

Ab initio study of the radiation pressure on dielectric and magnetic media

Brandon A. Kemp, Tomasz M. Grzegorzczuk, and Jin Au Kong

Research Laboratory of Electronics, Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, USA

bkemp@mit.edu

Abstract: The Maxwell stress tensor and the distributed Lorentz force are applied to calculate forces on lossless media and are shown to be in excellent agreement. From the Maxwell stress tensor, we derive analytical formulae for the forces on both a half-space and a slab under plane wave incidence. It is shown that a normally incident plane wave pushes the slab in the wave propagation direction, while it pulls the half-space toward the incoming wave. Zero tangential force is derived at a boundary between two lossless media, regardless of incident angle. The distributed Lorentz force is applied to the slab in a direct way, while the half-space is dealt with by introducing a finite conductivity. In this regard, we show that the ohmic losses have to be properly accounted for, otherwise differing results are obtained. This contribution, together with a generalization of the formulation to magnetic materials, establishes the method on solid theoretical grounds. Agreement between the two methods is also demonstrated for the case of a 2-D circular dielectric particle.

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OCIS codes: (260.2110) Electromagnetic Theory; (140.7010) Trapping; (290.5850) Particle Scattering

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1. Introduction

Recently, the radiation pressure exerted by an electromagnetic field impinging on a medium was derived by the direct application of the Lorentz law [1]. This method applies the Lorentz force

to bound currents distributed throughout the medium and bound charges at the surface of the medium. The approach allows for the computation of force at any point inside a dielectric [1, 2, 3, 4] and has been shown applicable to numerical methods, such as the finite-difference-time-domain (FDTD) [4]. A similar approach was taken to study the radiation pressure on a dielectric surface [5] and a semiconductor exhibiting the photon drag effect [6]. Comparison with the established stress tensor approach has not been done previously. The purpose of this paper is to compare this approach with the application of the Maxwell stress tensor [7, 8] to lossless media.

In this paper, two important generalizations of the method proposed in [1] are introduced that allow for the calculation of forces on lossless media. First, the force on a magnetic polarization vector and a bound magnetic charge density are introduced to the distributed Lorentz force to model magnetic materials. The second generalization allows for the discrimination of the force on free carriers in a slightly conducting medium, which may not contribute to the total force on the bulk medium. In addition to generalizing the Lorentz method, we also derive closed-form expressions from the Maxwell stress tensor for the force on a lossless slab and a half-space. At normal incidence, the force on a slab is in the wave propagation direction, while the force on a half-space pulls it toward the incoming wave. At any incident angle, the tangential force at a boundary is shown to be zero. The force on a 2D dielectric cylinder is calculated by both methods and shown to be in agreement. Furthermore, we contrast the two methods in their relative advantages and disadvantages.

2. Force calculation methods

The time-average forces of electromagnetic waves incident on dielectric and magnetic bodies are calculated from the total complex field vectors due to both the incident waves and the scattered waves from the bodies. The Maxwell stress tensor approach to compute these forces is considered first. Second, the direct application of the Lorentz force is generalized from [1, 4].

2.1. The Maxwell stress tensor

The momentum conservation theorem is derived from the Maxwell equations and the Lorentz force and is given by [8]

$$\vec{f}(\vec{r}, t) = -\frac{\partial \vec{G}(\vec{r}, t)}{\partial t} - \nabla \cdot \vec{T}(\vec{r}, t), \quad (1)$$

where \vec{r} and t refer to position and time, respectively, $\vec{G}(\vec{r}, t) = \vec{D}(\vec{r}, t) \times \vec{B}(\vec{r}, t)$ is the momentum density vector, $\vec{T}(\vec{r}, t)$ is the time-domain Maxwell stress tensor, and $f(\vec{r}, t)$ is the force density in $[N/m^3]$. The momentum density vector is fundamentally defined in terms of the electric flux density $\vec{D}(\vec{r}, t)$ and the magnetic flux density $\vec{B}(\vec{r}, t)$. By integrating over a volume V enclosed by a surface S and using the divergence theorem, the total force can be written as

$$\vec{F}(t) = -\frac{\partial}{\partial t} \int_V dV \vec{G}(\vec{r}, t) - \oint_S dS [\hat{n} \cdot \vec{T}(\vec{r}, t)]. \quad (2)$$

We are generally interested in the time average force, which is

$$\vec{F} = -\frac{1}{2} \text{Re} \left\{ \oint_S dS [\hat{n} \cdot \vec{T}(\vec{r})] \right\}, \quad (3)$$

since the time average of the first term on the right-hand side of equation (2) is zero. The complex Maxwell stress tensor for lossless media is

$$\vec{T}(\vec{r}) = \frac{1}{2} (\vec{D} \cdot \vec{E}^* + \vec{B}^* \cdot \vec{H}) \vec{I} - \vec{D} \vec{E}^* - \vec{B}^* \vec{H}, \quad (4)$$

where \bar{I} is the 3×3 identity matrix and $(^*)$ denotes the complex conjugate. Eqs. (3) and (4) are applied later to calculate the force on media.

2.2. Lorentz force on a host medium

The Lorentz force is applied to both bound currents due to the polarization of a dielectric and bound charges at the boundaries resulting from discontinuity of $\hat{n} \cdot \epsilon_0 \bar{E}$ [1]. The time-average Lorentz force in $[N/m^3]$ is found to be

$$\bar{f} = \frac{1}{2} \text{Re} \{ \rho_e \bar{E}^* + \bar{J} \times \bar{B}^* + \rho_m \bar{H}^* + \bar{M} \times \bar{D}^* \}, \quad (5)$$

where $(^*)$ denotes a complex conjugate. In Eq. (5), bound magnetic current \bar{M} and bound magnetic charge ρ_m have been added to the formulation of [1] to account for permeable media. The magnetic flux \bar{B} is modeled by a magnetic polarization vector \bar{P}_m such that $\bar{B} = \mu_r \mu_0 \bar{H} = \mu_0 \bar{H} - \bar{P}_m$ and it follows that

$$\bar{P}_m = -\mu_0 (\mu_r - 1) \bar{H}. \quad (6)$$

By application of Gauss's law, the bound magnetic charges are given by $\rho_m = \nabla \cdot \bar{P}_m = \nabla \cdot \mu_0 \bar{H}$. Invoking charge continuity leads to an expression for the bound magnetic current density, which can be used directly in equation (5)

$$\bar{M} = -i\omega \bar{P}_m = i\omega \mu_0 (\mu_r - 1) \bar{H}. \quad (7)$$

By duality a similar expression for the bound electric current density is obtained as in [1]

$$\bar{J} = -i\omega \bar{P}_e = -i\omega \epsilon_0 (\epsilon_r - 1) \bar{E}, \quad (8)$$

where ϵ_r can be complex to account for material losses [4]. The charge distributions in equation (5) are found at a medium boundary by considering the discontinuity in the normal fields. For example, at a boundary between two media referenced by the subscripts 0 and 1,

$$\rho_e = \hat{n} \cdot (\bar{E}_1 - \bar{E}_0) \epsilon_0 \quad \rho_m = \hat{n} \cdot (\bar{H}_1 - \bar{H}_0) \mu_0, \quad (9)$$

where \hat{n} is a unit vector normal to the surface pointing from region 0 to region 1. When applying Eq. (9), the average of the normal field vectors across the boundary should be used [1]. To get the total force on a material body from Eq. (5), the contribution from distributed current densities \bar{J} and \bar{M} are integrated over the volume of the medium and the effect of bound charge densities ρ_e and ρ_m are included at the medium boundaries.

3. Lossless slab

First, the problem of TE incidence on a lossless slab is considered, as shown in Fig. 1. The forces from a TM polarized wave are directly obtained from the duality principle. The forces are calculated by using the two methods which are referred to as stress tensor and Lorentz, from section 2.1 and section 2.2, respectively.

The force on an isotropic slab characterized by $\mu_1 = \mu_r \mu_0$ and $\epsilon_1 = \epsilon_r \epsilon_0$ is evaluated in free space ($\mu_2 = \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$, $\epsilon_2 = \epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$) due to an incident TE plane wave

$$\bar{E}_i = \hat{y} E_i e^{ik_0 z} e^{ik_x x}, \quad (10)$$

where E_i is the incident field magnitude (see Fig. 1). The total fields in the three regions are found by application of the boundary conditions [8].

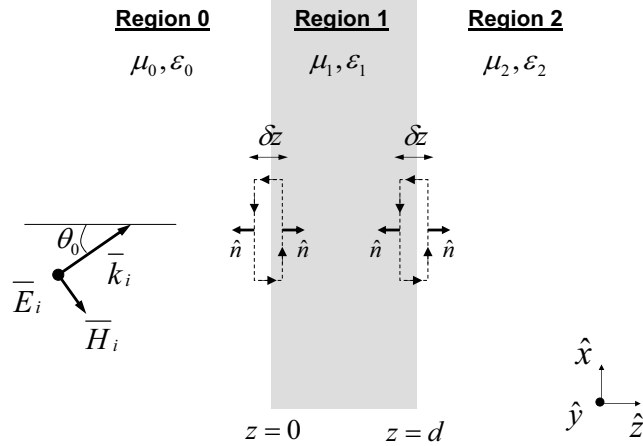


Fig. 1. A plane wave is incident onto a slab of thickness d and incident angle θ_0 . The integration path for the application of the Maxwell stress tensor to calculate the force on a slab is shown by the dotted lines. The path is shrunk so that $\delta z \rightarrow 0$. The integration is performed along the surface on both sides of a boundaries.

The Maxwell stress tensor is applied through Eq. (3) by selecting the integration path shown in Fig. 1. This integration path is chosen such that the fields are evaluated at the boundary as $\delta z \rightarrow 0$. The force per unit area on the slab of thickness d is, therefore, given by

$$\bar{F} = \frac{1}{2} \text{Re} \{ \hat{z} \cdot \bar{T}(z=0^-) - \hat{z} \cdot \bar{T}(z=0^+) + \hat{z} \cdot \bar{T}(z=d^-) - \hat{z} \cdot \bar{T}(z=d^+) \}, \quad (11)$$

where $\bar{T}(z=z_0)$ is the Maxwell stress tensor of equation (4) evaluated at the point $z=z_0$. The contributions from the fields inside the slab are restricted to the terms

$$\hat{z} \cdot \bar{T}(z=0^+) = \hat{z} \left[\frac{\epsilon_1}{2} |E_y(z=0^+)|^2 + \frac{\mu_1}{2} (|H_x(z=0^+)|^2 - |H_z(z=0^+)|^2) \right] + \hat{x} [-\mu_1 H_z(z=0^+) H_x^*(z=0^+)], \quad (12a)$$

$$\hat{z} \cdot \bar{T}(z=d^-) = \hat{z} \left[\frac{\epsilon_1}{2} |E_y(z=d^-)|^2 + \frac{\mu_1}{2} (|H_x(z=d^-)|^2 - |H_z(z=d^-)|^2) \right] + \hat{x} [-\mu_1 H_z(z=d^-) H_x^*(z=d^-)]. \quad (12b)$$

By substitution of the fields in the slab, it can be shown that

$$\hat{z} \cdot \bar{T}(z=0^+) = \hat{z} \cdot \bar{T}(z=d^-), \quad (13)$$

and the radiation pressure on the slab reduces to

$$\bar{F} = \frac{1}{2} \text{Re} \{ \hat{z} \cdot \bar{T}(z=0^-) - \hat{z} \cdot \bar{T}(z=d^+) \}. \quad (14)$$

Therefore, the force on a lossless slab can be computed solely from the knowledge of the fields outside the slab. This is to be expected and can be generalized to media of arbitrary geometry since the divergence of the stress tensor applied to continuous, lossless media is

zero. Simplification of Eq. (14) with knowledge of the reflected and transmitted fields gives the closed-form force per unit area on the slab as

$$\bar{F} = \hat{z} \frac{\epsilon_0}{2} E_i^2 \cos^2 \theta_0 [1 + |R_{slab}|^2 - |T_{slab}|^2], \quad (15)$$

where θ_0 is the incident angle and R_{slab} and T_{slab} are the slab reflection and transmission coefficients, respectively [8]. This analytic expression shows that the \hat{x} -component of the force is zero ($F_x = 0$), regardless of incident angle. At normal incidence, equation (15) can be written simply as the summation of force components due to momentum conservation,

$$\bar{F}_i = \hat{z} \frac{\epsilon_0}{2} E_i^2, \quad (16a)$$

$$\bar{F}_r = \hat{z} \frac{\epsilon_0}{2} E_i^2 |R_{slab}|^2, \quad (16b)$$

$$\bar{F}_t = -\hat{z} \frac{\epsilon_0}{2} E_i^2 |T_{slab}|^2, \quad (16c)$$

where \bar{F}_i , \bar{F}_r , and \bar{F}_t are the forces due to the incident, reflected, and transmitted wave momentums, respectively.

Equation (5) is used to calculate the Lorentz force on the slab, and the results are compared with Eq. (15). Figure 2 shows excellent agreement between the two methods applied to compute the force on a lossless dielectric slab ($\mu_r = 1, \epsilon_r = 4$). The maxima and minima in the force are due to the periodic dependence of the reflection coefficient R_{slab} and the transmission coefficient T_{slab} on the slab thickness. Minimum force is observed for slab thicknesses equal to multiples of half wavelength $n \frac{\lambda_1}{2}$ and maximum force is observed for slab thicknesses equal to odd multiples of quarter wavelength $(2n + 1) \frac{\lambda_1}{4}$, where λ_1 is the wavelength of the electromagnetic wave inside the slab and $n \in [0, 1, 2, \dots]$. For comparison, note that for $d = \lambda_1/4$, $f = 3.182 [pN/m^2]$ is calculated by the Lorentz method and $f = 3.184 [pN/m^2]$ is calculated by the application of the Maxwell stress tensor, which is in reasonable agreement with $f = 3.188 [pN/m^2]$ previously reported [4].

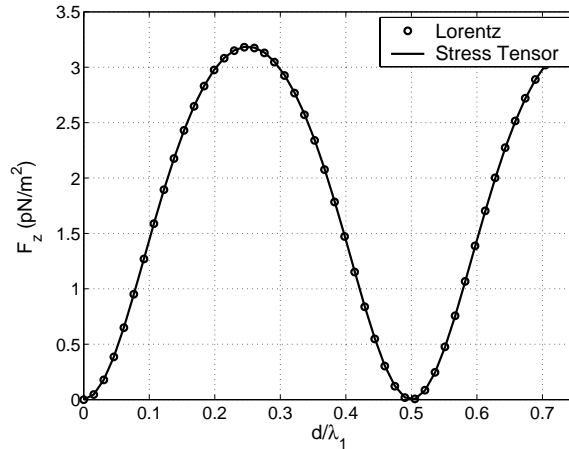


Fig. 2. Force density from a normal incident wave onto a lossless slab as a function of slab thickness d . The free space wavelength is $\lambda_0 = 640nm$ and $\epsilon_r = 4, \mu_r = 1, E_i = 1$. Shown are the forces calculated from the distributed Lorentz force (circles) and the Maxwell stress tensor (line). The background medium (region 0 and region 2) is free space.

Figure 3 shows the force on a permeable slab ($\mu_r = 4, \epsilon_r = 1$) as a function of incident angle θ_0 , measured from the surface normal. This example requires the calculation of both magnetic currents from Eq. (7) and magnetic surface charge from Eq. (9), which are used to compute the Lorentz force. As seen in Fig. 3, there is excellent agreement between the two force calculation methods at all incident angles. The Brewster angle for total transmission of a TE incident wave is given by [8]

$$\theta_b = \tan^{-1} \sqrt{\frac{\mu_1}{\mu_0}}, \quad (17)$$

and is calculated for this example to be $\theta_b = 63.4^\circ$. This is evident in Fig. 3 by the zero force at this particular angle.

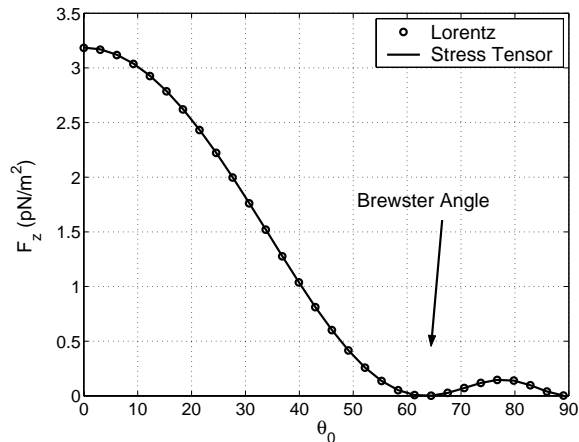


Fig. 3. Force density from an oblique incident wave on a quarter-wave slab ($d = \lambda_1/4 = 80nm$) as a function of incident angle θ_0 . The free space wavelength is $\lambda_0 = 640nm$ and $\epsilon_r = 1, \mu_r = 4, E_i = 1$. Shown are the forces calculated from the distributed Lorentz force (circles) and the Maxwell stress tensor (line). The background medium (region 0 and region 2) is free space.

The force is shown as a function of dielectric constant ϵ_r for a lossless dielectric slab in Fig. 4. Again, there is excellent agreement between the two force calculation methods. It is seen that the force goes to zero as expected when the slab is impedance matched to free space $\epsilon_r = 1$. For all positive permittivities, including the region $0 \leq \epsilon_r \leq 1$, the force is in the positive \hat{z} -direction, indicating that the force is pushing the slab.

4. Semi-infinite half-space

Calculation of the Lorentz force on a half-space is more involved than what might be expected. For a lossless half-space, the fields propagate inside the medium without attenuation. This poses a problem in finding the radiation pressure by integrating over the Lorentz force from $z = 0$ to $z \rightarrow \infty$. Previously, this issue has been sidestepped by introducing a small amount of loss in the medium, applying the distributed Lorentz force, and allowing the losses to approach zero after integration [1, 5]. The problem with this approach is that not all of the force on free charges can be attributed to force on the bulk medium. Some of the wave energy may be lost, such as ohmic losses in a conducting medium, which must be considered when calculating the total force on a material body. This problem will be addressed subsequently.

First, we derive the exact solution to the half-space problem of an incident TE plane wave by applying the Maxwell stress tensor to the boundary as described in section 2.1. The path of

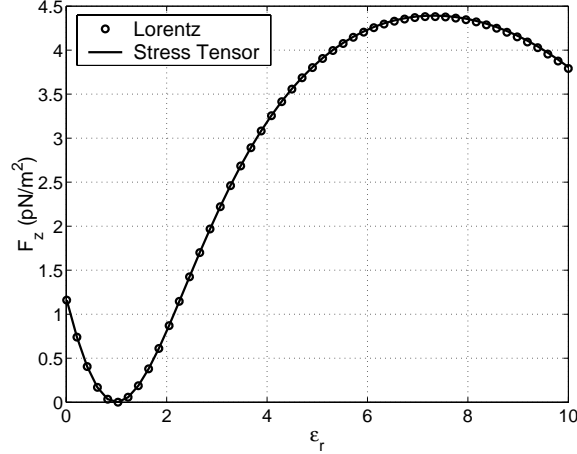


Fig. 4. Force density from a normal incident wave on a lossless slab ($d = 80nm$) as a function of the relative permittivity ϵ_r . The free space wavelength is $\lambda_0 = 640nm$ and $\mu_r = 1$, $E_i = 1$. Shown are the forces calculated from the distributed Lorentz force (circles) and the Maxwell stress tensor (line). The background media (region 0 and region 2) is free space.

integration for the Maxwell stress tensor is the same as that shown in Fig. 1 except that there is only one boundary at $z = 0$. Again we allow the integration to just enclose the boundary such that $\delta z \rightarrow 0$. The force per unit area on the half space medium is given by

$$\bar{F} = \frac{1}{2} Re \{ \hat{z} \cdot \bar{T}(z = 0^-) - \hat{z} \cdot \bar{T}(z = 0^+) \}. \quad (18)$$

The contributions on the two sides of the interface are

$$\hat{z} \cdot \bar{T}(z = 0^-) = \hat{z} \left[\frac{\epsilon_0}{2} |E_y(z = 0^-)|^2 + \frac{\mu_0}{2} (|H_x(z = 0^-)|^2 - |H_z(z = 0^-)|^2) \right] + \hat{x} [-\mu_0 H_z(z = 0^-) H_x^*(z = 0^-)], \quad (19a)$$

$$\hat{z} \cdot \bar{T}(z = 0^+) = \hat{z} \left[\frac{\epsilon_1}{2} |E_y(z = 0^+)|^2 + \frac{\mu_1}{2} (|H_x(z = 0^+)|^2 - |H_z(z = 0^+)|^2) \right] + \hat{x} [-\mu_1 H_z(z = 0^+) H_x^*(z = 0^+)], \quad (19b)$$

and the force tangential to the boundary is seen to be

$$F_x = \frac{1}{2} Re \left\{ -\mu_0 H_z(z = 0^-) H_x^*(z = 0^-) + \mu_1 H_z(z = 0^+) H_x^*(z = 0^+) \right\}. \quad (20)$$

Upon substitution of the fields on both sides of the boundary, the tangential force is simplified to

$$F_x = \frac{1}{2} E_i^2 \frac{k_x}{\omega^2} Re \left\{ \left(\frac{k_{0z}}{\mu_0} \right)^* [(1 + R_{hs})(1 - R_{hs})^* - p_{01}^* |T_{hs}|^2] \right\} \quad (21)$$

where R_{hs} and T_{hs} are the half-space reflection and transmission coefficients, respectively and p_{01} is given by [8]

$$p_{01} = \frac{\mu_0 k_{1z}}{\mu_1 k_{0z}}. \quad (22)$$

The simplification to Eq. (21) is a direct result of phase matching, which requires k_x to be continuous across the boundary. Applying the formulas $T_{hs} = 1 + R_{hs}$ and $p_{01}T_{hs} = 1 - R_{hs}$ obtained from the boundary conditions, it immediately follows that

$$p_{01}^*|T_{hs}|^2 = (1 + R_{hs})(1 - R_{hs})^* \quad (23)$$

so that the tangential force is zero ($F_x = 0$). This result is a direct consequence of the boundary conditions and phase matching at a single interface, and it can be generalized to multiple interfaces, including the slab of section 3, the details of which were omitted for brevity. This differs from previous reports of derived nonzero tangential force at a planar boundary [1, 4].

The radiation pressure on a lossless half-space medium is found from Eq. (18) to be

$$\bar{F} = \hat{z} \frac{\epsilon_0}{2} E_i^2 [\cos^2 \theta_0 (1 + R_{hs}^2) - \epsilon_r \cos^2 \theta_1 T_{hs}^2], \quad (24)$$

where ϵ_r is the relative permittivity of the half-space and θ_1 is the transmitted angle found from Snell's law. Note that this equation also accounts for variations in relative permeability μ_r , which appears in the reflection and transmission coefficients. Again we can see the contribution from the incident, reflected, and transmitted wave momentums on the force at the medium interface.

Equation (24) is applied to evaluate the radiation pressure on the half-space medium due to a normal incident plane wave. By taking the limit $\epsilon_r \rightarrow \infty$, it is simple to show that the radiation pressure approaches

$$\lim_{\epsilon_r \rightarrow \infty} \bar{F} = -\hat{z} \epsilon_0 E_i^2. \quad (25)$$

This limit is seen in Fig. 5 where $E_i = 1V/m$ such that $F_z \rightarrow -\epsilon_0(1V/m) = -8.85 [pN/m^2]$ for very large ϵ_r . Thus, the force of a normally incident plane wave on a lossless dielectric half-space is pulling toward the incoming wave, while in the slab case the plane wave pushes the medium in the wave propagating direction. Likewise, the analytical limit of $\epsilon_r \rightarrow 0$ gives

$$\lim_{\epsilon_r \rightarrow 0} \bar{F} = +\hat{z} \epsilon_0 E_i^2, \quad (26)$$

which is also seen in Fig. 5.

Equation (24) differs from the TE incident results $F_z = \frac{\epsilon_0}{2} E_i^2 (1 + R_{hs}^2) \cos^2 \theta_0$ and $F_x = \frac{\epsilon_0}{2} E_i^2 (1 - R_{hs}^2) \sin \theta_0 \cos \theta_0$ previously reported from the distribution of Lorentz force in a lossless media [1]. The previously reported results produce different values from Eq. (24) for three cases in particular. First, for an impedance matched interface such as a fictive interface between two free space media, $R_{hs} = 0$, which would yield a nonzero value of $F_z = \hat{z} \frac{\epsilon_0}{2} E_i^2$. Since such a fictive boundary could be placed anywhere, it would indicate that forces in free space are present everywhere. Second, a nonzero tangential force $F_x \neq 0$ at oblique incidence [1, 4] disagrees with Eq. (24). Third, the predicted normal force F_z is always positive for a lossless dielectric, which contradicts the known theory that the force of a normally incident plane wave on a lossless half-space is pulling toward the incident wave [8].

The differing results obtained from the distribution of Lorentz force and the Maxwell stress tensor do not imply that either result is incorrect. Instead, we interpret the force on a semi-infinite half-space obtained from the method of [1] to be the total Lorentz force on all bound and free charges and currents with the assumption that the fields attenuate to zero as $z \rightarrow \infty$ due to some finite loss, which holds once the force density is integrated, even if the losses approach zero. To illustrate this point, the force on a lossless half-space due to a normally incident wave can be computed from the distributed Lorentz force by considering the ohmic loss due to the

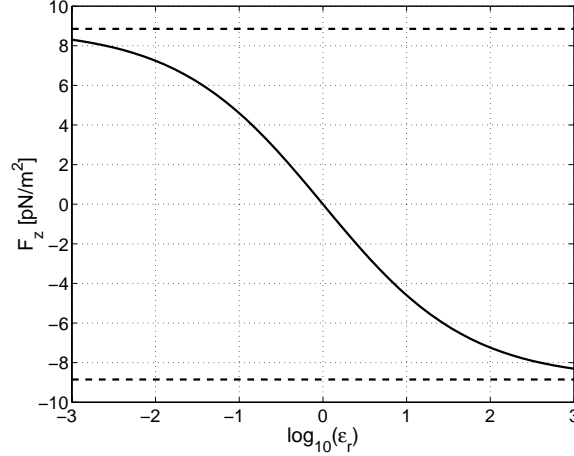


Fig. 5. Force density from a normal incident wave on a lossless half-space medium as calculated from the Maxwell stress tensor. The \hat{z} -directed force is shown as a function of the relative permittivity ϵ_r . The free space wavelength is $\lambda_0 = 640\text{nm}$ and $\mu_r = 1$, $E_i = 1$. The incident media (region 0) is free space. The dotted lines denote $\pm\epsilon_0 E_i^2 = 8.85 \text{ pN/m}^2$

conductivity σ in a slightly lossy medium. Some of the energy transferred from the wave to the conduction current \bar{J}_c is dissipated as ohmic loss. The time average power dissipated is [8]

$$P_c = \frac{1}{2} \text{Re} \left\{ \int_V dV \bar{J}_c(\bar{r}) \cdot \bar{E}^*(\bar{r}) \right\}, \quad (27)$$

which must be subtracted from the Lorentz force calculated on the conduction current \bar{J}_c . Therefore, the total radiation pressure on a dielectric half-space due to a normally incident plane wave is

$$\bar{F} = \lim_{\sigma \rightarrow 0} \frac{1}{2} \text{Re} \left\{ \int_0^\infty dz \left[\bar{J} \times \bar{B}^* - \hat{z} \frac{1}{v_e} (\bar{J}_c \cdot \bar{E}^*) \right] \right\}, \quad (28)$$

where \bar{J} is the current from Eq. (8), $\bar{J}_c = \sigma \bar{E}$ is the conduction current, and v_e is the energy velocity. After integration, the \hat{z} -directed force reduces to

$$F_z = \frac{\epsilon_0}{2} E_i^2 (1 + R_{hs}^2) - \lim_{\sigma \rightarrow 0} \frac{1}{2} \frac{\sigma}{v_e 2k_I} E_i^2 T_{hs}^2, \quad (29)$$

where k_I is the imaginary part of the wavenumber ($k_I = \text{Im}\{k_1\}$). As $\sigma \rightarrow 0$, the energy velocity and attenuation constant k_I can be approximated by [8]

$$v_e \approx \frac{1}{\sqrt{\mu_1 \epsilon_1}} \quad k_I \approx \frac{\sigma}{2} \sqrt{\frac{\mu_1}{\epsilon_1}}. \quad (30)$$

Thus, the total force on the dielectric half-space reduces to

$$F_z = \frac{\epsilon_0}{2} E_i^2 (1 + R_{hs}^2) - \frac{\epsilon_1}{2} E_i^2 T_{hs}^2 \quad (31)$$

where the first term is a result of the calculated Lorentz force [1] and the second term is derived from the ohmic losses due to the finite conductivity. This expression is in agreement with Eq. (24) at normal incidence $\theta_0 = 0$, which is derived from the Maxwell stress tensor without approximation.

5. Cylindrical particle

In this section, the force on a dielectric particle due to the superposition of three incident plane waves is calculated by the two methods of section 2. The fields are calculated by the Mie theory as in [9] and verified through comparison with results from the commercial package CST Microwave Studio ®. The force distribution $[N/m^3]$ inside the particle is then calculated using the Lorentz method. The distributed Lorentz force is numerically integrated throughout the interior surface of the particle to obtain the total force $[N/m]$. This value is then compared with the force obtained from the Maxwell stress tensor by integrating the tensor around a circle completely enclosing the 2D particle.

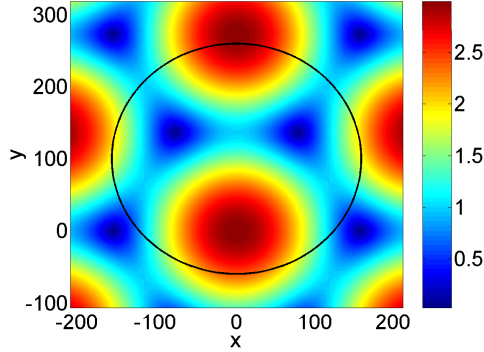


Fig. 6. The incident electric field magnitude $[V/m]$ is due to three plane waves of free space wavelength $\lambda_0 = 532$ nm propagating at angles $\{\pi/2, 7\pi/6, 11\pi/6\}$. The overlaid particle is a 2D polystyrene cylinder ($\epsilon_p = 2.56\epsilon_0$) of radius $a = 0.3\lambda_0$ with center position $(x_0, y_0) = (0, 100)$ [nm] embedded in water ($\epsilon_b = 1.69\epsilon_0$).

Three plane waves of free space wavelength $\lambda_0 = 532$ nm are incident in the (xy) plane with angles, $\{\pi/2, 7\pi/6, 11\pi/6\}$ [rad]. The particle is a polystyrene cylinder ($\epsilon_p = 2.56\epsilon_0$) of radius $a = 0.3\lambda_0$ with center position $(x_0, y_0) = (0, 100)$ [nm], and the background medium is water ($\epsilon_b = 1.69\epsilon_0$). The magnitude of the total incident electric field is shown in Fig. 6 with the position of the dielectric particle overlaid. It is seen that the particle is of the same size as a typical gradient trap in the Rayleigh regime (seen from the periodicity in the interference pattern), but poses no problem for the method used here. Figure 7 shows the total field magnitudes resulting from the incident fields and the scattered fields. The symmetry of the fields with respect to x produces zero net force in the x -direction, as we shall see.

The distributed Lorentz force is found from Eq. (5) to be

$$\bar{f} = \frac{1}{2} \text{Re} \{ -i\omega \bar{P}_e \times \mu_0 H^* \}, \quad (32)$$

where the electric polarization vector is $\bar{P}_e = \hat{z}(\epsilon_p - \epsilon_b)E_z$. The distributed Lorentz force components (f_x and f_y) inside the particle are shown in Fig. 8. It is seen that f_x is symmetric about $x = 0$, as expected. However, the distribution of f_y is more complex, and it is not immediately apparent from the distributed force if the total force F_y is positive or negative.

The total force is obtained from the Lorentz force of Fig. 8 by simple numerical integration and found to be $\bar{F} = \hat{y}2.11 \cdot 10^{-18}$ $[N/m]$. Subsequently, the Maxwell stress tensor of Eq. 4 was integrated along a circular path of radius $2a$ centered at (x_0, y_0) , resulting in a force of $\bar{F} = \hat{y}2.12 \cdot 10^{-18}$ $[N/m]$. Note that the exact path of integration for the Maxwell stress tensor is not important as long as it completely encloses the cylinder. The excellent agreement between the two force calculation methods for this example further establishes the method of [1] on

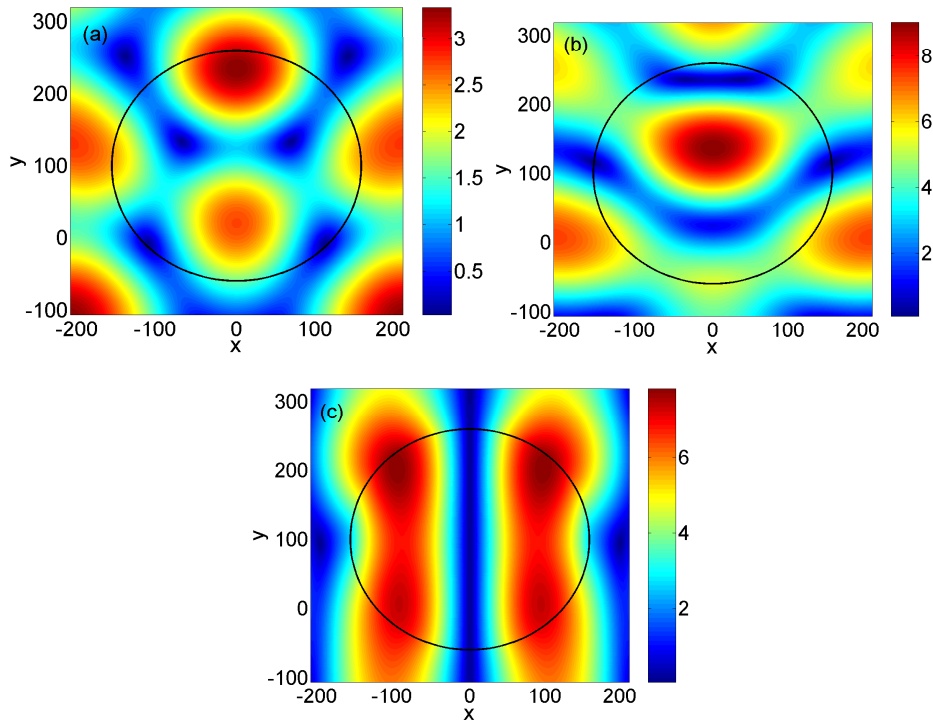


Fig. 7. The total field is calculated from Mie theory due to the three incident plane waves on a polystyrene cylinder ($\epsilon_p = 2.56\epsilon_0$) of radius $a = 0.3\lambda_0$ with center position $(x_0, y_0) = (0, 100)$ [nm] embedded in water ($\epsilon_b = 1.69\epsilon_0$). Shown are (a) $|\vec{E}_z|$ [V/m], (b) $|\vec{H}_x|$ [$10^{-3}H/m$], and (c) $|\vec{H}_y|$ [$10^{-3}H/m$].

a sound theoretical basis. However, there are some differences between the two methods. As shown in Fig. 8, the force distribution inside a medium can be easily calculated and displayed by the application of the Lorentz force. However, the stress tensor approach only yields the total force on the medium, thus it cannot be used to produce the plots in Fig. 8. The primary advantage of the stress tensor is the reduction of computation. For the 2D cylindrical particle, the surface integral used to calculate the total force from the distributed Lorentz force was reduced to a line integral for the application of the Maxwell stress tensor.

6. Conclusions

The distributed Lorentz force method of [1] was generalized to include contribution from magnetic material. This method was shown to provide results equivalent to those obtained from the application of the Maxwell stress tensor, which was used to derive simple closed form expressions for the force on a lossless slab in equation (15) and the force on a lossless half-space in equation (24). It was shown that in both cases, the force tangential to the interface is zero. A normally incident plane wave pushes a dielectric slab in the wave propagation direction and pulls a dielectric half-space toward the incident wave [8].

It was shown that the previous application of the distributed Lorentz force to the half-space problem lacks to account for the losses not manifested as force on the bulk medium. We argued that the difference between the force on a half-space medium that we calculated using the Maxwell stress tensor and the force derived from the Lorentz force previously reported [1, 4, 5]

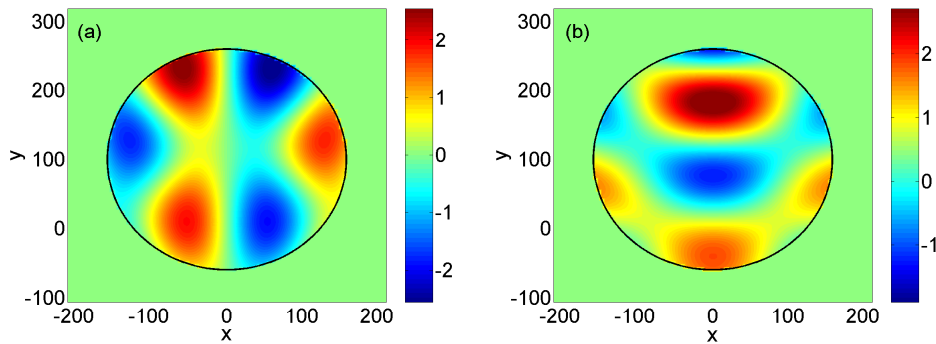


Fig. 8. The distributed Lorentz force for a polystyrene cylinder ($\epsilon_p = 2.56\epsilon_0$) of radius $a = 0.3\lambda_0$ with center position $(x_0, y_0) = (0, 100)$ [nm] embedded in water ($\epsilon_b = 1.69\epsilon_0$) due to three incident plane waves are shown. The individual components are (a) f_x [$10^{-4}N/m^3$] and (b) f_y [$10^{-4}N/m^3$]. The total force is obtained by integrating the distributed Lorentz force inside the particle.

is simply due to the loss of energy by mechanisms such as heat or absorption. As an example, Eq. (24) was derived for the normal incidence case from the Lorentz force with consideration given to ohmic energy loss in the conduction current.

Although the two force calculation methods can be applied to calculate the total force on media, there are inherent advantages and disadvantages for each method. Through numerous examples, [4] demonstrated that the distributed Lorentz method is easily applied to finite-difference time domain (FDTD) computational and can be used to find the forces distributed throughout a host medium. In particular, the tension and compression forces due to a light beam inside a dielectric medium were calculated. The Maxwell stress tensor is not suitable for this particular task because it is generally applied to media boundaries. However, the advantage of the Maxwell stress tensor is that it reduces a volume integral in three dimensions to a surface integral, thus reducing the computation required to calculate the total force on a material object. We have demonstrated the validity of these results by comparing the Lorentz force on a 2D particle with the force obtained from the Maxwell stress tensor.

Acknowledgments

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