Optical Momentum Transfer to Absorbing Mie Particles

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The momentum transfer to absorbing particles is derived from the Lorentz force density without prior assumption of the momentum of light in media. We develop a view of momentum conservation rooted in the stress tensor formalism that is based on the separation of momentum contributions to bound and free currents and charges consistent with the Lorentz force density. This is in contrast with the usual separation of material and field contributions. The theory is applied to predict a decrease in optical momentum transfer to Mie particles due to absorption, which contrasts the common intuition based on the scattering and absorption by Rayleigh particles.

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The momentum of light in optically dense media has been the center of a debate in physics for nearly a century [1,2]. Although the so-called Abraham-Minkowski controversy originated out of relativistic formulations, the primary issue of the radiation pressure exerted on the interface of a dielectric boundary can be studied independently of material motion [3]. The momentum density vector derived from the macroscopic electromagnetic wave theory [4] for a nonmagnetic medium is $\bar{G} = \bar{D} \times \bar{B} = \epsilon_0\mu_0\bar{E} \times \bar{H} + \bar{P} \times \mu_0\bar{H}$, where the wave momentum density is expressed as the sum of the electromagnetic momentum density $\epsilon_0\mu_0\bar{E} \times \bar{H}$ and a mechanical momentum density resulting from the dielectric polarization $\bar{P} = \bar{D} - \epsilon_0\bar{E}$ in the presence of a field [5]. The debate of the radiation pressure of normally incident light from free space onto a dielectric interface can be demonstrated by momentum conservation at the interface. The difference in the radiation pressure resulting from either $\bar{D} \times \bar{B}$ or $\epsilon_0\mu_0\bar{E} \times \bar{H}$ transmitted into the dielectric is significant; an outward force results from the former, while an inward force results from the latter [6].

Recently, the pressure of light on lossless media as calculated by both the application of the Lorentz force directly to bound currents and charges [7–9] and the momentum conservation theorem [4] were shown to be in agreement [10]. Application of the Lorentz force directly may be regarded as more fundamental, but it is computationally expensive compared to momentum conservation [11]. However, there still exist some questions as to the application of momentum conservation to the radiation pressure in lossy materials.

In this Letter, we rigorously treat the optical momentum transfer to lossy media in the framework of the macroscopic electromagnetic theory. We apply the Lorentz force directly to bound and free currents and charges, thus avoiding *a priori* assumptions regarding the form of wave momentum. The separation of the total Lorentz force in terms of forces on bound currents and charges ($\bar{F}_b$) and on free currents ($\bar{F}_c$) provides insight into the mechanisms of momentum transfer in lossy media. We show an equivalent interpretation in terms of momentum conservation that distinguishes two processes of momentum transfer resulting from the wave reflection or transmission at the boundary and the attenuation in the medium. Our proposed view renders a more direct description of experiments than the usual separation of wave momentum into electromagnetic and material contributions [3,5,12–14]. The Lorentz force density and momentum conservation are equivalently applied to explain relevant experimental observations and to calculate the radiation pressure on absorbing Mie particles. In contrast to the scattering plus absorption forces derived for small particles, we predict that absorption can reduce the total optical momentum transfer to certain particles due to the balance between the force on free currents and the force on bound currents and charges.

The Lorentz force is applied directly to bound and free currents and charges, which are used to model lossy media with complex permittivity $\epsilon = \epsilon_R + i\epsilon_I$ and permeability $\mu = \mu_R + i\mu_I$ in a background of (ϵ_0, μ_0). The time-average Lorentz force density on bound currents and charges due to harmonic excitation with $e^{-i\omega t}$ dependence is [10]

$$\bar{f}_b = \frac{1}{2}\mathrm{Re}\{\epsilon_0(\nabla\cdot\bar{E})\bar{E}^* + \mu_0(\nabla\cdot\bar{H})\bar{H}^* - i\omega(\epsilon_R - \epsilon_0)\bar{E}\times\bar{B}^* + i\omega(\mu_R - \mu_0)\bar{H}\times\bar{D}^*\}, \quad (1)$$

where Re{} represents the real part of a complex quantity and $*$ denotes the complex conjugate. The leading two terms in (1) contribute via a surface force density on bound electric and magnetic charges, while the final two terms represent the volume force density on bound electric and magnetic currents [10,11]. In addition to (1), the force density on free currents,

$$\bar{f}_c = \frac{1}{2}\mathrm{Re}\{\omega\epsilon_I\bar{E}\times\bar{B}^* - \omega\mu_I\bar{H}\times\bar{D}^*\}, \quad (2)$$

extends the recent analysis of the photon drag effect [15] to include magnetization, oblique incidence, and arbitrary polarization. The total time-average force on the material $\bar{F} = \bar{F}_c + \bar{F}_b$ results from integration of the time-average force densities over the entire medium.

The connection of (1) and (2) to momentum conservation can be shown by considering a normally incident electromagnetic wave with complex wave number $k_z = k_{zR} + ik_{zI}$ transmitted into a medium occupying the half-space $z > 0$. Substitution of the transmitted field $\bar{E} = \hat{y}E_0 e^{-k_{zI}z} e^{ik_{zR}z}$ into (1) and (2) yields

$$\bar{f}_b = -\hat{z}\tfrac{1}{2}k_{zI}[(\epsilon_R - \epsilon_0)|\bar{E}|^2 + (\mu_R - \mu_0)|\bar{H}|^2], \quad (3a)$$

$$\bar{f}_c = \hat{z}\tfrac{1}{2}k_{zR}[\epsilon_I|\bar{E}|^2 + \mu_I|\bar{H}|^2], \quad (3b)$$

where $\bar{H}$ is the magnetic field determined from Faraday's law. The negative sign leading the left-hand side of (3a) indicates that the force on the bound currents is opposite to the incident wave propagation direction when the medium is optically dense. The force density on free currents can be written as

$$\bar{f}_c = \hat{z}\frac{1}{2}\frac{n\omega}{c}[\epsilon_I|\bar{E}|^2 + \mu_I|\bar{H}|^2] = -\hat{z}\frac{1}{2}\,\mathrm{Re}\left\{\frac{n}{c}\nabla\cdot\bar{S}\right\}, \quad (4)$$

where $n = ck_{zR}/\omega$ is the index of refraction, c is the speed of light in vacuum, and $\bar{S} = \bar{E}\times\bar{H}^*$ is the complex Poynting vector resulting from the application of Poynting's theorem to the second equality. The result of (4) is interpreted as two means of momentum transfer. First the transfer of momentum at the boundary is due entirely to $\bar{F}_b$ since electromagnetic power is conserved in the reflection or transmission (i.e., $\mathrm{Re}\{\nabla\cdot\bar{S}\} = 0$). Second, the transfer of momentum to free currents due to the attenuation of the wave in the medium is given by the divergence of the momentum $\bar{p} = n\bar{S}/c$ in (4). Recent experiments confirm this result by showing that the observed transfer of momentum to an atom in a dilute gas is directly proportional to the macroscopic refractive index [16]. This dependence on n has also been observed in the photon drag measurements [17] and was recently analyzed in Refs. [15,18]. We conclude that the direct dependence of absorbed momentum on the refractive index n holds for both dielectric and magnetic media.

The connection of momentum transfer to bound and free currents is formalized by applying the momentum conservation theorem via the Maxwell stress tensor. The time-average force on currents and charges enclosed by a surface of area A with unit normal $\hat{n}$ is

$$\bar{F} = -\frac{1}{2}\,\mathrm{Re}\left\{\oint_A dA[\hat{n}\cdot\bar{\bar{T}}(\bar{r})]\right\}, \quad (5)$$

where the complex stress tensor is given by [11]

$$\bar{\bar{T}} = \tfrac{1}{2}(\bar{D}\cdot\bar{E}^* + \bar{B}^*\cdot\bar{H})\bar{\bar{I}} - \bar{D}\bar{E}^* - \bar{B}^*\bar{H}. \quad (6)$$

In (6), $\bar{D}\bar{E}^*$ and $\bar{B}^*\bar{H}$ are dyadic products and $\bar{\bar{I}}$ is the (3×3) identity matrix. The tensor (6) can be applied to distinguish between $\bar{F}_b$ and $\bar{F}_c$ by noting the relationship in (4). As an example, we consider the force calculation on a particle of radius a. The total Lorentz force $\bar{F}$ on bound and free currents is found by integrating a surface in (5) that just encloses the entire particle so that $\bar{\bar{T}}$ is evaluated at $r = a^+$ as shown in Fig. 1(a), and the tensor in (6) reduces to the free-space Maxwell stress tensor [19]. The force on free currents $\bar{F}_c$ is found by integrating the stress tensor (6) along the interior of the particle boundary at $r = a^-$ as shown in Fig. 1(b) such that all free currents are enclosed. The force on bound currents and charges is $\bar{F}_b = \bar{F} - \bar{F}_c$. The choice of integration paths for $\bar{F}$, $\bar{F}_b$, and $\bar{F}_c$ allow for the description of electromagnetic forces in media consistent with the direct application of the Lorentz force in (1) and (2).

The two methods are applied to model known experiments by considering the general solution of a TE electromagnetic wave obliquely incident on the surface of an infinite nonmagnetic medium ($\mu = \mu_0$) occupying the region $z > 0$. The radiation pressure on bound currents is found by integrating the Maxwell stress tensor in (6) along a path that just encloses the boundary as in Ref. [10] and, for a weakly absorbing dielectric, is

$$\bar{F}_b = \hat{z}E_i^2\left[\frac{\epsilon_0}{2}(1 + |R|^2)\cos^2\theta_i - \frac{\epsilon_R}{2}|T|^2\cos^2\theta_t\right], \quad (7)$$

where R is the reflection coefficient, T is the transmission coefficient, and θ_i and θ_t are the incident and transmitted angles, respectively. Thus, the total force on bound currents is normal to the surface and directed toward the incoming wave for $\epsilon_R > \epsilon_0$, while the tangential component of wave momentum is conserved across the boundary due to phase matching. To formally prove this assertion, it is necessary to consider the tangential component of the Lorentz force density

$$\hat{x}\cdot\bar{f}_b = \tfrac{1}{2}\mathrm{Re}\{-i(\epsilon_R - \epsilon_0)k_x^*|E_i|^2|T|^2 e^{-2k_{zI}z}\}, \quad (8)$$

which is due to the transmitted field $\bar{E} = \hat{y}E_i T e^{-k_{zI}z} e^{ik_{zR}z} e^{ik_x x}$. The tangential force density on bound currents in (8) is zero when k_x is real, yielding a normal pressure on a half-space void of free currents. The total radiation pressure on an absorbing half-space is found from (5) with the tensor integration extended to $z \to \infty$

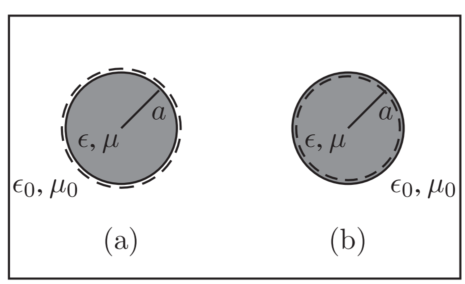

FIG. 1. Integration path for (5) applied to a lossy particle with radius a and (ϵ, μ) in a background of (ϵ_0, μ_0). (a) An integration path that completely encloses the particle gives the total Lorentz force $\bar{F}$. (b) The integration path just inside the boundary gives the force on the free carriers $\bar{F}_c$.

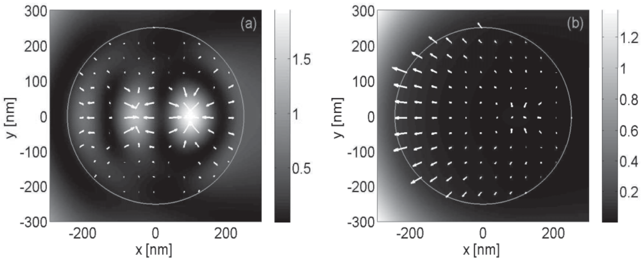

FIG. 2. Lorentz force density on bound currents (arrows) overlain on electric field intensity $|E_z|^2$ $[(V/m)^2]$ resulting from a $\hat{z}$ polarized plane wave of unit amplitude incident from free space with wavelength $\lambda_0 = 1064$ nm onto a dielectric cylinder. (a) The lossless cylinder is defined by $\epsilon = 16\epsilon_0$. ($\max(|f_b|) = 1.25 \times 10^{-6}$ $[\mathrm{N/m^3}]$) (b) The lossy cylinder, described by $\epsilon = (16 + 10i)\epsilon_0$, contains an additional force density on free currents. ($\max(|f_b|) = 3.00 \times 10^{-9}$ $\mathrm{N/m^3}$).

$$\hat{z} \cdot \bar{F} = \frac{\epsilon_0}{2} E_i^2 (1 + |R|^2) \cos^2\theta_i, \tag{9a}$$

$$\hat{x} \cdot \bar{F} = \frac{\epsilon_0}{2} E_i^2 (1 - |R|^2) \cos\theta_i \sin\theta_i, \tag{9b}$$

which is in agreement with the Lorentz force density integrated over the region $z \in [0, \infty)$ as in Refs. [7,9]. The result (9b) was originally demonstrated in 1905 by Poynting [20], who observed a tangential force given by $\frac{\epsilon_0}{2} E_i^2 \sin\theta_i \cos\theta_i$ for a nearly perfect absorbing medium ($R \approx 0$ and $\epsilon_I \neq 0$) and zero tangential force for the reflection from a mirror $|R| = 1$. Thus, the radiation pressure is normal to the surface of a perfect reflector and is given by the force on free currents at the surface $\bar{F} = \bar{F}_c = \hat{z}\epsilon_0 |E_i|^2 \cos^2\theta_i$. If the dielectric constant of the background medium is increased by $\sqrt{n}$, then the force on the reflector increases to $\sqrt{n}\epsilon_0 |E_i|^2 \cos^2\theta_i = \frac{n}{c} S \cos^2\theta_i$, which has been observed for mirrors submerged in dielectric liquids [21,22].

The total fields due to plane-wave incidence on a 2D particle are found from Mie theory applied to an absorbing infinite cylinder as in Refs. [23,24]. We consider two separate problems of $\bar{E}_{\mathrm{inc}} = \hat{z} e^{ik_0 x}$ incident from free space onto a lossless cylinder and onto a lossy cylinder, each of diameter 0.5 μm. The choice of an infinite dielectric cylinder incident by a TE wave allows for a complete description of the Lorentz force by the distribution inside the particle since the force on bound charges at the boundary is zero. The total electric field intensity is shown in Fig. 2(a) for the lossless dielectric particle. The total force is the force on bound currents $\bar{F} = \bar{F}_b = \hat{x}4.02 \times 10^{-18}$ N/m, which is computed by integrating the force density (1) over the area of the particle or, equivalently, by integrating the stress tensor of (6) along the circular path shown in Fig. 1(a). The integration is performed by simple numerical integration as in Ref. [11]. Although equivalent in results, the former approach provides the viewpoint that the particle is pulled toward the resulting high intensity focus, while the latter gives the usual intuition of a particle being pushed by the transfer of wave momentum. The total electric field intensity and force density on bound currents for a lossy particle is shown in Fig. 2(b). The resulting force density $\bar{F}_b = -\hat{x}2.05 \times 10^{-18}$ N/m indicates that the bound currents are pulled toward the incident wave, which is offset by a positive momentum transfer to free currents represented by $\bar{F}_c = \hat{x}5.80 \times 10^{-18}$ N/m, found equivalently by applying an integration path to the stress tensor shown in Fig. 1(b). The total pressure on the particle is $\bar{F} = \hat{x}3.75 \times 10^{-18}$ N/m, which is less than the total force on the transparent particle.

A physically realistic situation of an electromagnetic wave impinging on a spherical particle is studied using Mie theory. For $\epsilon_R/\epsilon_0 = 2$, Fig. 3(a) shows that a maximum optical momentum transfer occurs for a value of ϵ_I near maximum absorption (i.e. the penetration depth is on the order of the particle diameter). In contrast, a particle with large value for ϵ_R can exhibit reduced momentum transfer due to significant wave attenuation in the sphere as shown in Fig. 3(b). A further decrease in F for the high contrast sphere is observed as ϵ_I approaches the limit of a perfect reflector. The later point is made by comparing the adiabatic momentum transfers to the transparent dielectric sphere and to the reflecting sphere of equal size. However, the combined effect of F_b and F_c is required to explain the radiation pressure increase of Fig. 3(a) or decrease of

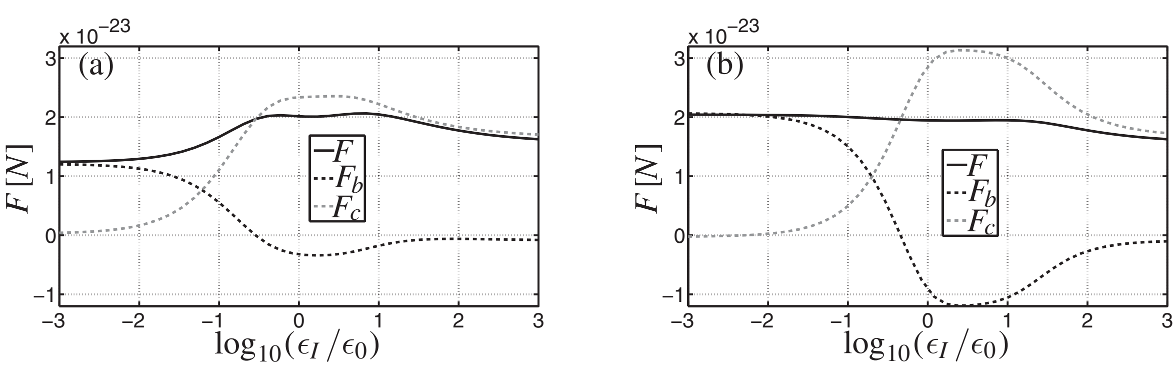

FIG. 3. Forces on a 2 μm diameter sphere due to a plane wave of unit amplitude. The wave is incident from free space with wavelength $\lambda_0 = 1064$ nm onto a nonmagnetic sphere with (a) $\epsilon = 2\epsilon_0 + i\epsilon_I$ and (b) $\epsilon = 16\epsilon_0 + i\epsilon_I$.

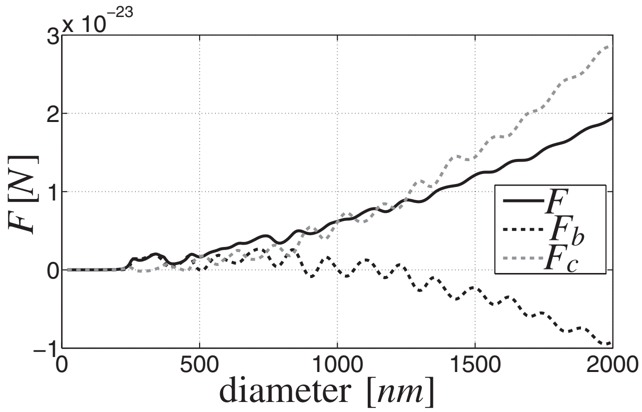

FIG. 4. Force versus diameter for a dielectric sphere ($\epsilon/\epsilon_0 = 16 + i$) incident by a unit amplitude plane wave. The free-space wavelength of the incident wave is $\lambda_0 = 1064$ nm.

Fig. 3(b) due to absorption. This separation of F into F_b and F_c is further investigated by plotting the force versus sphere diameter for constant material parameters in Fig. 4. For small spheres, the power absorption is small since the diameter is much less than the penetration depth. When the diameter is of the order of the penetration depth, the force on free currents becomes significant due to the direct dependence upon n given by (4).

In this Letter, we have provided a perspective of momentum transfer in lossy media in agreement with the distribution of Lorentz force and relevant experiments. In the case of an absorbing Mie particle, the contributions from $\bar{F}_b$ and $\bar{F}_c$ sum to give the total force on the particle. The particles we consider consist of $\epsilon_R = 16\epsilon_0$, a value typical for semiconductors, and $\epsilon_R = 2$, which is representative of many insulators. A novelty of our results is the reduction of optical momentum transfer to particles due to absorption, which requires high dielectric contrast with the background medium and an attenuation length on the order of particle diameter. These results differ from the expected result of scattering plus absorption forces resulting from Rayleigh particles [25]. Because a detailed understanding of both $\bar{F}_b$ and $\bar{F}_c$ are required to describe the physics involved, the theory presented here is fundamental to the understanding of optical momentum transfer to absorbing particles.

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