quad ik\,\vec{\boldsymbol{n}}$

偶极辐射场：

$$\vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}} = ik\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{p}}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{\boldsymbol{n}}\times\ddot{\vec{\boldsymbol{p}}}$$

由 $\nabla\times\vec{\boldsymbol{\mathcal{H}}} = \frac{\partial\vec{\boldsymbol{\mathcal{D}}}}{\partial t} \quad \Longrightarrow \quad \vec{\boldsymbol{\mathcal{E}}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\boldsymbol{B}}$，故

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\boldsymbol{B}} = \frac{i}{\omega\epsilon_0\mu_0}(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{\boldsymbol{p}}}\times\vec{\boldsymbol{n}})\times\vec{\boldsymbol{n}}$$

整理：利用 $k = \omega/c$ 和谐变场：$-i\omega\,\mathcal{X} = \dot{\mathcal{X}}$，并设 $\vec{\boldsymbol{p}}$ 沿 $\hat{e}_z$ 且初相为 0

$$\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{r}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{n})\,\frac{e^{ikr}}{r}, \qquad \vec{n} = \frac{\vec{r}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{n}$$

偶极辐射场：

$$\vec{\mathcal{B}} = \nabla \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{n} \times \dot{\vec{p}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n} \times \ddot{\vec{p}}$$

$$\text{由 } \nabla \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} \quad \Longrightarrow \quad \vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla \times \vec{\mathcal{B}}\text{，故}$$

$$\vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla \times \vec{\mathcal{B}} = \frac{i}{\omega\epsilon_0\mu_0}(ik\vec{n}) \times \vec{\mathcal{B}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}} \times \vec{n}) \times \vec{n}$$

整理：利用 $k = \omega/c$ 和谐变场：$-i\omega\,\mathcal{X} = \dot{\mathcal{X}}$，并设 $\vec{p}$ 沿 $\hat{e}_z$ 且初相为 0

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

$$\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{r}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{n})\,\frac{e^{ikr}}{r}, \qquad \vec{n} = \frac{\vec{r}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{n}$$

偶极辐射场：

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\vec{n}\times\vec{\mathcal{A}} = ik\vec{n}\times\vec{\mathcal{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{n}\times\dot{\vec{p}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}$$

$$\text{由 } \nabla\times\vec{\mathcal{H}} = \frac{\partial\vec{\mathcal{D}}}{\partial t} \quad \Longrightarrow \quad \vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\mathcal{B}}\text{，故}$$

$$\vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla\times\vec{\mathcal{B}} = \frac{i}{\omega\epsilon_0\mu_0}(ik\vec{n})\times\vec{\mathcal{B}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$$

整理：利用 $k = \omega/c$ 和谐变场：$-i\omega\,\mathcal{X} = \dot{\mathcal{X}}$，并设 $\vec{p}$ 沿 $\hat{e}_z$ 且初相为 0

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\;\hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\;\hat{e}_\theta,$$

讨论

$$\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}} \qquad \nabla\frac{e^{ikr}}{r} = \frac{ik\vec{r}}{r}\frac{e^{ikr}}{r} = (ik\,\vec{n})\,\frac{e^{ikr}}{r}, \qquad \vec{n} = \frac{\vec{r}}{r}$$

对任意势或场 X 求导时，对 $\frac{1}{r}$ 的求导必导致 $\frac{X}{r}$ 项，远小于对 e^{ikr} 求导的 $ik\,X$ 项

$$\Longrightarrow \quad \text{在求远区 } (r \gg \lambda) \text{ 的辐射场时：} \nabla \quad \Longrightarrow \quad ik\,\vec{n}$$

偶极辐射场：

$$\vec{\mathcal{B}} = \nabla \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}} = ik\vec{n} \times \vec{\mathcal{A}}^{(0)} == \frac{\mu_0}{4\pi}\frac{ike^{ikr}}{r}\vec{n} \times \dot{\vec{p}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n} \times \ddot{\vec{p}}$$

$$\text{由 } \nabla \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} \quad \Longrightarrow \quad \vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla \times \vec{\mathcal{B}}\text{，故}$$

$$\vec{\mathcal{E}} = \frac{i}{\omega\epsilon_0\mu_0}\nabla \times \vec{\mathcal{B}} = \frac{i}{\omega\epsilon_0\mu_0}(ik\vec{n}) \times \vec{\mathcal{B}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}} \times \vec{n}) \times \vec{n}$$

整理：利用 $k = \omega/c$ 和谐变场：$-i\omega\,\mathcal{X} = \dot{\mathcal{X}}$，并设 $\vec{p}$ 沿 $\hat{e}_z$ 且初相为 0

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

讨论

(1) 偶极辐射的磁场沿纬线，电场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直，$\vec{E} \times \vec{B}$ 沿 $\vec{n}$ 方向

Let there be light

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，
等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流:

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle \vec{S} \rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}} \times \vec{\mathcal{B}}^*\right]$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，
等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle \vec{S} \rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}} \times \vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\,\vec{n}$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}} \times \vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint \vec{n}\cdot\langle\vec{S}\rangle\, r^2\, \mathrm{d}\Omega$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint \vec{n}\cdot\langle\vec{S}\rangle\, r^2\, \mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3}$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle \vec{S} \rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}} \times \vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle \vec{P} \rangle = \oint \vec{n}\cdot\langle \vec{S} \rangle\, r^2\, \mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3},$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \; \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \; \hat{e}_\theta,$$

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\text{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\,\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint \vec{n}\cdot\langle\vec{S}\rangle\, r^2\, \mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$$

平均辐射功率：$$\langle\vec{P}\rangle = \oint\vec{n}\cdot\langle\vec{S}\rangle\, r^2\,\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \qquad \vec{p} = \vec{p}_0 e^{-i\omega t}$$

讨论

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint\vec{n}\cdot\langle\vec{S}\rangle\, r^2\,\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}| \sin\theta \; \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}| \sin\theta \; \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\text{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint \vec{n}\cdot\langle\vec{S}\rangle\, r^2\, \mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。是否存在球对称的经典电磁辐射？

Let there be light

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint\vec{n}\cdot\langle\vec{S}\rangle\, r^2\,\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。是否存在球对称的经典电磁辐射？ 不存在 —— Am J Phys **70** 715

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article J Opt A 2, R15 (2000)

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint\vec{n}\cdot\langle\vec{S}\rangle\, r^2\,\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。是否存在球对称的经典电磁辐射？ 不存在 —— Am J Phys **70** 715

思考： 从推迟势出发，试证若电荷不守恒，则可能存在球对称的经典电磁辐射。

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi,\quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint\vec{n}\cdot\langle\vec{S}\rangle\, r^2\,\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3},\quad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。是否存在球对称的经典电磁辐射？ 不存在 —— Am J Phys **70** 715

思考： 从推迟势出发，试证若电荷不守恒，则可能存在球对称的经典电磁辐射。

(2) 辐射功率与频率四次方成正比 —— 瑞利 (Rayleigh) 定律。可解释夕阳红，晴空蓝

$$\vec{\mathcal{B}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^3 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\phi, \quad \vec{\mathcal{E}} = \frac{e^{ikr}}{4\pi\epsilon_0 c^2 r}|\ddot{\vec{p}}|\sin\theta\ \hat{e}_\theta,$$

(2) e^{ikr} 为相位滞后因子，表明辐射场以有限速度传播

(3) $e^{ikr-i\omega t}$ 表明辐射场是沿径向外传的球面波。等相面为球面，但等相面上振幅不相等。距偶极子很远处，等相面近似为平面，辐射场近似为平面波

Review article *J Opt A 2, R15 (2000)*

Figure 16. The dragonfly *Orthetrum caledonicum* (Libellulidae), male. The blue colour results from Rayleigh scattering.

平均辐射能流：$\langle\vec{S}\rangle = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\left[\vec{\mathcal{E}}\times\vec{\mathcal{B}}^*\right] = \frac{|\ddot{\vec{p}}|^2}{32\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{r^2}\vec{n}$

平均辐射功率：$\langle\vec{P}\rangle = \oint \vec{n}\cdot\langle\vec{S}\rangle\, r^2\, \mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3} = \frac{1}{4\pi\epsilon_0}\frac{\omega^4|\vec{p}_0|^2}{3c^3}, \qquad \vec{p} = \vec{p}_0 e^{-i\omega t}$

讨论

(1) 辐射能流非球对称，沿垂直于偶极子方向最强。 —— 最低阶的偶极辐射是非球对称的。是否存在球对称的经典电磁辐射？ 不存在 —— Am J Phys **70** 715

思考： 从推迟势出发，试证若电荷不守恒，则可能存在球对称的经典电磁辐射。

(2) 辐射功率与频率四次方成正比 —— 瑞利 (Rayleigh) 定律。可解释夕阳红，晴空蓝蜻蜓背上的蓝色也来自瑞利散射。如上图。

四、磁偶极辐射和电四极辐射

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \left[\dfrac{1}{r^2} - \dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}\,e^{ikr}$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2} - \frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\left[\dfrac{1}{r^2}-\dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \left[\dfrac{1}{r^2} - \dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}}$

对稳恒电流：$\displaystyle\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5)

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right]\cdot\left[\dfrac{1}{r^2}-\dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \left[\dfrac{1}{r^2} - \dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}} = \vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}}$

对时变电流：

$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \cdot \vec{\boldsymbol{n}}$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')$$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right] \cdot \left[\dfrac{1}{r^2} - \dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}}$

对稳恒电流：$\displaystyle\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\displaystyle\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}} = \vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' \cdot \vec{\boldsymbol{n}} = \vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') = \frac{1}{2}\vec{\boldsymbol{n}} \cdot \Big[\overbrace{\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'}^{\text{对稳恒电流此两项化为 } \vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}}}\Big]$$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] \cdot \left[\frac{1}{r^2} - \frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] \cdot \vec{\boldsymbol{n}} = \vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' \cdot \vec{\boldsymbol{n}} = \vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}} \cdot \Big[\overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项化为 } \vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}}}\Big]$$

$$\frac{1}{2}\vec{\boldsymbol{n}} \cdot [\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'] = \frac{1}{2}\Big\{(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - [\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\vec{\boldsymbol{r}}'\Big\} = -\frac{1}{2}\vec{\boldsymbol{n}} \times [\vec{\boldsymbol{r}}' \times \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]$$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2} - \frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\Big[\overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项化为 } \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}}\Big]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right] = \frac{1}{2}\Big\{(\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - [\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\vec{\boldsymbol{r}}'\Big\} = -\frac{1}{2}\vec{\boldsymbol{n}}\times[\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]$$

两边同时积分：$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\frac{1}{r^2}-\frac{ik}{r}\right]\frac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\Big[\overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')+\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')-\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项化为 }\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}}\Big]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')-\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'] = \frac{1}{2}\Big\{(\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - [\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\vec{\boldsymbol{r}}'\Big\} = -\frac{1}{2}\vec{\boldsymbol{n}}\times[\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]$$

两边同时积分：$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')-\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}']\,\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},\qquad \vec{\boldsymbol{m}} = \frac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\dfrac{1}{r^2} - \dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\Big[\overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项化为 } \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}}\Big]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right] = \frac{1}{2}\Big\{(\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - [\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\vec{\boldsymbol{r}}'\Big\} = -\frac{1}{2}\vec{\boldsymbol{n}}\times[\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]$$

两边同时积分：$\dfrac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},\qquad \vec{\boldsymbol{m}} = \dfrac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

$$\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\right] = \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\right](\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}') + \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\cdot\nabla'\right](\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')$$

$$= -\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')$$

两边同时积分，左边化为面积分为 0

四、磁偶极辐射和电四极辐射

看矢势级数展开的第二项：$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\left[\dfrac{1}{r^2}-\dfrac{ik}{r}\right]\dfrac{\vec{\boldsymbol{r}}}{r}e^{ikr}$

当 $r \gg \lambda \gg l'$ 时，$|\vec{\boldsymbol{A}}^{(1)}| \ll |\vec{\boldsymbol{A}}^{(0)}|$

远区：$r \gg 1/k \sim \lambda \quad \Longrightarrow \quad \vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\dfrac{ik\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$

对稳恒电流：$\int [\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\mathrm{d}\tau' = 0$ (§4.4 p5) $\Longrightarrow \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}$

对时变电流：

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\Big[\overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项积分为 0}} + \overbrace{\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'}^{\text{对稳恒电流此两项化为 } \vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}}\Big]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right] = \frac{1}{2}\Big\{(\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}')\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - [\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]\vec{\boldsymbol{r}}'\Big\} = -\frac{1}{2}\vec{\boldsymbol{n}}\times[\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')]$$

两边同时积分：$\dfrac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}, \qquad \vec{\boldsymbol{m}} = \dfrac{1}{2}\int\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$

$$\nabla'\cdot\left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\right] = \left[\nabla'\cdot\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\right](\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}') + \left[\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\cdot\nabla'\right](\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')$$

$$= -\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')$$

两边同时积分，左边化为面积分为 0

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

Let there be light

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,' + \vec{\boldsymbol{r}}\,'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,')\vec{\boldsymbol{r}}\,'\right]$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},\quad \vec{\boldsymbol{m}} = \frac{1}{2}\int\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

Let there be light

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},\quad \vec{\boldsymbol{m}} = \frac{1}{2}\int\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}, \quad \vec{\boldsymbol{m}} = \frac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int \rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\overleftrightarrow{\boldsymbol{D}}' = \int 3\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau = \overleftrightarrow{\boldsymbol{D}} + \int \rho(\vec{\boldsymbol{r}}')r'^2\overleftrightarrow{\boldsymbol{I}}\,\mathrm{d}\tau', \quad \overleftrightarrow{\boldsymbol{D}} = \int \rho(\vec{\boldsymbol{r}}')[3\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}' - r'^2\overleftrightarrow{\boldsymbol{I}}]\,\mathrm{d}\tau'$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}, \quad \vec{\boldsymbol{m}} = \frac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\overleftrightarrow{\boldsymbol{D}}' = \int 3\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau = \overleftrightarrow{\boldsymbol{D}} + \int\rho(\vec{\boldsymbol{r}}')r'^2\overleftrightarrow{\boldsymbol{I}}\,\mathrm{d}\tau', \quad \overleftrightarrow{\boldsymbol{D}} = \int\rho(\vec{\boldsymbol{r}}')[3\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}' - r'^2\overleftrightarrow{\boldsymbol{I}}]\,\mathrm{d}\tau'$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\dot{\overleftrightarrow{\boldsymbol{D}}} + \frac{1}{6}\vec{\boldsymbol{n}}\mathcal{Q}\right], \qquad \mathcal{Q} = \int\rho(\vec{\boldsymbol{r}}')r'^2\,\mathrm{d}\tau'$$

Let there be light

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}, \quad \vec{\boldsymbol{m}} = \frac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\overleftrightarrow{\boldsymbol{D}}' = \int 3\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau = \overleftrightarrow{\boldsymbol{D}} + \int\rho(\vec{\boldsymbol{r}}')r'^2\overleftrightarrow{\boldsymbol{I}}\,\mathrm{d}\tau', \quad \overleftrightarrow{\boldsymbol{D}} = \int\rho(\vec{\boldsymbol{r}}')[3\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}' - r'^2\overleftrightarrow{\boldsymbol{I}}]\,\mathrm{d}\tau'$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\dot{\overleftrightarrow{\boldsymbol{D}}} + \frac{1}{6}\vec{\boldsymbol{n}}\mathcal{Q}\right], \qquad \mathcal{Q} = \int\rho(\vec{\boldsymbol{r}}')r'^2\,\mathrm{d}\tau'$$

在求辐射场上时，$\nabla \implies ik\,\vec{\boldsymbol{n}}, \quad \vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{A}} = ik\,\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}}$

$$\vec{\mathcal{A}}^{(1)}(\vec{r}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{j}(\vec{r}\,')\vec{r}\,'\,\mathrm{d}\tau'\right]\cdot\vec{n}$$

$$\vec{j}(\vec{r}\,')\vec{r}\,'\cdot\vec{n} = \vec{n}\cdot\vec{r}\,'\vec{j}(\vec{r}\,') = \frac{1}{2}\vec{n}\cdot\left[\vec{r}\,'\vec{j}(\vec{r}\,') + \vec{j}(\vec{r}\,')\vec{r}\,' + \vec{r}\,'\vec{j}(\vec{r}\,') - \vec{j}(\vec{r}\,')\vec{r}\,'\right]$$

$$\frac{1}{2}\vec{n}\cdot\int\left[\vec{r}\,'\vec{j}(\vec{r}\,') - \vec{j}(\vec{r}\,')\vec{r}\,'\right]\mathrm{d}\tau' = -\vec{n}\times\vec{m}, \quad \vec{m} = \frac{1}{2}\int \vec{r}\,'\times\vec{j}(\vec{r}\,')\,\mathrm{d}\tau'$$

$$\int\left[\vec{r}\,'\vec{j}(\vec{r}\,') + \vec{j}(\vec{r}\,')\vec{r}\,'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{r}\,'\vec{r}\,')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int \rho(\vec{r}\,')\vec{r}\,'\vec{r}\,'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\overleftrightarrow{\boldsymbol{D}}' = \int 3\rho(\vec{r}\,')\vec{r}\,'\vec{r}\,'\,\mathrm{d}\tau = \overleftrightarrow{\boldsymbol{D}} + \int \rho(\vec{r}\,')r'^2\overleftrightarrow{\boldsymbol{I}}\,\mathrm{d}\tau', \quad \overleftrightarrow{\boldsymbol{D}} = \int \rho(\vec{r}\,')[3\vec{r}\,'\vec{r}\,' - r'^2\overleftrightarrow{\boldsymbol{I}}]\,\mathrm{d}\tau'$$

$$\vec{\mathcal{A}}^{(1)}(\vec{r}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[-\vec{n}\times\vec{m} + \frac{1}{6}\vec{n}\cdot\dot{\overleftrightarrow{\boldsymbol{D}}} + \frac{1}{6}\vec{n}\mathcal{Q}\right], \qquad \mathcal{Q} = \int \rho(\vec{r}\,')r'^2\,\mathrm{d}\tau'$$

在求辐射场上时，$\nabla \implies ik\,\vec{n}$， $\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}}$

$\vec{\mathcal{A}}^{(1)}(\vec{r})$ 中的最后一项正比于 $\vec{n}\times\vec{n} = 0$，对辐射场没贡献。

考虑到对谐变场，$\dfrac{\partial\mathcal{X}}{\partial t} \implies -i\omega\,\mathcal{X} = -ikc\mathcal{X}$，$\vec{\mathcal{A}}^{(1)}(\vec{r})$ 可写成

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right]\cdot\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{n}} = \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') = \frac{1}{2}\vec{\boldsymbol{n}}\cdot\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}' + \vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]$$

$$\frac{1}{2}\vec{\boldsymbol{n}}\cdot\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') - \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = -\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}}, \quad \vec{\boldsymbol{m}} = \frac{1}{2}\int \vec{\boldsymbol{r}}'\times\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\,\mathrm{d}\tau'$$

$$\int\left[\vec{\boldsymbol{r}}'\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') + \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\right]\mathrm{d}\tau' = \int\left[-\frac{\partial\rho}{\partial t}(\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}')\right]\mathrm{d}\tau' = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau'\right] = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\overleftrightarrow{\boldsymbol{D}}'$$

$$\overleftrightarrow{\boldsymbol{D}}' = \int 3\rho(\vec{\boldsymbol{r}}')\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}'\,\mathrm{d}\tau = \overleftrightarrow{\boldsymbol{D}} + \int\rho(\vec{\boldsymbol{r}}')r'^2\overleftrightarrow{\boldsymbol{I}}\,\mathrm{d}\tau', \quad \overleftrightarrow{\boldsymbol{D}} = \int\rho(\vec{\boldsymbol{r}}')[3\vec{\boldsymbol{r}}'\vec{\boldsymbol{r}}' - r'^2\overleftrightarrow{\boldsymbol{I}}]\,\mathrm{d}\tau'$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = -\frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\dot{\overleftrightarrow{\boldsymbol{D}}} + \frac{1}{6}\vec{\boldsymbol{n}}\mathcal{Q}\right], \qquad \mathcal{Q} = \int\rho(\vec{\boldsymbol{r}}')r'^2\,\mathrm{d}\tau'$$

在求辐射场上时，$\nabla \implies ik\,\vec{\boldsymbol{n}}$， $\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{A}} = ik\,\vec{\boldsymbol{n}}\times\vec{\boldsymbol{A}}$

$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 中的最后一项正比于 $\vec{\boldsymbol{n}}\times\vec{\boldsymbol{n}} = 0$，对辐射场没贡献。

考虑到对谐变场，$\dfrac{\partial\mathcal{X}}{\partial t} \implies -i\omega\,\mathcal{X} = -ikc\mathcal{X}$，$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})$ 可写成

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\ddot{\overleftrightarrow{\boldsymbol{D}}}\right], \qquad \text{比较} \quad \vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}} \times \dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}} \cdot \ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}} \times \dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}} \cdot \ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

讨论

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\quad \left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

讨论

(1) 稳恒时，电荷激发多极电场：$\varphi(\vec{\boldsymbol{r}}) \sim \frac{1}{r}\left[c_0 + c_1\frac{d}{r} + c_2\left(\frac{d}{r}\right)^2 + \dots\right]$, $\quad \vec{\boldsymbol{E}} = -\nabla\varphi \sim \frac{1}{r^2}$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

讨论

(1) 稳恒时，电荷激发多极电场：$\varphi(\vec{\boldsymbol{r}}) \sim \frac{1}{r}\left[c_0 + c_1\frac{d}{r} + c_2\left(\frac{d}{r}\right)^2 + \ldots\right]$, $\vec{\boldsymbol{E}} = -\nabla\varphi \sim \frac{1}{r^2}$

电流激发多极磁场：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) \sim \frac{1}{r}\left[\vec{\boldsymbol{c}}_1\frac{d}{r} + \vec{\boldsymbol{c}}_2\left(\frac{d}{r}\right)^2 + \ldots\right]$, $\vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} \sim \frac{1}{r^3}$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\quad \left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

讨论

(1) 稳恒时，电荷激发多极电场：$\varphi(\vec{\boldsymbol{r}}) \sim \frac{1}{r}\left[c_0 + c_1\frac{d}{r} + c_2\left(\frac{d}{r}\right)^2 + \ldots\right]$, $\quad \vec{\boldsymbol{E}} = -\nabla\varphi \sim \frac{1}{r^2}$

电流激发多极磁场：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) \sim \frac{1}{r}\left[\vec{\boldsymbol{c}}_1\frac{d}{r} + \vec{\boldsymbol{c}}_2\left(\frac{d}{r}\right)^2 + \ldots\right]$, $\quad \vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} \sim \frac{1}{r^3}$

谐变时，远区 $(r \gg \lambda)$ 电荷电流分布激发多极辐射场： $\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}}$, $\quad \vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\epsilon_0\mu_0}$

$$\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} + \frac{1}{6}\vec{\boldsymbol{n}}\cdot\ddot{\overleftrightarrow{\boldsymbol{D}}}\right]$$

比较 $\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$, $\quad \left|\vec{\boldsymbol{A}}^{(1)}(\vec{\boldsymbol{r}})\right| \Big/ \left|\vec{\boldsymbol{A}}^{(0)}(\vec{\boldsymbol{r}})\right| \sim kl'$

讨论

(1) 稳恒时，电荷激发多极电场：$\varphi(\vec{\boldsymbol{r}}) \sim \dfrac{1}{r}\left[c_0 + c_1\dfrac{d}{r} + c_2\left(\dfrac{d}{r}\right)^2 + \ldots\right]$, $\quad \vec{\boldsymbol{E}} = -\nabla\varphi \sim \dfrac{1}{r^2}$

电流激发多极磁场：$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) \sim \dfrac{1}{r}\left[\vec{\boldsymbol{c}}_1\dfrac{d}{r} + \vec{\boldsymbol{c}}_2\left(\dfrac{d}{r}\right)^2 + \ldots\right]$, $\quad \vec{\boldsymbol{B}} = \nabla\times\vec{\boldsymbol{A}} \sim \dfrac{1}{r^3}$

谐变时，远区 $(r \gg \lambda)$ 电荷电流分布激发多极辐射场：$\quad \vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}}$, $\quad \vec{\boldsymbol{\mathcal{E}}} = \dfrac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\epsilon_0\mu_0}$

$$\vec{\boldsymbol{\mathcal{A}}}(\vec{\boldsymbol{r}}) \sim \frac{e^{ikr}}{r}\left[(kl')\,\vec{\boldsymbol{c}}_1 + (kl')^2\,\vec{\boldsymbol{c}}_2 + \ldots\right], \qquad k = 2\pi/\lambda = \omega/c$$

$$\vec{\mathcal{A}}^{(1)}(\vec{r}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\left[-\vec{n}\times\dot{\vec{m}} + \frac{1}{6}\vec{n}\cdot\ddot{\overleftrightarrow{D}}\right]$$

比较 $\vec{\mathcal{A}}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{p}},\qquad \left|\vec{\mathcal{A}}^{(1)}(\vec{r})\right|\Big/\left|\vec{\mathcal{A}}^{(0)}(\vec{r})\right| \sim kl'$

讨论

(1) 稳恒时，电荷激发多极电场：$\varphi(\vec{r}) \sim \frac{1}{r}\left[c_0 + c_1\frac{d}{r} + c_2\left(\frac{d}{r}\right)^2 + \ldots\right]$， $\vec{E} = -\nabla\varphi \sim \frac{1}{r^2}$

电流激发多极磁场：$\vec{A}(\vec{r}) \sim \frac{1}{r}\left[\vec{c}_1\frac{d}{r} + \vec{c}_2\left(\frac{d}{r}\right)^2 + \ldots\right]$， $\vec{B} = \nabla\times\vec{A} \sim \frac{1}{r^3}$

谐变时，远区 $(r \gg \lambda)$ 电荷电流分布激发多极辐射场： $\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}}$， $\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0}$

$$\vec{\mathcal{A}}(\vec{r}) \sim \frac{e^{ikr}}{r}\left[(kl')\,\vec{c}_1 + (kl')^2\,\vec{c}_2 + \ldots\right],\qquad k = 2\pi/\lambda = \omega/c$$

(2) 磁偶与电四极辐射属同一项，都为 $(kl')^2$ 量级，在电荷电流分布较小时小于电偶极辐射
一般情况下，磁偶辐射与电四极辐射两项同时出现，仅对某些特殊情况，才只有一项。
例如当圆电流环各处电流以相同振幅位相变化时，只有磁偶辐射项

(3) 磁偶辐射场

$$\vec{\boldsymbol{A}}_m(\vec{\boldsymbol{r}}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}} \times \dot{\vec{\boldsymbol{m}}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}},$$

其中：$\dot{\vec{\boldsymbol{m}}} = -i\omega\vec{\boldsymbol{m}}$

(3) 磁偶辐射场

$$\vec{\boldsymbol{A}}_m(\vec{\boldsymbol{r}}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}} \times \dot{\vec{\boldsymbol{m}}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}} \times \vec{\boldsymbol{m}},$$

其中：$\dot{\vec{\boldsymbol{m}}} = -i\omega\vec{\boldsymbol{m}}$

$$\vec{\boldsymbol{B}} = \nabla \times \vec{\boldsymbol{A}} = ik\,\vec{\boldsymbol{n}} \times \vec{\boldsymbol{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{\boldsymbol{m}}} \times \vec{\boldsymbol{n}}) \times \vec{\boldsymbol{n}}$$

(3) 磁偶辐射场

$$\vec{\boldsymbol{\mathcal{A}}}_m(\vec{\boldsymbol{r}}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}}\times\dot{\vec{\boldsymbol{m}}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}}\times\vec{\boldsymbol{m}},\qquad \text{其中：}\dot{\vec{\boldsymbol{m}}} = -i\omega\vec{\boldsymbol{m}}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\,\vec{\boldsymbol{n}}\times\vec{\boldsymbol{\mathcal{A}}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{\boldsymbol{m}}}\times\vec{\boldsymbol{n}})\times\vec{\boldsymbol{n}}$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{\boldsymbol{n}}\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\epsilon_0\mu_0} = c(\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{\boldsymbol{n}}\times\ddot{\vec{\boldsymbol{m}}}$$

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m}, \qquad \text{其中：}\dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c, \quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}}, \quad c\vec{\mathcal{B}}\to -\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}, \qquad \vec{\mathcal{E}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m}, \qquad \text{其中：}\dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c,\quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}},\quad c\vec{\mathcal{B}}\to-\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}, \qquad \vec{\mathcal{E}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(5) 若取磁偶极方向为 $\hat{e}_z$，磁偶辐射的电场沿纬线，磁场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m}, \qquad \text{其中：} \dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c, \quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}}, \quad c\vec{\mathcal{B}}\to -\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}, \qquad \vec{\mathcal{E}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(5) 若取磁偶极方向为 $\hat{e}_z$，磁偶辐射的电场沿纬线，磁场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直

仍有 $\vec{E}\times\vec{B}$ 沿 $\vec{n}$，辐射角分布服从 $\sin^2\theta$，辐射功率正比于 ω^4

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m}, \qquad \text{其中：}\dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c,\quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}},\quad c\vec{\mathcal{B}}\to-\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}},\qquad \vec{\mathcal{E}} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(5) 若取磁偶极方向为 $\hat{e}_z$，磁偶辐射的电场沿纬线，磁场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直

仍有 $\vec{E}\times\vec{B}$ 沿 $\vec{n}$，辐射角分布服从 $\sin^2\theta$，辐射功率正比于 ω^4

但辐射功率与电偶极辐射功率之比为：$P_m/P_e\sim(kl')^2\sim(l'/\lambda)^2$

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m},\qquad \text{其中：}\dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c,\quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}},\quad c\vec{\mathcal{B}}\to-\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}},\qquad \vec{\mathcal{E}} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(5) 若取磁偶极方向为 $\hat{e}_z$，磁偶辐射的电场沿纬线，磁场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直

仍有 $\vec{E}\times\vec{B}$ 沿 $\vec{n}$，辐射角分布服从 $\sin^2\theta$，辐射功率正比于 ω^4

但辐射功率与电偶极辐射功率之比为：$P_m/P_e\sim(kl')^2\sim(l'/\lambda)^2$

(6) 电四极子辐射功率正比于 ω^6

(3) 磁偶辐射场

$$\vec{\mathcal{A}}_m(\vec{r}) = -\frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\dot{\vec{m}} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{n}\times\vec{m}, \qquad \text{其中：} \dot{\vec{m}} = -i\omega\vec{m}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\,\vec{n}\times\vec{\mathcal{A}} = \frac{\mu_0}{4\pi c^2}\frac{e^{ikr}}{r}(\ddot{\vec{m}}\times\vec{n})\times\vec{n}$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = \frac{-k\,\vec{n}\times\vec{\mathcal{B}}}{\omega\epsilon_0\mu_0} = c(\vec{\mathcal{B}}\times\vec{n}) = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}\vec{n}\times\ddot{\vec{m}}$$

(4) 磁偶极辐射场可通过对偶关系从电偶极辐射场得到

对偶关系：$\vec{p}\to\vec{m}/c, \quad \vec{\mathcal{E}}\to c\vec{\mathcal{B}}, \quad c\vec{\mathcal{B}}\to-\vec{\mathcal{E}}$

电偶极辐射场：$\vec{\mathcal{B}} = -\dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{rc}\vec{n}\times\ddot{\vec{p}}, \qquad \vec{\mathcal{E}} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}(\ddot{\vec{p}}\times\vec{n})\times\vec{n}$

(5) 若取磁偶极方向为 $\hat{e}_z$，磁偶辐射的电场沿纬线，磁场沿经线，$\vec{E}$，$\vec{B}$，$\vec{n}$ 两两垂直

仍有 $\vec{E}\times\vec{B}$ 沿 $\vec{n}$，辐射角分布服从 $\sin^2\theta$，辐射功率正比于 ω^4

但辐射功率与电偶极辐射功率之比为：$P_m/P_e \sim (kl')^2 \sim (l'/\lambda)^2$

(6) 电四极子辐射功率正比于 ω^6

(7) 实际计算中若需要考虑多极项，常利用 $\dfrac{e^{ikR}}{R}$ 的另一展开式

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^{*}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^{*}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') e^{ikR}}{R} \, \mathrm{d}\tau'$$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y^*_{nm}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R} \mathrm{d}\tau'$$

$$= ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') Y^*_{nm}(\theta', \phi') j_n(kr') \, \mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^*(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R} \mathrm{d}\tau'$$

$$= ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') Y_{nm}^*(\theta', \phi') j_n(kr') \, \mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^{*}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') e^{ikR}}{R} \mathrm{d}\tau'$$

$$= ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') Y_{nm}^{*}(\theta', \phi') j_n(kr') \,\mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

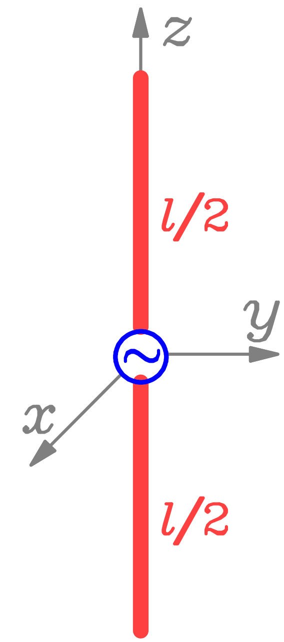
五、线型天线辐射

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^*(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R} \, \mathrm{d}\tau'$$

$$= ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}') Y_{nm}^*(\theta', \phi') j_n(kr') \, \mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

五、线型天线辐射

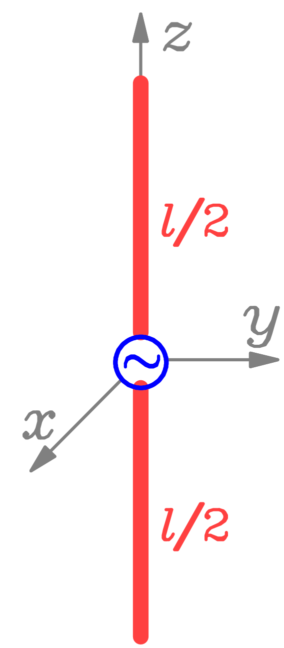


(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^{*}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi} \int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') e^{ikR}}{R} \mathrm{d}\tau' = ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}\,') Y_{nm}^{*}(\theta', \phi') j_n(kr') \, \mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

五、线型天线辐射

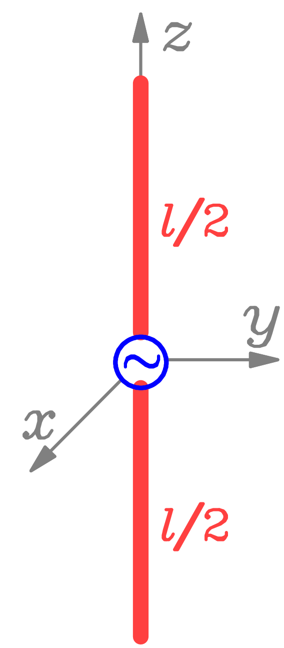
如图，半径远小于长度的直圆柱。



(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_{nm}^*(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau'$$
$$= ik\mu_0 \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')Y_{nm}^*(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right] h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$

五、线型天线辐射

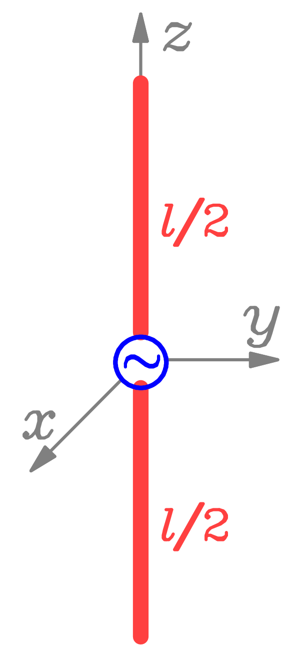


如图，半径远小于长度的直圆柱。 设输入信号谐变，天线线电流也谐变。

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_{nm}^{*}(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau'$$
$$= ik\mu_0 \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')Y_{nm}^{*}(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right] h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$

五、线型天线辐射



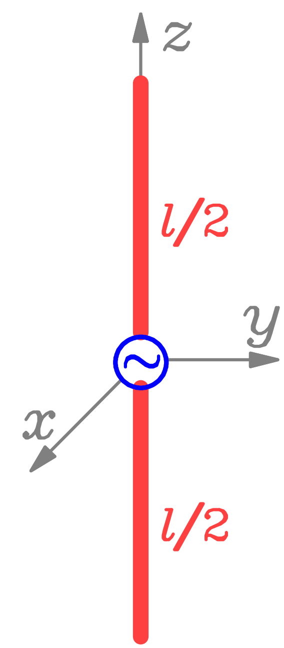
如图，半径远小于长度的直圆柱。　设输入信号谐变，天线线电流也谐变。

天线端点 $z = \pm\dfrac{l}{2}$ 电流为 0：$I(z,t) = I(z)e^{-i\omega t}$，$I(z') = I_0 \sin k(\frac{l}{2} - |z|)$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y_{nm}^{*}(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\mathcal{A}}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$
$$= ik\mu_0 \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int \vec{j}(\vec{r}\,')Y_{nm}^{*}(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right] h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$

五、线型天线辐射



如图，半径远小于长度的直圆柱。　设输入信号谐变，天线线电流也谐变。

天线端点 $z=\pm\frac{l}{2}$ 电流为 0：$I(z,t)=I(z)e^{-i\omega t}$，$I(z')=I_0\sin k(\frac{l}{2}-|z|)$

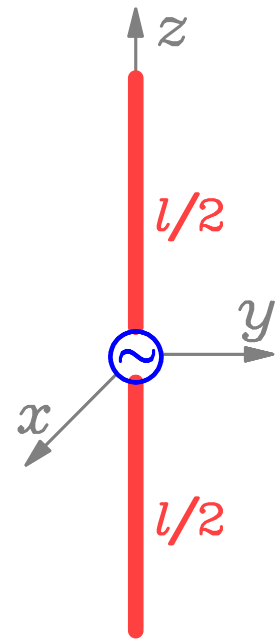
推迟势：$\vec{\mathcal{A}} = \mathcal{A}\,\hat{e}_z = \frac{\mu_0\,\hat{e}_z}{4\pi}\int\frac{I(z')e^{ikR}}{R}\mathrm{d}z'$，　$R=\sqrt{r^2+z'^2-2z'r\cos\theta}$

(7) $$\frac{e^{ikR}}{R} = 4\pi ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y^*_{nm}(\theta', \phi') Y_{nm}(\theta, \phi) j_n(kr') h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\mathcal{A}}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}\,')e^{ikR}}{R} \mathrm{d}\tau'$$

$$= ik\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[\int \vec{j}(\vec{r}\,') Y^*_{nm}(\theta', \phi') j_n(kr') \, \mathrm{d}\tau' \right] h_n^{(1)}(kr) Y_{nm}(\theta, \phi)$$

五、线型天线辐射



如图，半径远小于长度的直圆柱。　设输入信号谐变，天线线电流也谐变。

天线端点 $z = \pm\frac{l}{2}$ 电流为 0：$I(z,t) = I(z)e^{-i\omega t}$，$I(z') = I_0 \sin k(\frac{l}{2} - |z|)$

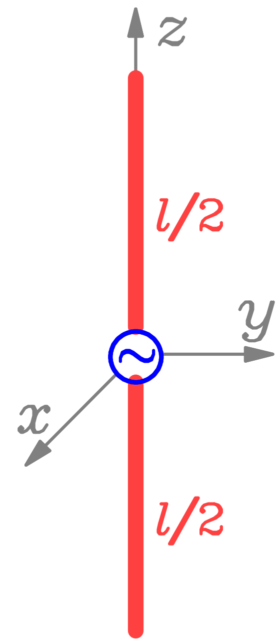
推迟势：$\vec{\mathcal{A}} = \mathcal{A}\,\hat{\boldsymbol{e}}_z = \frac{\mu_0\,\hat{\boldsymbol{e}}_z}{4\pi} \int \frac{I(z')e^{ikR}}{R} \mathrm{d}z'$，　$R = \sqrt{r^2 + z'^2 - 2z'r\cos\theta}$

远场：$r \gg l \quad \Longrightarrow \quad R = r - z'\cos\theta$

(7) $$\frac{e^{ikR}}{R} = 4\pi i k \sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y^*_{nm}(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau'$$
$$= ik\mu_0 \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')Y^*_{nm}(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right] h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$

五、线型天线辐射



如图，半径远小于长度的直圆柱。 设输入信号谐变，天线线电流也谐变。

天线端点 $z = \pm\dfrac{l}{2}$ 电流为 0： $I(z,t) = I(z)e^{-i\omega t}$， $I(z') = I_0 \sin k(\frac{l}{2} - |z|)$

推迟势： $\vec{\boldsymbol{A}} = \mathcal{A}\,\hat{\boldsymbol{e}}_z = \dfrac{\mu_0\,\hat{\boldsymbol{e}}_z}{4\pi}\displaystyle\int \frac{I(z')e^{ikR}}{R}\mathrm{d}z'$, $\quad R = \sqrt{r^2 + z'^2 - 2z'r\cos\theta}$

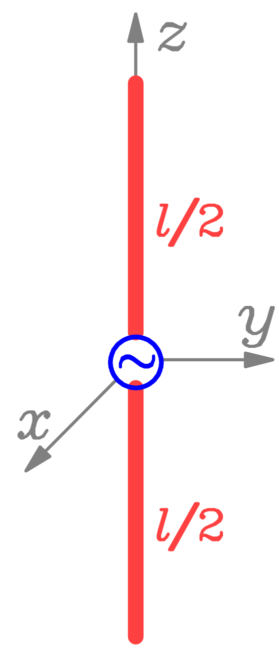
远场： $r \gg l \quad\Longrightarrow\quad R = r - z'\cos\theta$ 　　　　从而：

(7) $$\frac{e^{ikR}}{R} = 4\pi ik\sum_{n=0}^{\infty}\sum_{m=-n}^{n} Y^*_{nm}(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr), \qquad r > r'$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')e^{ikR}}{R}\,\mathrm{d}\tau'$$

$$= ik\mu_0\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int \vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}')Y^*_{nm}(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right] h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$

五、线型天线辐射



如图，半径远小于长度的直圆柱。　设输入信号谐变，天线线电流也谐变。

天线端点 $z=\pm\dfrac{l}{2}$ 电流为 0：$I(z,t)=I(z)e^{-i\omega t}$，$I(z')=I_0\sin k(\frac{l}{2}-|z|)$

推迟势：$\vec{\boldsymbol{A}} = \mathcal{A}\,\hat{\boldsymbol{e}}_z = \dfrac{\mu_0\,\hat{\boldsymbol{e}}_z}{4\pi}\displaystyle\int\frac{I(z')e^{ikR}}{R}\mathrm{d}z'$，$\quad R=\sqrt{r^2+z'^2-2z'r\cos\theta}$

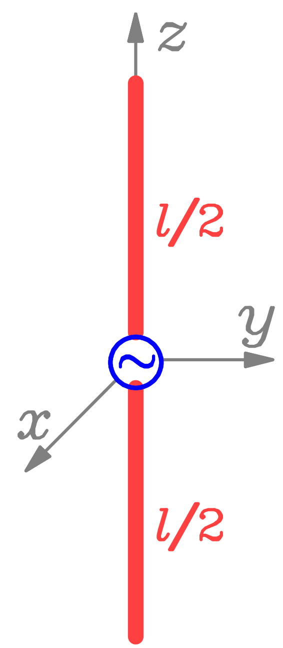
远场：$r\gg l \quad\Longrightarrow\quad R = r - z'\cos\theta$ 　　　从而：

$$\mathcal{A} = \frac{\mu_0\,\hat{\boldsymbol{e}}_z}{4\pi}\int\frac{I_0\sin k(\frac{l}{2}-|z|)e^{ik(r-z'\cos\theta)}}{r-z'\cos\theta}\mathrm{d}z'$$

$$(7)\quad \frac{e^{ikR}}{R}=4\pi ik\sum_{n=0}^{\infty}\sum_{m=-n}^{n}Y^*_{nm}(\theta',\phi')Y_{nm}(\theta,\phi)j_n(kr')h_n^{(1)}(kr),\qquad r>r'$$

$$\vec{\mathcal{A}}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{j}(\vec{r}\,')e^{ikR}}{R}\,\mathrm{d}\tau'$$

$$=ik\mu_0\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\int\vec{j}(\vec{r}\,')Y^*_{nm}(\theta',\phi')j_n(kr')\,\mathrm{d}\tau'\right]h_n^{(1)}(kr)Y_{nm}(\theta,\phi)$$



五、线型天线辐射

如图，半径远小于长度的直圆柱。 设输入信号谐变，天线线电流也谐变。

天线端点 $z=\pm\frac{l}{2}$ 电流为 0： $I(z,t)=I(z)e^{-i\omega t}$， $I(z')=I_0\sin k(\frac{l}{2}-|z|)$

推迟势： $\vec{\mathcal{A}}=\mathcal{A}\,\hat{e}_z=\frac{\mu_0\,\hat{e}_z}{4\pi}\int\frac{I(z')e^{ikR}}{R}\mathrm{d}z'$, $\quad R=\sqrt{r^2+z'^2-2z'r\cos\theta}$

远场： $r\gg l\quad\Longrightarrow\quad R=r-z'\cos\theta$ 从而：

$$\mathcal{A}=\frac{\mu_0\,\hat{e}_z}{4\pi}\int\frac{I_0\sin k(\frac{l}{2}-|z|)e^{ik(r-z'\cos\theta)}}{r-z'\cos\theta}\mathrm{d}z'=\frac{\mu_0 I_0\,\hat{e}_z}{4\pi r}\int\sin k(\frac{l}{2}-|z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z'$$

Let there be light

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r} \int \sin k(\tfrac{l}{2} - |z|) e^{ik(r - z'\cos\theta)} \mathsf{d}z'$$

Let there be light

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r} \int \sin k(\tfrac{l}{2} - |z|) e^{ik(r - z' \cos\theta)} \mathsf{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi k r} \frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

Let there be light

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r - z'\cos\theta)}\mathsf{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi k r}\, \frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\mathcal{B}} = \nabla \times \vec{\mathcal{A}} = ik\vec{n} \times (\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

Let there be light

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r} \int \sin k(\tfrac{l}{2} - |z|) e^{ik(r - z' \cos\theta)} \mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi k r} \frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla \times \vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}} \times (\mathcal{A}\, \hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\, \hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0}$$

Let there be light

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\vec{n}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{n})\times\vec{\mathcal{B}}}{\omega\mu_0\epsilon_0} = c\vec{\mathcal{B}}\times\vec{n}$$

$\Longrightarrow$ $\vec{\mathcal{E}}$, $\vec{\mathcal{B}}$, $\vec{n}$ 两两垂直，$\vec{\mathcal{E}}\times\vec{\mathcal{B}}$ 沿 $\vec{n}$

辐射能流：$$\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\mathcal{E}}\times\vec{\mathcal{H}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin\theta}\right]^2$$

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi kr}\,\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi cr^2}\left[\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin\theta}\right]^2$$

讨论

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi kr}\,\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla \times \vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}} \times (\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}}) \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}} \times \vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin\theta}\right]^2$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi$

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi kr}\,\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$，$\vec{\boldsymbol{\mathcal{B}}}$，$\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi cr^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin\theta}\right]^2$$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi \quad\Longrightarrow\quad \langle\vec{\boldsymbol{S}}\rangle = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi cr^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla \times \vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}} \times (\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}}) \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}} \times \vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$，$\vec{\boldsymbol{\mathcal{B}}}$，$\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin\theta}\right]^2$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi \quad\Longrightarrow\quad \langle\vec{\boldsymbol{S}}\rangle = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

辐射功率：$P = \oint\langle\vec{\boldsymbol{S}}\rangle\cdot\hat{\boldsymbol{e}}_r r^2\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{I_0^2}{2c}\left[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)\right]$

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\frac{kl}{2}}{\sin\theta}\right]^2$$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi \quad\Longrightarrow\quad \langle\vec{\boldsymbol{S}}\rangle = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\dfrac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

辐射功率：$P = \oint\langle\vec{\boldsymbol{S}}\rangle\cdot\hat{\boldsymbol{e}}_r r^2\mathrm{d}\Omega = \dfrac{1}{4\pi\epsilon_0}\dfrac{I_0^2}{2c}[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)]$

辐射阻抗：$P = \dfrac{I_0^2}{2}R_{\mathrm{rad}} \quad\Longrightarrow\quad R_{\mathrm{rad}} = \dfrac{1}{4\pi\epsilon_0}[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)] = 73.1\ \Omega$

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla\times\vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}}\times(\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}})\times\vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}}\times\vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$\langle\vec{\boldsymbol{S}}\rangle = \dfrac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}}\times\vec{\boldsymbol{\mathcal{H}}}^*) = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\left[\dfrac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin\theta}\right]^2$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi \quad\Longrightarrow\quad \langle\vec{\boldsymbol{S}}\rangle = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\,\dfrac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

辐射功率：$P = \oint\langle\vec{\boldsymbol{S}}\rangle\cdot\hat{\boldsymbol{e}}_r r^2\mathrm{d}\Omega = \dfrac{1}{4\pi\epsilon_0}\dfrac{I_0^2}{2c}\left[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)\right]$

辐射阻抗：$P = \dfrac{I_0^2}{2}R_{\mathrm{rad}} \quad\Longrightarrow\quad R_{\mathrm{rad}} = \dfrac{1}{4\pi\epsilon_0}[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)] = 73.1\ \Omega$

辐射阻抗描述天线辐射能力，辐射阻抗越大，输入同样电流，辐射功率越大，辐射能力越好

$$\mathcal{A} = \frac{\mu_0 I_0\,\hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2}-|z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\,\hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right)-\cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\mathcal{B}} = \nabla\times\vec{\mathcal{A}} = ik\vec{n}\times(\mathcal{A}\,\hat{e}_z) = -ik\mathcal{A}\,\hat{e}_\phi$$

$$\vec{\mathcal{E}} = \frac{i\nabla\times\vec{\mathcal{B}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{n})\times\vec{\mathcal{B}}}{\omega\mu_0\epsilon_0} = c\vec{\mathcal{B}}\times\vec{n}$$

$\Longrightarrow$ $\vec{\mathcal{E}}$, $\vec{\mathcal{B}}$, $\vec{n}$ 两两垂直，$\vec{\mathcal{E}}\times\vec{\mathcal{B}}$ 沿 $\vec{n}$

辐射能流：$\langle\vec{S}\rangle = \dfrac{1}{2}\mathrm{Re}(\vec{\mathcal{E}}\times\vec{\mathcal{H}}^*) = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\left[\dfrac{\cos\left(\dfrac{kl}{2}\cos\theta\right)-\cos\dfrac{kl}{2}}{\sin\theta}\right]^2$

讨论

(1) 半波天线：$l=\lambda/2 \quad\Longrightarrow\quad kl=\pi \quad\Longrightarrow\quad \langle\vec{S}\rangle = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\dfrac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

辐射功率：$P = \oint\langle\vec{S}\rangle\cdot\hat{\boldsymbol{e}}_r r^2\mathrm{d}\Omega = \dfrac{1}{4\pi\epsilon_0}\dfrac{I_0^2}{2c}\left[\ln(2\pi\gamma)-\mathrm{Ci}(2\pi)\right]$

辐射阻抗：$P=\dfrac{I_0^2}{2}R_{\mathrm{rad}} \quad\Longrightarrow\quad R_{\mathrm{rad}} = \dfrac{1}{4\pi\epsilon_0}[\ln(2\pi\gamma)-\mathrm{Ci}(2\pi)] = 73.1\ \Omega$

辐射阻抗描述天线辐射能力，辐射阻抗越大，输入同样电流，辐射功率越大，辐射能力越好

(2) 短天线：$l\ll\lambda \quad\Longrightarrow\quad I(z') = I_0\sin k(\tfrac{l}{2}-|z|)$

$$\mathcal{A} = \frac{\mu_0 I_0\, \hat{\boldsymbol{e}}_z}{4\pi r}\int \sin k(\tfrac{l}{2} - |z|)e^{ik(r-z'\cos\theta)}\mathrm{d}z' = \frac{\mu_0 I_0 e^{ikr}\, \hat{\boldsymbol{e}}_z}{2\pi k r}\,\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin^2\theta}$$

$$\vec{\boldsymbol{\mathcal{B}}} = \nabla \times \vec{\boldsymbol{\mathcal{A}}} = ik\vec{\boldsymbol{n}} \times (\mathcal{A}\,\hat{\boldsymbol{e}}_z) = -ik\mathcal{A}\,\hat{\boldsymbol{e}}_\phi$$

$$\vec{\boldsymbol{\mathcal{E}}} = \frac{i\nabla \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = \frac{i(ik\vec{\boldsymbol{n}}) \times \vec{\boldsymbol{\mathcal{B}}}}{\omega\mu_0\epsilon_0} = c\vec{\boldsymbol{\mathcal{B}}} \times \vec{\boldsymbol{n}}$$

$\Longrightarrow$ $\vec{\boldsymbol{\mathcal{E}}}$, $\vec{\boldsymbol{\mathcal{B}}}$, $\vec{\boldsymbol{n}}$ 两两垂直，$\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{B}}}$ 沿 $\vec{\boldsymbol{n}}$

辐射能流：$$\langle\vec{\boldsymbol{S}}\rangle = \frac{1}{2}\mathrm{Re}(\vec{\boldsymbol{\mathcal{E}}} \times \vec{\boldsymbol{\mathcal{H}}}^*) = \frac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\frac{I_0^2}{2\pi c r^2}\left[\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right) - \cos\dfrac{kl}{2}}{\sin\theta}\right]^2$$

讨论

(1) 半波天线：$l = \lambda/2 \quad\Longrightarrow\quad kl = \pi \quad\Longrightarrow\quad \langle\vec{\boldsymbol{S}}\rangle = \dfrac{\hat{\boldsymbol{e}}_r}{4\pi\epsilon_0}\dfrac{I_0^2}{2\pi c r^2}\,\dfrac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

辐射功率：$$P = \oint\langle\vec{\boldsymbol{S}}\rangle\cdot\hat{\boldsymbol{e}}_r r^2\mathrm{d}\Omega = \frac{1}{4\pi\epsilon_0}\frac{I_0^2}{2c}\left[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)\right]$$

辐射阻抗：$P = \dfrac{I_0^2}{2}R_{\text{rad}} \quad\Longrightarrow\quad R_{\text{rad}} = \dfrac{1}{4\pi\epsilon_0}[\ln(2\pi\gamma) - \mathrm{Ci}(2\pi)] = 73.1\ \Omega$

辐射阻抗描述天线辐射能力，辐射阻抗越大，输入同样电流，辐射功率越大，辐射能力越好

(2) 短天线：$l \ll \lambda \quad\Longrightarrow\quad I(z') = I_0\sin k(\tfrac{l}{2} - |z|) = I_0'(1 - \tfrac{2|z|}{l})$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|)$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I'_0(1 - \frac{2|z|}{l})$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}}$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \dfrac{\mu_0}{4\pi} \dfrac{e^{ikr}}{r} \dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \displaystyle\int_{-l/2}^{l/2} I(z')\, \hat{\boldsymbol{e}}_z \mathrm{d}z' = \dfrac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, {I_0'}^2 \left(\dfrac{l}{\lambda}\right)^2$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{\mathcal{A}}}^{(0)} = \dfrac{\mu_0}{4\pi} \dfrac{e^{ikr}}{r} \dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \displaystyle\int_{-l/2}^{l/2} I(z')\, \hat{\boldsymbol{e}}_z \mathrm{d}z' = \dfrac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{\mathcal{A}}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, I_0'^{\,2} \left(\dfrac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathsf{rad}} = \dfrac{\pi}{6} Z_0 (\dfrac{l}{\lambda})^2$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\mathcal{A}}^{(0)} = \dfrac{\mu_0}{4\pi} \dfrac{e^{ikr}}{r} \dot{\vec{p}} \qquad \dot{\vec{p}} = \displaystyle\int_{-l/2}^{l/2} I(z')\, \hat{e}_z \mathrm{d}z' = \dfrac{I_0' l\ \hat{e}_z}{2}$

由 $\vec{\mathcal{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, I_0'^2 \left(\dfrac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathsf{rad}} = \dfrac{\pi}{6} Z_0 (\dfrac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}\,{I_0'}^2\left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathsf{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\, \hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}\, I_0'^2 \left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathsf{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \displaystyle\int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \dfrac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, I_0'^{\,2}\left(\dfrac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \dfrac{\pi}{6} Z_0 (\dfrac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}\, I_0'^2 \left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$　对短天线，天线越长，辐射（接受）能力越强。
由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I'_0(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \displaystyle\int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \dfrac{I'_0 l\ \hat{\boldsymbol{e}}_z}{2}$

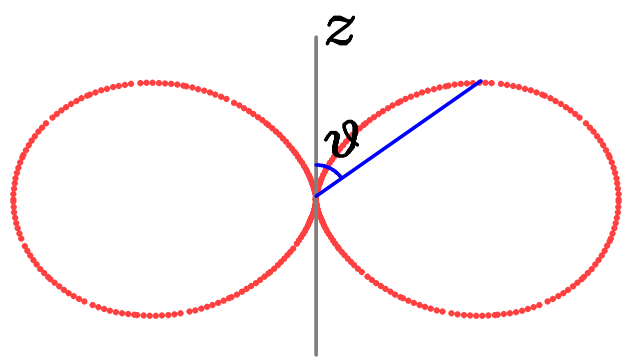
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, I_0'^2 \left(\dfrac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \dfrac{\pi}{6} Z_0 (\dfrac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\, \hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

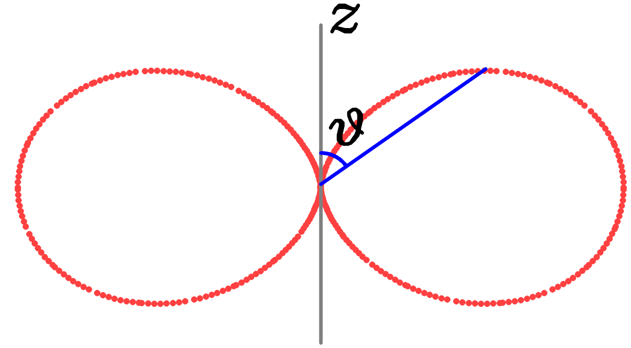
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}} I_0'^2 \left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathrm{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



$$\langle f(\theta, \phi) \rangle \propto \sin^2 \theta$$

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

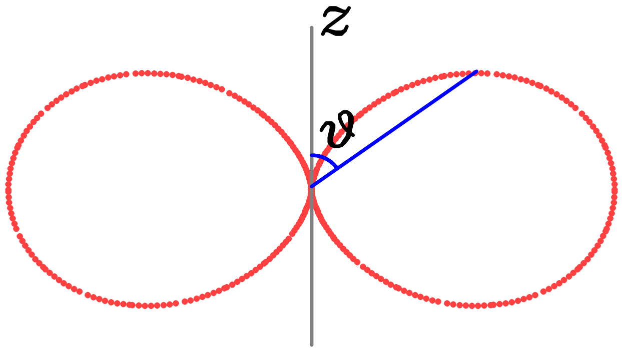
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}I_0'^2\left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\rm rad} = \frac{\pi}{6}Z_0(\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$　对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

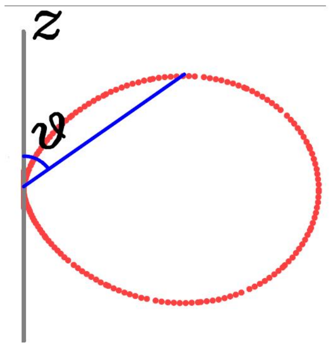
(3) 提高偶极天线的辐射能力：

以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



$$\langle f(\theta,\phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图



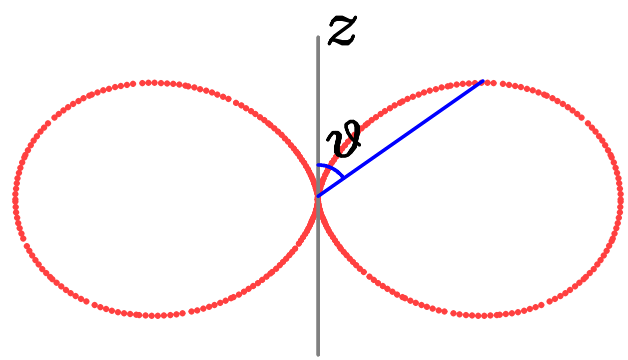
(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \displaystyle\int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \dfrac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \dfrac{\pi}{12}\sqrt{\dfrac{\mu_0}{\epsilon_0}}\, I_0'^2 \left(\dfrac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\mathrm{rad}} = \dfrac{\pi}{6} Z_0 (\dfrac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 　对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。



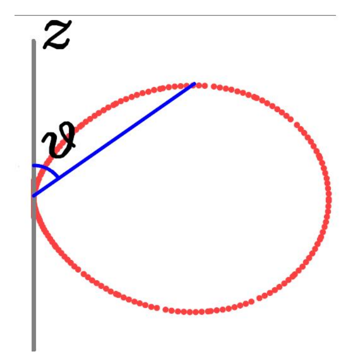
(3) 提高偶极天线的辐射能力：

以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：

$$\langle f(\theta,\phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？



(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

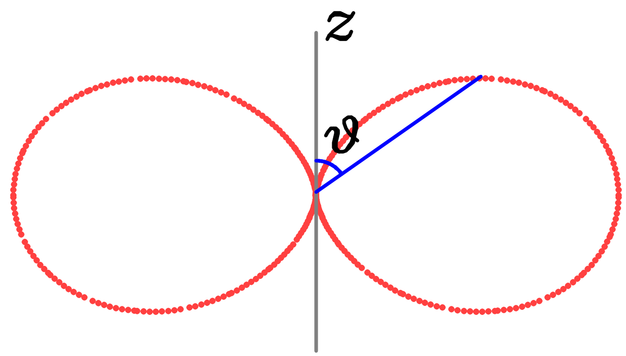
这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}} I_0'^2 \left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

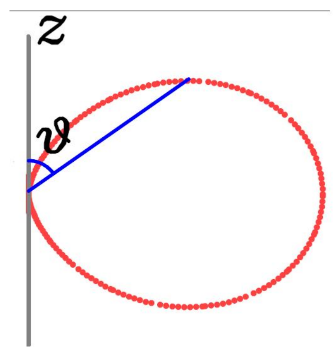


以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：

$$\langle f(\theta, \phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？ NO!



(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

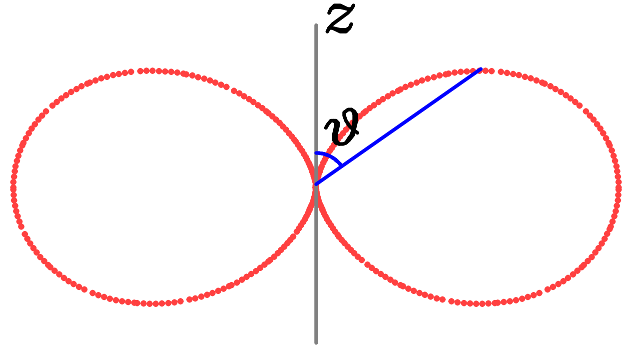
这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}I_0'^2\left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6}Z_0(\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$　对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

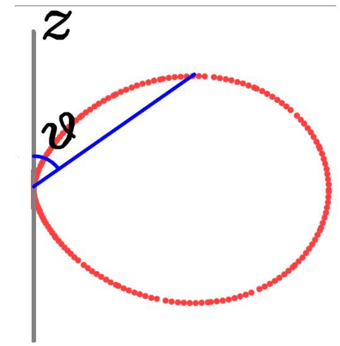


以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：

$$\langle f(\theta,\phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？　NO!　—— why?



(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

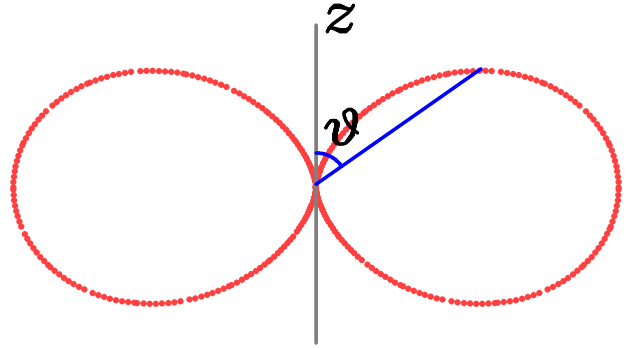
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}I_0'^2\left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6}Z_0(\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

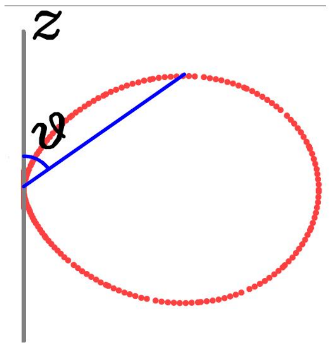
以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



$$\langle f(\theta,\phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？ NO! —— why?



如何改进

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

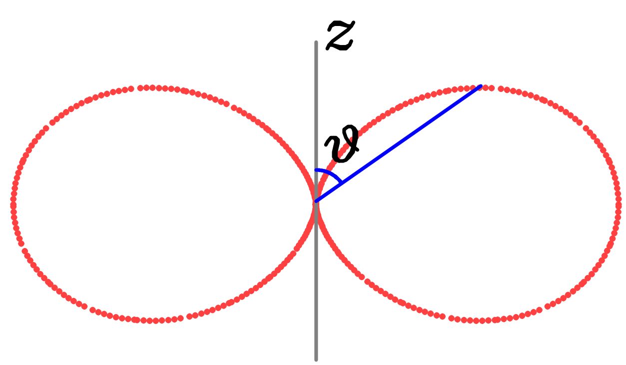
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}}I_0'^2\left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6}Z_0(\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

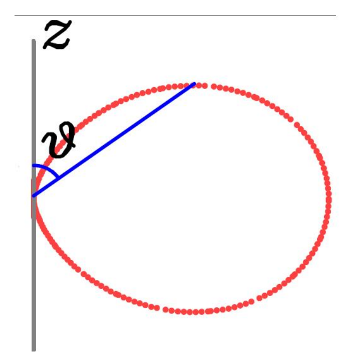
以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



$$\langle f(\theta, \phi)\rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？ NO! —— why?



如何改进 —— 光子晶体、electromagnetic metamaterials

(2) 短天线：$l \ll \lambda \quad \Longrightarrow \quad I(z') = I_0 \sin k(\frac{l}{2} - |z|) = I_0'(1 - \frac{2|z|}{l})$

这时可用电偶极辐射近似：$\vec{\boldsymbol{A}}^{(0)} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\vec{\boldsymbol{p}}} \qquad \dot{\vec{\boldsymbol{p}}} = \int_{-l/2}^{l/2} I(z')\,\hat{\boldsymbol{e}}_z \mathrm{d}z' = \frac{I_0' l\ \hat{\boldsymbol{e}}_z}{2}$

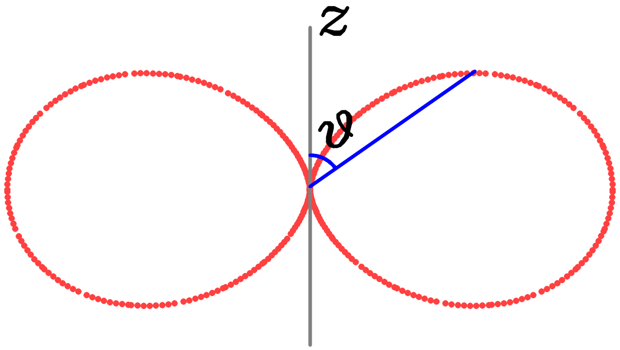
由 $\vec{\boldsymbol{A}}^{(0)}$ 可求得辐射场及辐射功率：$P = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\epsilon_0}} I_0'^2 \left(\frac{l}{\lambda}\right)^2 \quad \Longrightarrow \quad R_{\text{rad}} = \frac{\pi}{6} Z_0 (\frac{l}{\lambda})^2$

Z_0 为真空中的波阻抗，$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ 对短天线，天线越长，辐射（接受）能力越强。

由于短天线为电偶极辐射，故也称偶极天线。

(3) 提高偶极天线的辐射能力：

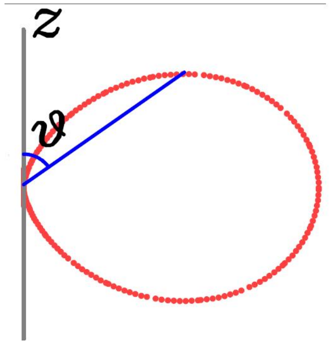
以上对短天线的讨论假设了天线置于真空中，辐射角分布如图：



$$\langle f(\theta, \phi) \rangle \propto \sin^2\theta$$

对于手机，人们希望向外辐射增强，向内减弱。如下图

是否在如图偶极天线左边放一金属板即可？ NO! —— why?

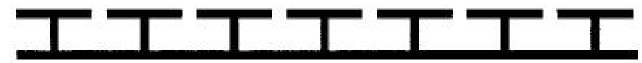


如何改进 —— 光子晶体、electromagnetic metamaterials

Yablonovitch, IEEE Trans Microwave Theory and Technology, **47**, 2059 (1999)

Let there be light

Yablonovitch, IEEE Trans Microwave Theory and Technology, **47**, 2059 (1999)



(a)