空腔中的电磁波一定为波导中的电磁本征态。取TM_{mn}模式，电磁场的一个分量为

$$E_z = E_0 e^{ik_z z} \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-i\omega t}$$

加上端面以后，由于端面会对电磁波进行反射，因此分量写成

$$E_z = \left(E_0 e^{ik_z z} + E_0^{'} e^{ik_z z}\right)\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-i\omega t}$$

由此得到电场的x分量和y分量分别为

$$E_x = i\frac{m\pi}{ak_c^{\ 2}}\left(k_z E_0 e^{ik_z z} - k_z E_0^{'} e^{-ik_z z}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-i\omega t}$$

$$E_y = i\frac{n\pi}{bk_c^{\ 2}}\left(k_z E_0 e^{ik_z z} - k_z E_0^{'} e^{-ik_z z}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-i\omega t}$$

端面处边界条件为

$$\overrightarrow{e_n} \times \overrightarrow{E} \,\big|_{z=0,d} = 0 \qquad \overrightarrow{e_n} \bullet \overrightarrow{B} \,\big|_{z=0,d} = 0$$

即

$$E_x \,\big|_{z=0} = E_x \,\big|_{z=d} = E_y \,\big|_{z=0} = E_y \,\big|_{z=d} = 0$$

可得

$$k_z = \frac{p\pi}{d} \qquad E_0 = E_0^{'}$$

因此矩形谐振腔内TM波场的空间分布为：

$$E_x = -2\frac{m\pi}{a}\frac{p\pi}{d}\frac{E_0}{k_c{}^2}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$E_y = -2\frac{n\pi}{b}\frac{p\pi}{d}\frac{E_0}{k_c{}^2}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$E_z = 2E_0\sin\left(\frac{m\pi}{a}\right)\sin\left(\frac{n\pi}{b}\right)\cos\left(\frac{p\pi}{d}\right)e^{-i\omega t}$$

$$B_x = -2i\frac{n\pi}{b}\frac{k_0E_0}{ck_c{}^2}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$B_y = 2i\frac{m\pi}{a}\frac{k_0E_0}{ck_c{}^2}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$B_z = 0$$

$$E_x = -2\frac{m\pi}{a}\frac{p\pi}{d}\frac{E_0}{k_c^{\ 2}}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$E_y = -2\frac{n\pi}{b}\frac{p\pi}{d}\frac{E_0}{k_c^{\ 2}}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$E_z = 2E_0\sin\left(\frac{m\pi}{a}\right)\sin\left(\frac{n\pi}{a}\right)\cos\left(\frac{p\pi}{a}\right)e^{-i\omega t}$$

$$B_x = -2i\frac{n\pi}{b}\frac{k_0E_0}{ck_c^{\ 2}}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)e^{-i\omega t}$$

$$B_y = 2i\frac{m\pi}{a}\frac{k_0E_0}{ck_c^{\ 2}}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$B_z = 0$$

TM波

对TM波，m,n都不可为0，否则磁场部分全为0

对TE波，p不可为0，否则电场部分全部为0；m,n不可同时为0，否则电场部分全部为0
因此m,n,p中最多只有一个可以等于0

对于立方体，a=b=c

$$\frac{\omega}{c} = \frac{\pi}{a}\sqrt{m^2 + n^2 + p^2}$$

TE波

$$B_x = -2iB_0\frac{m\pi}{a}\frac{k_z}{k_c^{\ 2}}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{a}y\right)\cos\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$B_y = -2iB_0\frac{n\pi}{a}\frac{k_z}{k_c^{\ 2}}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)\cos\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$B_z = 2iB_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{a}y\right)\sin\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$E_x = 2B_0\frac{n\pi}{a}\frac{ck_0}{k_c^{\ 2}}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)\sin\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$E_y = -2B_0\frac{m\pi}{a}\frac{ck_0}{k_c^{\ 2}}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{a}y\right)\sin\left(\frac{p\pi}{a}z\right)e^{-i\omega t}$$

$$E_z = 0$$

简并度的讨论

角频率ω	允许的模式	简并度
$\frac{c\pi}{a}\sqrt{2}$	TM110 TE101 TE011	3
$\frac{c\pi}{a}\sqrt{3}$	TM111 TE111	2
$\frac{c\pi}{a}\sqrt{5}$	TM210 TM120 TE102 TE201 TE012 TE021	6
$\frac{c\pi}{a}\sqrt{6}$	TM112 TM121 TM211 TE112 TE121 TE211	6
$\frac{c\pi}{a}\sqrt{9}$	TM221 TM212 TM122 TE221 TE212 TE122	6
$\frac{c\pi}{a}\sqrt{10}$	TM310 TM130 TE103 TE301 TE013 TE031	6
$\frac{c\pi}{a}\sqrt{11}$	TM311 TM131 TM113 TE311 TE131 TE113	6
……		

对m,n,p可为i,j,k的情况

{m,n,p}={i,j,k}		简并度
i,j,k≠0	i=j=k	2
	i=j≠k	6
	i,j,k互不相等	12
i=0　j,k≠0	j=k	3
	j≠k	6
i=0,j=0		0
i=j=k=0		0

若$\omega \gg \frac{\pi c}{a}$，令$r \triangleq \frac{\omega a}{\pi c}$

$r=\sqrt{m^2+n^2+p^2} \gg 1$时，$r$到$r+dr$区间内的简并度约为

$$G(r)dr = 12 \times \frac{4\pi r^2}{1\times 1\times 1}dr = 48\pi r^2 dr$$