

### Problem set 4

Due: Wednesday, November 10

#### 1. (Bertrand competition with product differentiation)

1. Two firms, A and B, produce differentiated products and are engaged in Bertrand competition. Suppose the demand function for the firm a is:

$$q_a = 24 - \theta p_a + p_b$$

and similarly for firm b is

$$q_b = 24 - \theta p_b + p_a$$

$\theta$  is a measure of how different the firms' products are. Marginal costs are assumed to be zero.

(a) Set up the maximization problem for firm a, derive the reaction function and solve for the optimal prices and equilibrium profits.

(b) Compare your results from part (a) to the standard Bertrand case.

#### Linear City Model With a monopoly

2. Friars Ice Cream, Inc. has a monopoly on the sales of ice cream in a linear Hotelling town. It currently has one retail store located at the far left end of town (at  $x = 0$ ) and consumers are evenly distributed on the unit interval. Each consumer places a value of  $\bar{s}$  on the consumption of a single cup of ice cream. Because of the perishable nature of ice cream, the transportation cost incurred by a consumer is quadratic in the distance he must travel to Friars: a consumer incurs transportation cost =  $t \times (\text{distance})^2$  where distance is the one-way distance he must travel to make a purchase. The monopolist has no marginal cost of supplying a consumer. Assume  $t = 1$ .

a) Express the effective price (i.e. price inclusive of the transport cost) paid by a consumer of a purchase from Friars' store located at a distance of  $x$ , and then the net utility received from that purchase.

b) Find the profit-maximizing price Friars Ice Cream, Inc. will charge and determine the extent of the market it will serve. Hint: Write price as a function of the distance to the "marginal" customer (i.e. the customer who will be just indifferent between buying and not buying)

- c) Show that the monopolist will serve the entire town provided  $\bar{s} \geq 3$ .
- d) Find an expression that ensures the firm will breakeven when it serves the entire town.

Suppose now that Friars has the option of opening up a second location but, because of zoning restrictions, the only possibility is at the opposite end of town ( $x = 1$ ).

- e) If it opened this second store, would Friars charge the same price at the two locations, or different prices and hence engage in price discrimination? Explain your answer.

### 3. Exercise 7.6, Oz Shy questions 1, 2,3

#### **Hotelling model with heterogeneous firms**

4. Imagine that the two hair salons located on main street have different constant unit costs. In particular one salon has a unit cost of \$10, whereas the other salon has a constant unit cost of \$20. The low cost salon, Cheap-Cuts is located at the east end of the town,  $x = 0$ . The high cost salon, The Ritz, is located at the west end of the town  $x = 1$ . There are 100 potential customers who live along the one mile stretch and they are uniformly spread out along the mile. Consumers are willing to pay \$50 for a haircut done at their home. If a consumer has to travel to get a haircut, then a travel cost of \$5 per mile is incurred. Each salon wants to set a price for a haircut that maximizes the salon's profit.
- The demand functions facing the two salons are not affected by the fact that one salon is high cost and the other salon is low cost. However the salon's best response functions are affected. Compute the best response function for each salon. How does an increase in the unit cost of one salon affect the other salon's best response?
  - Work out the Nash equilibrium in prices for this model. Compare these prices to the case where both firms have unit cost equal to \$10. Explain why prices changed in the way they did.

#### **Salop Circular City Model**

5. Consider a market characterized by competition around a unit circle, where consumers are uniformly distributed on the circle (their density is equal to 1). Consumers wish to buy one unit of the good and have a transport cost  $t$  of \$16 per unit distance. Each consumer will purchase exactly one unit from the lowest-effective-price firm provided that their effective price, is less than their reservation price  $s$  of \$50, and zero otherwise.

Also, assume marginal cost of production  $c$  is \$8 and that there is a fixed cost  $f$  of \$1 for a firm to locate on the circle. In the first stage, potential entrants simultaneously choose whether or not to enter. Let  $n$  denote the number of entering firms. Firms are automatically located equidistant from one another on the circle.

a. Given  $n$  firms, so that the first stage of the game has already occurred, solve for the equilibrium price  $p$ ?

(b) What is the equilibrium number of firms in the market? What is the equilibrium price now that you determine the equilibrium number of firms?

### **Fixed Variety Model**

6. Consider the following demand equations in a differentiated product model.

$$q_1 = 100 - 1.5p_1 + p_2 \text{ and } q_2 = 100 - 1.5p_2 + p_1.$$

- (a). Express this system of demand equation in the inverse demand form.
- (b). Based on the measure of differentiation that we derived in class obtain the measure for this system of demand equation. Explain how would you represent increasing homogeneity of the products.

(c). Suppose producers of both 1 and 2 have constant marginal cost equal to zero. Solve for the equilibrium when they compete as Cournot firms. Represent the Cournot Equilibrium on a graph.

(d). How does the equilibrium in this case compare to the case where products are exactly homogeneous?