QUESTIONS AND ANSWERS

Contributions to this section, both Questions and Answers, are welcomed. Please submit four copies to the editorial office. Please include a *title* for each submission, include name and address at the end, and put references in the standard format used in the American Journal of Physics. For further suggestions, sample Questions and Answers, and requested form for both Questions and Answers, see Robert H. Romer, "Editorial: 'Questions and Answers,' a new section of the American Journal of Physics," Am. J. Phys. **62** (6) 487–489 (1994).

Questions at any level and on any appropriate AJP topic, including the "quick and curious" question, are encouraged.

Question #51. Applications of third-order and fifth-order differential equations

First, an observation. Most of the fundamental equations in physics are second-order partial differential equations (PDEs), or, equivalently, may be expressed as pairs of first-order PDEs. Newton's second law, wave equations, the Schrödinger equation, and Einstein's field equations of general relativity are second-order PDEs. Examples of pairs of first-order PDEs include Hamilton's equations, Maxwell's equations, and the Dirac equation (which can be "derived" by factoring the Klein–Gordon equation with hypercomplex numbers to give the positive- and negative-energy Dirac equations). Beyond second order, there is a fourth-order equation that describes the deflection of a cantilevered beam.

Now, on to the question. Are there any useful applications in physics (or anywhere else, for that matter) of a third-order or fifth-order differential equation? Since the calculus of variations can be extended to functionals whose Euler–Lagrange equations are PDEs of order higher than two, is the scarcity of such applications telling us something important, not about mathematics, but about physical symmetries?

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Question #52. Group velocity and energy propagation

One of the most counterintuitive aspects of linear dispersive wave propagation is that the energy propagates at the group velocity, not the phase velocity. Most references that we have come across demonstrate this for nondissipative systems in one of the following ways: (1) by performing calculations of the energy flux in specific systems, such as surface gravity waves on water; (2) by postulating a damping mechanism and then letting the damping tend to zero at the end of the calculation; or (3) by considering the asymptotic form of an arbitrary initial disturbance.[1–3] Method (1) is not general, (2) appears a bit contrived, and (3) applies only to the limit of long times, at distances far from the initial disturbance.

Does anyone know of a general proof of this strange property of dispersive systems? We are also looking for "hand-waving proofs"—those phrased in terms that a scientifically literate layperson could follow.

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[1]L. Brillouin, *Wave Propagation and Group Velocity* (Academic, New York, 1960), pp. 96–100.
[2]J. Lighthill, *Waves in Fluids* (Cambridge U.P., Cambridge, 1978), pp. 254–260.
[3]G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974), pp. 374–379.

Answer to Question #33 ["Underwater vision of dolphins and terns," Clifford E. Swartz, Am. J. Phys. 64(1), 13 (1996)]

Seeing clearly both in and out of water is crucial for the survival of many animals. Learning how various species accomplish this feat—and why it is a feat at all—is well within the grasp of introductory physics students. Exploration of the topic can enhance students' understanding of basic optics and highlight the relevance of physics to the life sciences.

The Nature of the Problem. The vertebrate eye has two parts that are involved in focusing light onto the retina—the lens and the cornea. The lens is internal; the cornea, the outermost part of the eye, is in contact with the environment. For air-dwelling animals, the cornea is the eye's main refractive element. For water dwellers, the cornea is optically ineffective, however, since its index of refraction is essentially the same as that of water. The main challenge faced by animals moving back and forth between air and water is therefore one of compensating for the alternate gain and loss of corneal refraction. Without special adaptations, air dwellers are very farsighted (hyperopic) underwater, and aquatic organisms are very nearsighted (myopic) in air.

Humans can avoid underwater farsightedness by wearing a diving mask, which traps air next to the corneas and allows them to retain their refractive properties. While no other organisms sport diving masks (to our knowledge), various fascinating adaptations for amphibious vision have evolved among vertebrates. We survey four main strategies below.

Accommodation. The most common means of focusing clearly (achieving emmetropic vision) in both water and air is via accommodation. Accommodation involves altering the focal point of the lens, and it enables organisms to focus on objects over a range of viewing distances. In general, fishes and amphibians accommodate by moving their lenses backward and forward (closer to and farther from the retina), whereas reptiles, birds, and mammals accommodate by changing their lenses' shape.[1] Substantial variation in accommodation ability exists among vertebrates.

Some amphibious organisms are such extraordinarily good accommodators that they can focus clearly in and out of water by this means alone. Cormorants[2] and dippers,[3] two types of diving birds, can vary the refractive power of their lenses by a remarkable 40–50 diopters (vs 16 diopters for an average adolescent human[1] and 8–10 diopters for falcons, chickens, and pigeons[3]). Dippers owe their excellent accom-

