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Bank capital, interbank contagion, and bailout policy

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ABSTRACT

This paper develops a theoretical framework in which asset linkages in a syndicated loan agreement can infect a healthy bank when its partner bank fails. We investigate how capital constraints affect the choice of the healthy bank to takeover or liquidate the exposure held jointly with the failing bank, and how the bank's ex ante optimal capital holding and possibility of contagion are affected by anticipation of bail-out policy, capital requirements and the joint exposure. We identify a range of factors that strengthen or weaken the possibility of contagion and bailout. Recapitalization with common stock rather than preferred equity injection dilutes existing shareholder interests and gives the bank a greater incentive to hold capital to cope with potential contagion. Increasing the minimum regulatory capital does not necessarily reduce contagion, while the requirement of holding conservation capital buffer could increase the bank's resilience to avoid contagion.

1. Introduction

There is a longstanding and ongoing debate about whether government bailout is necessary during a financial crisis and, if so, in what form it should be provided. Some believe that government bailout of banks will save banks and their projects, minimizing a domino effect in the financial system and the loss of employment: "*Bailing out Wall Street bankers is necessary to keep the US economy from crumbling even further and taking American workers down with it.*" (Barack Obama, US president, 29 September 2008).

However, others believe that banks can self-adjust, finding a new equilibrium without help from the government: *"Bailout is not necessary. The banking industry can handle this mess internally and does not need subsidies."* (Bert Ely, a leading expert on banking and finance in the Washington policy community, 24 September 2008).

Therefore, the banks' ability to self-adjust plays a key role in government bailout decisions. Given the potential drawbacks of government bailout, it is important to understand whether and to what extent banks can absorb external shocks internally during a financial crisis. Improved understanding of this issue can help the authorities better balance the benefits of government bailout, in containing the contagion of a financial crisis, from its substantial costs.[1]

In this paper, we develop a theoretical framework in which a healthy bank (Bank 1) can become infected when its partner bank (Bank 2) in a joint exposure to a syndicated loan fails and defaults on its share of loan. We analyze the impact of Bank 1's capital holding and the size of its exposure on contagion or continuation of joint exposures. Furthermore, we investigate how Bank 1's capital prior to the crisis and possibility of contagion are affected by anticipated bailout and regulation policies and a number of important factors related to Bank 1's exposure.

Our study employs the inventory theoretic framework of bank capital, which advocates that banks maintain a buffer of capital in

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[1] Government bailout increases the federal budget deficit and may even drag the country into a fiscal crisis. Hellmann et al. (2000) cite a World Bank study showing that the costs related to financial crises can reach 40 percent of GDP. During the 2008 global financial crisis, the US government spent $250 billion to recapitalize the banks under the Troubled Asset Relief Program (TARP). European governments intervened to rescue financial institutions, such as Fortis by the Benelux countries ($16 billion), Dexia by Belgium, France, and Luxembourg (€150 billion), Hypo Real Estate Bank by Germany (€50 billion), ING by Dutch government (€35 billion), and others.

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excess of regulatory requirements to reduce future costs of illiquidity and recapitalization.[2] In our model, two banks jointly make a syndicated loan for an indivisible project. When an external shock leads the partner bank to discontinue its business operations, Bank 1 has two options: (a) accepting the liquidation of the syndicated project and receiving a comparatively low liquidation value, or (b) taking over all of the interest of Bank 2 in the indivisible project. Bank 1 also anticipates that the government may inject common equity or preferred equity into it if Bank 2 becomes distressed. If Bank 1's capital level after taking over or liquidating the distress loan is lower than the regulatory capital requirement, the bank will be liquidated with the loss of all future dividends payments to shareholders. Thus, the failure of Bank 2 forces Bank 1 into liquidation and contagion occurs.

In our analysis, we first provide the basic accounting analysis using balance sheet developments to examine when continuation of the joint project is possible, when contagion may emerge, and when bailout is needed to prevent contagion. Then we extend the analysis using the technique of dynamic stochastic optimization to investigate Bank 1's value to shareholders when it takes over or liquidates the joint project, and its value to shareholders prior to the shock allowing for the possible bank actions after the crisis. Bank 1's decision in the crisis is based on the relative values after taking over or liquidating the joint project. Then we characterize the optimal ex-ante capital holding and compare it with the regulatory capital requirement to examine whether contagion happens and how much capital in the form of common stock or preferred stock must be provided when bailout is necessary.

Our simulations show that contagion will not occur if the healthy bank properly anticipates Bank 2's failure and increases its ex-ante optimal capital holding to accommodate the joint project that may fail. However, if Bank 1 seriously underestimates the probability of the shock, its capital level will be lower than the regulatory requirement for taking over or liquidating the project, triggering contagion. In addition, if it has a high fraction of its assets invested in the joint project, a low bargaining power over the project, an exposure smaller than Bank 2's exposure in the joint project, or a large loss of market value of the project, its capital level is more likely to be lower than the required capital level to take over or liquidate the project. In sum, low capital ratios play a key role in promoting contagion and forcing liquidation. Interbank contagion can be minimized if the surviving banks are well capitalized and capable of making optimal choices in response to potential external shocks.

Our model provides several important policy implications. First, a higher anticipated probability of bailout will lead Bank 1 to hold less capital, reflecting the risk of moral hazard. Second, when the government injects funds in the form of common equity rather than preferred stock, it dilutes existing shareholder interests more and hence provides a stronger incentive for Bank 1 to hold more capital, reducing moral hazard. Third, increasing the minimum regulatory capital ratio per se may increase the possibility of contagion if Bank 1's increase of optimal capital buffer is not sufficient to match the increased capital requirement. Finally, the requirement of holding conservation capital buffer (as in Basel III) outside periods of stress could increase the bank's resilience to avoid contagion during the crisis. These results, collectively, provide theoretical support for the global government efforts to promote robust supervision and regulation of financial firms and give new insight into how this task can be best undertaken.[3]

Three contributions of our analysis are noted. First, our study adds to the theoretical bank contagion literature by examining interbank contagion due to banks' joint exposure to a common asset. In our model, contagion arises from uncertainties of banks' assets side, which differs from the common theoretical framework (such as bank-run models) for analyzing contagion from liabilities-side risk due to maturity mismatch. In the seminal paper by Diamond and Dybvig (1983), bank-run is caused by a shift in depositors' expectations due to some commonly observed factor such as a sunspot. In more realistic settings, Chari and Jagannathan (1988), Gorton (1985) rely on asymmetric information between the bank and its depositors on the true value of loans to induce bank runs, while Chen (1999) relies on Bayesian updating depositors who learn from interim bank failures that lead to bank runs. Allen and Gale (2000) propose that contagion arises because a liquidity shock in one region can spread throughout the economy due to interregional claims of one bank on other banks.

While the above bank contagion literature has focused mainly on deposit withdrawals as a propagation mechanism, a disturbance on the lending side can propagate and infect the system. This possibility deserves more attention from the theoretical perspective. Honohan (1999) shows disturbances can be transmitted through lending decisions due to banks over-committing to risky lending. Our paper adds to this strand of studies by examining contagion arising from lending-side risk, in particular, due to banks' joint exposure to a syndicated loan. This is supported by empirical evidence in Ivashina and Scharfstein (2010), who find that banks co-syndicated with Lehman suffered more stresses of liquidity, indicating that Lehman's failure put more of the funding burden on other members of the syndicate and exposed them to increased likelihood that more firms would draw on their credit lines.

Although our model deals with potential contagion arising from exposure to a syndicated loan agreement, the implications can be extended to more general situations of interbank linkages, for example, exposure to a common asset market such as sub-prime mortgage backed securities, or a situation with direct counterparty exposure. The counterparty contagion hypothesis predicts that firms with close business or credit relationships with a distressed firm will suffer adverse consequences from the financial troubles of the distressed firm (Davis and Lo, 2001; Jarrow and Yu, 2001).[4] Given the complexity of interbank linkages, counterparty risk is even more worrisome for financial institutions. In the spirit of our model, whether other banks will fail in the wake of the collapse of a counterparty bank depends on whether their optimal capital holding before the shock exceeds the minimum

[2] This strand of literature posits that banks treat their capital holding strategy as an inventory decision that allows them to be forward-looking by increasing their capital levels as necessary or adjusting their asset portfolios in response to any future breach of regulatory capital requirements. The buffer stock model of bank capital was first proposed by Baglioni and Cherubini (1994), later developed by Milne and Robertson (1996), Milne and Whalley (2001), Milne (2004), and in discrete time by Calem and Rob (1996). Peura and Keppo (2006) extend the continuous-time framework to take account of delays in raising capital. Milne and Robertson (1996) state that banks maintain extra capital in excess of minimum regulatory requirements in order to reduce the potential future costs of illiquidity and recapitalization. Milne (2002) further examines the implications of bank capital regulation as an incentive mechanism for portfolio choice. Milne (2004) argues that banks' risk-taking incentives depend on their capital buffer, not on the absolute level of capital. Our focus is different. We consider the bank's optimal capital decision and interbank contagion using the inventory framework.

[3] For example, the US Department of the Treasury states that "capital and liquidity requirements were simply too low. Regulators did not require firms to hold sufficient capital to cover trading assets, high-risk loans, and off-balance sheet commitments, or to hold increased capital during good times to prepare for bad times." (Financial regulatory reform: a new foundation, 2010. See http://www.financialstability.gov/docs/regs/FinalReport_web.pdf)

[4] Empirically the counterparty contagion hypothesis is supported by Hertzel et al. (2008), Jorion and Zhang (2009), Brunnermeier (2009), Chakrabarty and Zhang (2012), Iyer and Peydro (2011), among others. As Helwege (2009) points out, government bailout is necessary if counterparty contagion is a major contagion channel for financial firms. The related interbank contagion literature relies on contractual dependency such as a bilateral swap agreement to induce contagion when one party is unable to honor the contract (e.g., Gorton and Metrick, 2012). Another interbank contagion channel is when fire-sale of illiquid assets by one bank depresses asset prices and prompts financial distress at other institutions (e.g., Shleifer and Vishny (1992), Allen and Gale (1994), Diamond and Rajan (2005), Brunnermeier (2009), Wagner (2011)).

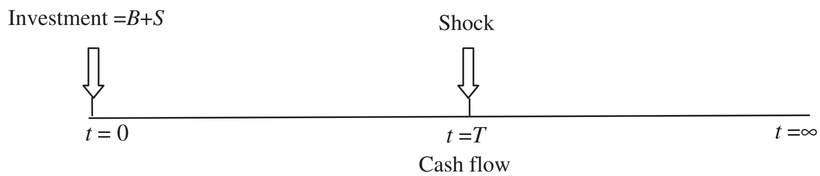

Fig. 1. Investment and cash flow for Project G.

capital requirement after the banks take action such as liquidating or taking over the assets associated with the failed bank.

Second, using the inventory buffer model of bank capital to study contagion allows us to model banks' precautionary risk management behaviors before crisis happens. Banks' optimal capital holding prior to the crisis is endogenously determined. Within the inventory framework, the bank manages inventory reserves in order to cope with uncertain outcomes. If the bank has sufficient inventory reserves to take over the joint assets of other banks, the failure of one bank does not necessarily lead to contagion. So when the risk of failure of other banks is properly understood, the possibility of contagion in the inventory setup becomes relatively remote. Government bailout is not always necessary if a bank can internally cope with potential contagion arising from asset linkages.[5]

An alternative is the conventional approach in which a bank' capital is a continuously binding constraint, similar to a household budget or a firm's feasible production set. With this approach, one bank's takeover of another bank's assets is impossible because this would violate the binding capital requirement. Liquidation of a joint project is the only possible outcome. If the bank invests a large share of assets in the project and the loss ratio is high, the failure of one bank leads directly to the failure of its partner banks in a joint project. In order to prevent such interbank contagion, it is necessary for the government to inject equity in other banks. However, this omits any possibility of continuing the joint project without government intervention. Hence contagion becomes excessively mechanical in the conventional set-up, which is inherently biased towards government bailout.

Third, our paper adds to the bailout literature by explicitly examining how government bailout policy (injection of common equity versus preferred equity) affects banks' ex ante capital buffer and possibilities of interbank contagion, and how banks' capital holding prior to the crisis, in turn, affects the level of government bailout. Earlier studies have addressed whether, when, and how to bail out a bank.[6] Our study complements the literature by providing a case for why a bailout is not always necessary to help a healthy bank survive contagion.

Spurred by the recent financial crisis, there is a growing literature on bank bailouts.[7] Acharya and Yorulmazer (2008) point out that granting liquidity to surviving banks to take over failed banks is preferable to bailing out failed banks because it induces banks to differentiate their risks. Kashyap et al. (2008) propose replacing capital requirements by mandatory capital insurance policy so that banks are forced to hoard liquidity. Chari and Kehoe (2010) show that regulation in the form of ex-ante restrictions on private contracts can increase welfare while ex-post bailouts trigger a bad continuation equilibrium of the policy game. Farhi and Tirole (2012) propose a model that banks choose to correlate their risk exposures in anticipation of imperfectly targeted government intervention to distressed institutions.

Our study is closely related to Philippon and Schnabl (2013), who analyze public intervention choices (buying equity, purchasing assets, and providing debt guarantees) to alleviate debt overhang among private firms. They find that with asymmetric information between firms and the government, buying equity dominates the two other interventions. We also consider bailout with equity injection, but our study further shows that common stock bailout is preferable ex ante to preferred equity bailout because it induces banks to target for a higher level of capital holding and thus reduces the government bailout budget.

Our results on bailout policy also complement the findings of Acharya et al. (2010) that government support to surviving banks conditional on their liquid asset holdings increases banks' incentive to hold liquidity, and that support to failed banks or unconditional support to surviving banks has the opposite effect. While their study stresses the role of banks' asset composition, our focus is the role of banks' capital holdings in anticipation of common stock or preferred equity bailout.

The rest of the paper is organized as follows. We introduce our benchmark model setup in Section 2. In Section 3, we provide the basic accounting analysis to examine when interbank contagion may emerge due to a failure of a partner bank. In Section 4 we derive the solution for a bank's optimal capital-asset ratio prior to the crisis for dealing with a partner bank's potential failure in anticipation of government intervention. Section 5 shows simulation results for the relationships between a number of public policy and banks' investment parameters and the level of ex-ante capital holding, possibility of contagion, and government bailout amounts. Section 6 concludes.

2. The model

In this section, we set up a framework to describe how a bank determines its optimal capital-asset ratio, assuming that banks maintain a buffer of capital that exceeds the regulatory requirement in order to reduce the potential future costs of illiquidity and recapitalization and the contagion effects of failure of its partner bank.

2.1. One project

We assume that a banking group enter into a syndicated loan agreement to finance part of an investment, B, in an indivisible Project G. Financing for the rest of the project, S, is obtained by issuing equity or debt, or comes from other sources.[8] Project G is being

[5] Our paper is also related to earlier work studying bank behavior under capital requirement constraints. Diamond and Rajan (2000) argue that the optimal bank capital structure reflects a tradeoff between the effects of bank capital on liquidity creation, the expected costs of bank distress, and the default risk of borrowers. Bolton and Freixas (2006) posit that bank lending is constrained by capital adequacy requirements. Hubbard et al. (2002) find that banks that maintain more capital charge a lower interest rate on loans. Jokipii and Milne (2008) show that capital buffers of the banks in the EU15 have a significant negative co-movement with the cycle, which exacerbates the pro-cyclical impact of Basel II.

[6] For example, Boot and Thakor (1993) model that a desire for the regulator to acquire a reputation as a capable monitor could distort bank closure policy. Dreyfus et al. (1994) discuss whether the setting of ceilings on the amount of deposit insurance coverage is optimal. Rochet and Tirole (1996) derive the optimal prudential rules while protecting the central banks from conducting undesired rescue operations. Gale and Vives (2002) argue that a bail out should be restricted ex ante due to moral hazard concerns. Gorton and Huang (2004) show that the government bailout for banks in distress provides more effective liquidity than private investors. Diamond and Rajan (2005) propose a robust sequence of intervention.

[7] See the review of theoretical literature by Philippon and Schnabl (2013), who also discuss several recent empirical work on bailouts such as Giannetti and Simonov (2011), Glasserman and Wang (2011).

[8] Given that our main research objective is not designing the capital structure for Project G, we assume that the market is perfect and that the financing methods available for rest of the investment do not affect the cash flow of Project G or the returns on investment B that the banking group receives.

implemented in two phases. At $t = 0$, the banks invest in Project G. After that, the assets in place generate cash flow, which gives the banks a return on their investment. Project G will repay the banks in full as long as the project is viable. However, a shock causes one bank in the banking group to go into distress and default on its share of the loan at time $t = T$, which arrives according to a Poisson process.[9] The other banks in the group has to decide whether to liquidate its own share of the loan in Project G (in which case the project will be liquidated as well) or to take over the failed bank's loan in Project G. Fig. 1 shows the timeline for the scenario.

2.2. Two banks

We assume that the banking group consists of two banks: Bank 1 and Bank 2. Bank i ($i = 1, 2$) holds a fixed amount of non-tradable assets valued at A_i at $t = 0$. The capital of Bank i, denoted by C_i, is the book value of its equity. The bank has raised the difference between assets and capital by issuing short-term deposits of $D_i = A_i - C_i$, assuming an infinitely elastic supply of deposits fully insured by the regulator. We assume that the original asset allocation of Bank i has been optimally made.

The total assets of Bank i can be divided into two components: l_iA_i and $(1 - l_i)A_i$, $(0 \leqslant l_i \leqslant 1)$, where l_iA_i is the amount lent by Bank i to Project G and $(1 - l_i)A_i$ is the amount invested in other projects.[10] According to our assumptions, $B = \sum_{i=1}^{2} l_iA_i$.[11]

Regulators constantly audit the net worth of a bank. If the net worth of a bank is lower than the minimum regulatory requirement, it has to be liquidated. Its debt holders will then be repaid in full out of deposit insurance, but its shareholders will receive nothing.

We make the following assumptions in line with Milne (2002, 2004) to obtain an analytical solution:

(1) The total existing assets of the banks are fixed, and the banks can adjust only their dividend payouts.
(2) The banks are able to finance all cash flow needed instantaneously by taking out deposit insurance or absorbing more deposits at zero cost.

Take Bank 1 as an example. At any time t, Bank 1 pays dividends at a rate θ subject to $\theta \geqslant 0$. Cash flow affects net worth C and hence deposits D according to

$$dC = [l_1AR_1 + (1 - l_1)AR_2 - \theta]dt + \sigma_1 l_1 A dZ_1 + \sigma_2(1 - l_1)AdZ_2 = -dD \tag{1}$$

where R_1 and R_2 denote the expected return of investment l_1A and $(1 - l_1)A$ in excess of the deposit interest rate, respectively, σ_1 and σ_2 denote the risk of investment l_1A and $(1 - l_1)A$, and Z_1 and Z_2 are Brownian motions, with the correlation coefficient of ω_{12}.[12] We assume that $R_1 > R_2$ and $\sigma_1 > \sigma_2$.

Bank 1 chooses θ to maximize the shareholders' value, measured by the expected discounted value of future dividends:

$$V(C) = \max_\theta E\left[\int_0^T e^{-\rho t}\theta_t dt + e^{-\rho T} H(C_T)\right] \tag{2}$$

where ρ represents both the discount factor ($\rho > 0$) and, because deposits are unremunerated, the excess cost of equity relative to bank debt. The first term in the brackets represents the cumulative discounted cash flow generated by the investment project before the shock occurs, and the second term in the brackets represents the discounted cash flow when the shock occurs. The specific form of $H(C_T)$ depends on which action Bank 1 takes when the shock happens. We discuss it in detail in Section 4. Regulators constantly compare the net worth C of Bank 1 with the minimum regulatory requirement $\overline{C} = A\tau$, in which τ is the required capital-asset ratio. If $C < \overline{C}$, the bank is liquidated. As a notational convenience, we normalize the model with reference to the assets of Bank 1 by assuming throughout that $A = 1$.

We introduce an additional parameter, $n = l_2/l_1$, to represent the relative shares of Bank 2 over Bank 1 in the joint project. So $l_2 = nl_1$ and parameter, n, parameterizes the relative size of exposure. Suppose the amount lent by Bank 1 to Project G is l, then the amount lent by Bank 2 is nl. The subscript on the proportion l of the bank's assets held in the joint project is dropped for convenience.

The bank's equity capital C is subject to the regulatory requirement that it does not fall below a minimum required ratio of bank assets i.e. $C \geqslant \tau$. Regulators constantly audit the net worth of a bank. If the net worth of a bank is lower than the minimum regulatory requirement, it has to be liquidated. Its debt holders will then be repaid in full out of deposit insurance, but its shareholders will receive nothing.

2.3. One shock

At a random time T, a shock (the systemic crisis) arriving according to a Poisson process causes Bank 2 to default on its share of the loan and require termination of the syndicated loan unless Bank 1 takes over the loan in its entirety. Bank 1 expects the intensity of the shock to be $\phi > \psi 0$.[13] Bank 1 has to decide whether to liquidate its own loan in Project G (in which case the project will be liquidated as well) or to take over the failed bank's loan in Project G.

Bank 1 also expects the government to offer an equity capital injection to Bank 1 in the form of preferred equity or common stock with the probability λ. We assume that government capital injection will give Bank 1 the new desired capital level C^*, depending on whether the joint project has been taken over or liquidated.[14] If Bank 1 accepts the bailout, it will choose the optimal injection amount K to maximize its shareholders' value after it takes over or liquidates Project G with the injected capital. If the bailout takes the form of preferred equity, the shareholders of preferred

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[9] We thank the referee for his/her suggestion of introducing a jump process for the shock.

[10] In our model, we assume that the capital structure decision is determined after the initial investment decision is made. In other words, l_i is given exogenously and the bank determines its optimal capital-asset ratio based on the given l_i. Because we are interested mainly in the impact of an external shock on the optimal capital holding of banks, endogenous selection of l_i will make the calculation more complicated. The assumption that the bank's original portfolio choice is independent of the capital structure is a possible limitation of our model.

[11] For simplicity, we choose the two-bank setting to examine the potential interbank contagion issue. With one surviving bank, contagion is possible and there is a potential role for one bank's takeover of the joint assets and for government bailout. Our model can be generalized to one failed bank and N-1 surviving banks, in which case:

$$B = \sum_{i=1}^{N} l_iA_i$$

For a fixed amount B, given that the larger the number of banks in the group, the lower the fraction l_i, we can examine the effect of the number of banks in the group by l_i. When l_i is small, the loan to Project G is a small fraction of investment for bank i. This occurs when the number of banks in the group is high. When l_i is large, the loan to Project G is a large fraction of investment for bank i. This occurs when the number of banks in the group is low.

[12] Our model setup, in which a bank holds two components of correlated assets, departs from that of Milne (2002, 2004), who treats all banks' assets homogeneously. As discussed later, both the size of the distressed loan as a fraction of total assets and the correlation coefficient are important in determining the bank's optimal capital holding.

[13] An important advantage of assuming a random asset maturity with a Poisson process is that at any point before the shock, the expected remaining time-to-shock is always $1/\phi$.

[14] In reality, in the event of a government bailout, the injected capital increases the bank's capitalization to well above the minimum regulatory requirement to protect its own interests and limit the possibility of failure. This assumption is also consistent with our framework, where capital plays an inventory role. We thank the referee for pointing this out.

Table 1
Symbols.

Symbol	Definition
A	Bank 1's non-tradable assets
C	Book value of Bank 1's equity
D	Bank 1's deposits, which is equal to the difference between A and C
l_1	Bank 1's investment ratio in Project G
θ	Bank 1's dividend payout rate ($\theta \geqslant 0$)
R_1, R_2	The expected returns of investment l_1A and investment $(1-l_1)A$ in excess of the deposit interest rate ($R_1 > R_2$)
σ_1, σ_2	The volatilities of investment l_1A and investment $(1-l_1)A$, $\sigma_1 > \sigma_2$
Z_1, Z_2	The Brownian motions of investment l_1A and investment $(1-l_1)A$
ω_{12}	The correlation coefficient of Z_1 and Z_2
ρ	The discount factor and the excess cost of equity relative to bank debt
$\overline{C}$	The minimum regulatory capital level for Bank 1 before the shock occurs
τ	The capital-asset ratio required by the regulator
n	The relative ratio of Bank 2's investment in Project G over Bank 1's investment in Project G
ϕ	The intensity of the Poisson process for the arrival of a financial crisis
λ	The probability of government bailout in the form of preferred equity or common stock
C^*	Bank 1's new desired capital level after the joint project has been taken over or liquidated
K	The optimal bailout amount for Bank 1 to maximize its shareholders' value after it takes over or liquidates Project G with the injected capital
ξ	The loss-given-default ratio (LGD) of Project G
x	The bargaining power of Bank 1 over Project G
y	The ratio of mark-to-market value of Project G over the initial investment value
$\widehat{C}$	The minimum regulatory capital level for Bank 1 to take over the loan
$\widehat{\widehat{C}}$	The minimum regulatory capital level for Bank 1 to liquidate the loan

equity will receive only the fixed dividend; they will not share in the upside gain should the bank recover. In contrast, since common stock shareholders will share in the upside potential, an injection of common equity will dilute existing shareholder interests.

Table 1 summarizes and explains the notation used in the model.

3. Bank's balance sheet development upon the shock

In this section we provide a preliminary accounting analysis of how parameter assumptions affect the possible balance sheet developments. We identify when continuation of the joint project is possible, when contagion will happen and when bail out can be used to prevent liquidation of joint projects. Doing this first provides helpful intuition and makes the subsequent technical exposition in Sections 4 and 5 easier to follow.

As described in Section 2, the initial balance sheet of Bank 1 can be formulated as:

Assets	*Liabilities*
$1-l$	D_t
l	$C_t = 1 - D_t$
1	1

Balance Sheet 1.

When the shock happens, Bank 2 defaults on its share of the loan. If Bank 1 decides to liquidate its loan in Project G, l, Project G will be liquidated. We assume ξ is the loss-given-default ratio (LGD) of Project G. If Bank 1 decides to take over the failed bank's loan in Project G, the amount paid for taking over the assets and continuing the joint project will depend on the bargaining power of Bank 1.

The lowest possible price will be the recovery value from liquidation $(1-\xi)nl$ (because bank 2 can still get this amount by refusing to sell the assets). The other extreme is if Bank 1 must pay full accounting value for the loan (if Bank 2 has all the bargaining power over the sale of the assets). Let $x \in [0,1]$ represent the bargaining power of Bank 1, the price paid for the assets in the joint project can then be written as $[x(1-\xi) + (1-x)]nl = (1-x\xi)nl$. If x is equal to 1, Bank 1 has stronger bargaining power, the actual payout could be the lowest one, $n(1-\xi)l$. If Bank 2 has a stronger bargaining power, x could be equal to 0 and the actual payout will be the greatest one, nl. We further take into account of "mark to market accounting", which could lead to a mark down in the valuation of the (impaired) joint project in the event of continuation from 1 to y. The lowest possible valuation would be the price paid for the assets; the highest possible valuation is the original accounting value, so $1 - x\xi < y \leqslant 1$.[15]

When the crisis occurs, Bank 1 faces a choice between two outcomes, liquidation or continuation without government support.

1. If the joint project is continued, then the bank must inject additional cash into the syndicated project (requiring it to raise additional deposits). The balance sheet now becomes as in Balance sheet 2 below.
   Asset $y(1+n)l$ is the mark-to-market value of Project G after Bank 1 takes over the project, liability $(1-x\xi)(nl)$ is the additional deposit raised by Bank 1, capital $(1-y)(1+n)l$ is the net change of capital level due to capital loss, arising from accounting mark down of taking over Project G offset by Bank 1's capital gain from its bargaining power, $(nl)x\xi$.

Assets	*Liabilities*
$1-l$	$D_T + (1-x\xi)(nl)$
$y(1+n)l$	$\overline{C}_T = 1 - D_T - [(1+n)(1-y) - nx\xi]l$
$1 + [n - (1+n)(1-y)]l$	$1 + [n - (1+n)(1-y)]l$

Balance Sheet 2.

This implies that the capital ratio alters from C_T to $\frac{C_T - [(1+n)(1-y) - nx\xi]l}{1 + [n-(1+n)(1-y)]l}$. We define $\Delta_1 = \frac{C_T - [(1+n)(1-y) - nx\xi]l}{1 + [n-(1+n)(1-y)]l} - C_T = \frac{[nx\xi - (1+n)(1-y)]l - C_T[n-(1+n)(1-y)]l}{1 + [n-(1+n)(1-y)]l}$. Now the capital ratio will fall ($\Delta_1 < 0$) if $x\xi n < (1-D_T)n + D_T(1+n)(1-y)$, i.e., if the bargaining gain from its bargaining power is

[15] Possessing bargaining power in the acquisition of the joint assets ($x > 0$), may allow the bank to increase its capital (provided that this bargaining gain exceeds any accounting mark down of asset values); and it is possible that the increase in capital is so large that the capital ratio of the bank actually rises rather than falls after it acquires the distressed assets. An illustration is the acquisition by Barclays Group of the assets of Lehman Brothers North America in 2008, which were immediately marked up the Barclays accounts as "negative good will" because they paid much less for the assets than their accounting value. We thank the referee for pointing this out.

not sufficient to offset the fall in the capital ratio from the increase in the balance sheet and the mark down in the value of assets. The bank will be unable to continue the joint project if $\frac{C_T-[(1+n)(1-y)-nx\xi]l}{\{1+[n-(1+n)(1-y)-1]\}} < \tau$ or if $C_T < (1+[n-(1+n)(1-y)]l)\tau - [nx\xi-(1+n)(1-y)]l = \hat{C}$.

2. The joint project is liquidated, in which case with a loss-given-default ratio of Project G, ξ, depositors are repaid the recovery from the liquidated loan $(1-\xi)l$ and the balance sheet of the bank becomes that presented below as balance sheet 3.

Assets	*Liabilities*
$1-l$	$D_T - l(1-\xi)$ $\overline{C}_T = 1 - D_T - l\xi$
$1-l$	$1-l$

Balance Sheet 3.

Capital falls by $l\xi$ and the capital ratio changes from $1-D_T$ to $\frac{1-D_T-l\xi}{1-l}$. The difference is $\Delta_2 = \frac{1-D_T-l\xi}{1-l} - (1-D_T) = \frac{l(C_T-\xi)}{1-l}$. This implies that the capital ratio will fall ($\Delta_2 < 0$) provided the loss given default ξ is greater than the capital ratio before failure, $1-D_T$. There will be contagion if the fall in capital is large enough to push bank 1 into liquidation i.e. if $C_T + \Delta_2 = C_T + l\frac{C_T-\xi}{1-l} < \tau$ or $C_T < (1-l)\tau + \xi l = \hat{\hat{C}}$.

From the above discussion, it is clear that the impact of the systemic crisis, and the choices available to the bank when such a crisis occurs, will vary according to the amount of capital it holds at the time of the crisis, C_T, the size of its exposure to the joint project relative to the bank's total assets l, the relative exposure of the two banks to the joint project n, the loss ratio of the project after liquidation ξ, the bank's bargaining power over the impaired assets x, and the accounting treatment of jointly held assets y. There are two critical levels of capital $\hat{C}$ and $\hat{\hat{C}}$. If $C \geqslant \hat{C}$, then Bank 1 will be able to take over the project and survive without government assistance. If $C \geqslant \hat{\hat{C}}$, then Bank 1 will be able to liquidate the joint project and survive without government assistance. But if both $C < \hat{C}$ and $C < \hat{\hat{C}}$ then there is contagion, and the failure of Bank 2 forces Bank 1 into liquidation without government assistance.[16]

In the following two sections, we use stochastic dynamic programming technique to examine the bank's choice between liquidation and continuation in the crisis with anticipation of bailout policy. We compare Bank 1's ex-post value to shareholders under different scenarios, which is then used to analyze the value function and capitalization decisions of the bank prior to the crisis. Then we conduct simulations to investigate the impact of anticipated shock intensity, public policy (bailout in the form of common stock or preferred stock, and regulatory capital requirements) and parameter values (e.g., the exposure to the distressed loan) on Bank 1's capitalization decisions, possibility of contagion, and government bailout amounts.

4. Endogenous capital holding decision

Using stochastic dynamic programming, we analyze the post-crisis value of Bank 1 to shareholders and the value prior to the crisis. There are two possible post-crisis value functions: $U(C)$ (for the case when the bank takes over project G) and $W(C)$ (for the case when the bank liquidates project G). These have simple closed form analytical solutions of the general form $A_1 \exp m_1 C + A_2 \exp m_2 C$. These obtain because, post-crisis, the only decision of the bank is to pay or retain dividends and to continue in operation until, eventually, capital falls to the minimum regulatory required level $\hat{C}$ and the bank must close. This is a standard problem of optimal balance sheet management, previously solved by Milne and Robertson (1996), Radner and Shepp (1996) and others. Optimal policy is barrier control, paying no dividends if $C < C^*$, and otherwise to make sufficient dividend payments to maintain $C \geqslant C^*$ for some target level of buffer capital C^*. m_1 and m_2 are constants determined by the relevant equation of motion for the post-crisis evolution of C and A_1 and A_2 are constants of integration. The three free parameters A_1, A_2 and C^* are determined by three boundary conditions applying at $\hat{C}$, $\hat{\hat{C}}$ and C^*. Appendix A states the equations of motion, the boundary conditions and the resulting closed form solutions.

4.1. Parameter restriction

As shown in the previous section, the regulatory capital required for Bank 1 to take over the distressed loan is $\hat{C} = \{(1+[n-(1+n)(1-y)]l\}\tau + [(1+n)(1-y)-nx\xi]l$; while the capital required for Bank 1 to liquidate the distressed loan is $\hat{\hat{C}} = (1-l)\tau + \xi l$. In comparison with the minimum capital requirement for Bank 1 before the shock occurs, i.e., $\overline{C} = \tau$, several possible relationships among $\overline{C}$, $\hat{C}$, and $\hat{\hat{C}}$ exist:

(1) $\overline{C} < \hat{C} < \hat{\hat{C}}$, that is, the regulatory capital requirement for Bank 1 before the shock is lower than that required to take over Project G, which is in turn lower than the amount required to liquidate Project G when the shock occurs.

(2) $\overline{C} < \hat{\hat{C}} < \hat{C}$, that is, the minimum regulatory capital requirement for Bank 1 before the shock is lower than that required to liquidate Project G, which is in turn lower than the amount required to take over Project G when the shock occurs.

(3) $\hat{\hat{C}} < \overline{C} < \hat{C}$, that is, the minimum regulatory capital requirement for Bank 1 before the shock is higher than that required to liquidate Project G, but lower than the amount required to take over Project G when the shock occurs.

It can be easily shown that if $\frac{(1+n)(1-y+y\tau)}{1+nx} < \xi < \frac{\tau}{x} + \frac{(1+n)(1-y)(1-\tau)}{nx}$ holds, the capital required to take over Project G will always be lower than the capital required to liquidate the project, i.e., $\overline{C} < \hat{C} < \hat{\hat{C}}$.[17] For example, if the regulatory capital ratio is 10%, $x = 0$, $n = 1$, and $y = 1$, Condition (1) will hold as long as the loss-given-default ratio of the bank loan is higher than 30%, which is supported by the empirical evidence.[18] We therefore choose this plausible condition and focus on the first case, $\overline{C} < \hat{C} < \hat{\hat{C}}$, in the subsequent analysis.

16 So far we assume that the government takes no action to avert the systemic crisis. However, the government may choose to intervene to prevent bank failure or liquidation of assets by providing additional capital, either as common equity capital or as preferred shares. In Section 4, we will solve for the bailout amounts that must be provided by the authorities to avoid contagion or maximize shareholder's value. In Section 5, we use simulations to show how the bailout amounts depend upon the parameters of the model.

17 The willingness to pay Bank 2 will be affected by capitalization of Bank 1. If Bank 1 has relatively low capital, the takeover will have a relatively small benefit to its own shareholders. The loss of value because of moving closer to minimum capital constraint is relatively large, compared to the benefit of acquiring a new positive cash flow. So Bank 1 is less willing to pay a high price to take over the distressed loan. We thank the referee for suggesting us to consider the complexity of the actual payout by Bank 1 to Bank 2 due to the bargaining process, the project's cash flow, and Bank 1's capitalization.

18 Gupton et al., 2000) examine 181 bank loan defaults (mostly syndicated loans) and find that the mean bank-loan value in default is 69.5% for Senior Secured loans and 52.1% for Senior Unsecured loans. Therefore the loss-given-default ratio (1-recovery rate), is 30.5% for Senior Secured bank loans and 47.9% for senior unsecured loans. Bank loans usually have a higher recovery rate than other forms of debt. Fitch (2005) reports historical recovery rates of Senior Unsecured bonds for 24 industries over the period 2000 to 2004. The mean of average loss-given-default ratio is 67% across industries.

4.2. Bank 1's problem

In anticipation of crisis, Bank 1's post-crisis value functions in different scenarios (liquidation of the joint project, continuation without bailout, or continuation with bailout) will determine Bank 1's choice of whether to liquidate or continue the joint asset, and hence its pre-crisis value function, $V(C)$, and its target level of capital, C^*. The application of standard techniques shows that the ex-ante value function V satisfies Hamilton–Jacobi–Bellman (HJB henceforth) differential equation in the following general form:

$$\rho V = \max_\theta \left\{ \begin{array}{l} \theta + [lR_1 + (1-l)R_2 - \theta]V_c \\ + \frac{1}{2}\left[\sigma_1^2 l^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{cc} \end{array} \right\} + \Lambda(C) \tag{3}$$

where $\Lambda(C)$ takes the following forms depending on the relationship between $C,\ \overline{C},\ \widehat{C}$, and $\widehat{\widehat{C}}$:

$$\phi \max\{W(C-\xi l) - V(C), U(C - [(1+n)(1-y) - nx\xi]l) - V(C)\} \quad \text{if } \widehat{C} < \widehat{\widehat{C}} < C \tag{i}$$

$$\phi[U(C - [(1+n)(1-y) - nx\xi]l) - V(C)] \quad \text{if } \widehat{C} < C < \widehat{\widehat{C}} \tag{ii}$$

$$\lambda\phi \left\{ \begin{array}{l} \max\{\max_{K_1 > \widehat{C} - C}[U(C - [(1+n)(1-y) - nx\xi]l + K_1) - K_1], \\ \max_{K_2 > \widehat{\widehat{C}} - C}[W(C - \xi l + K_2) - K_2], 0\} - V(C) \end{array} \right\} - (1-\lambda)\phi V(C) \quad \text{if } \overline{C} < C \leqslant \widehat{C} \text{ and bailout in the form of preferred equity} \tag{iii}$$

and

$$\lambda\phi \left\{ \begin{array}{l} \max\left\{\max_{K_1 > \widehat{C} - C}\left[\frac{C}{C+K_1}U(C - [(1+n)(1-y) - nx\xi]l + K_1)\right], \right. \\ \left. \max_{K_2 > \widehat{\widehat{C}} - C}\left[\frac{C}{C+K_2}W(C - \xi l + K_2)\right]\right\} - V(C) \end{array} \right\} - (1-\lambda)\phi V(C) \quad \text{if } \overline{C} < C \leqslant \widehat{C} \text{ and bailout in the form of common stock} \tag{iv}$$

The first term θ in brackets is the dividend payment per unit of time. The next two terms $[lR_1 + (1-l)R_2 - \theta]V_c + \frac{1}{2}\left[\sigma_1^2 l^2 + \sigma_2^2(1-l)^2 + 2l_1(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{cc}$ capture the expected change in the continuation value caused by fluctuation in the bank fundamental V.

Under Condition (i) and (ii) Bank 1 can choose to liquidate or take over Project G with no government intervention. Under Condition (i), the last term, $\Lambda(C)$, represents the expected impact, which occurs with probability ϕdt, on Bank 1's continuation value, $V(C)$, of taking over or liquidating Project G, whichever is better. W is Bank 1's value function if it selects to liquidate Project G, and U is Bank 1's value function if it selects to take over Project G. Under Condition (ii), the last term, $\Lambda(C)$, which occurs with probability ϕdt, represents the expected impact of taking over Project G on $V(C)$ (the detailed derivations of U and W functions are provided in Appendix A).

Under Condition (iii) and (iv), Bank 1 expects government bailout if crisis occurs with an expected probability of $\lambda\phi dt$. The last term, $\Lambda(C)$, represents the expected impact on $V(C)$ from the shock and the government bailout. $U(C - [(1+n)(1-y) - nx\xi]l + K_1)$ is Bank 1's value function if it accepts government capital and takes over Project G. $W(C - \xi l + K_2)$ is Bank 1's value function if it accepts the government's capital and liquidates Project G. K_1 (K_2) is the amount of capital injection that maximizes Bank 1's value function if it takes over (liquidates) Project G. Under Condition (iii), the shareholders of preferred stock will receive only the fixed dividend and will not share in the upside gain should the bank recover. Under Condition (iv), an injection of common equity dilutes existing shareholder interests and hence provides a stronger incentive for Bank 1 to hold more capital to cope with the failure of other banks. The term $-(1-\lambda)\phi V(C)$ reflects the expected effect on Bank 1's continuation value, $-V(C)$, from the shock and no government bailout, which occurs with probability $(1-\lambda)\phi dt$.

Finally, if $C \leqslant \overline{C}$, Bank 1's capital holding does not meet the regulatory requirement, and is also insufficient to take over or liquidate Project G. Bank 1 will be liquidated.

$$V(C) = 0 \tag{4}$$

4.3. Analytical solutions

Using the dynamic stochastic programming techniques, we find the following analytical solutions for the value function $V(C)$ and the optimal capital holding C^*(C^*_{pre} for the preferred equity bailout, and C^*_{com} for the common stock bailout).

(i, ii) If $\widehat{C} < \widehat{\widehat{C}} < C$,

$$\begin{aligned} V(C) = {} & \Pi_1 \exp(m_{1V}(C - \overline{C})) + \Pi_2 \exp(m_{2V}(C - \overline{C})) \\ & - \frac{\phi\Psi_1}{\frac{1}{2}\sigma_V^2 m_{1U}^2 + R_V m_{1U} - (\rho+\phi)} \exp(m_{1U}(C - \widehat{C})) \\ & - \frac{\phi\Psi_2}{\frac{1}{2}\sigma_V^2 m_{2U}^2 + R_V m_{2U} - (\rho+\phi)} \exp(m_{2U}(C - \widehat{C})) \end{aligned} \tag{5}$$

where

$$m_{1V} = \frac{-R_V + \sqrt{R_V^2 + 2(\rho+\phi)\sigma_V^2}}{\sigma_V^2}, \quad m_{2V} = \frac{-R_V - \sqrt{R_V^2 + 2(\rho+\phi)\sigma_V^2}}{\sigma_V^2},$$

$$\sigma_V^2 = \sigma_1^2(l)^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12},$$

$$R_V = lR_1 + (1-l)R_2$$

(iii) if $\overline{C} < C \leqslant \widehat{C}$ and bailout in the form of preferred equity:

$$V(C) = \Omega_1 \exp(m_{1v}(C - \overline{C})) + \Omega_2 \exp(m_{2v}(C - \overline{C})) + \frac{R_V\lambda\phi}{(\rho+\phi)^2} + \frac{\lambda\phi}{\rho+\phi}(C - \widetilde{C}) \tag{6}$$

when $C < \overline{C},\ V(C) = 0$.

(iv) if $\overline{C} < C \leqslant \widehat{C}$ and bailout in the form of common stock:

$$V(C) = \Omega_3 \exp(m_{1V}(C - \overline{C})) + \Omega_4 \exp(m_{2V}(C - \overline{C})) + \frac{R_V\lambda\phi\Omega_5}{(\rho+\phi)^2} + \frac{\lambda\phi\Omega_5}{\rho+\phi}C \tag{7}$$

where $\Omega_5 = \frac{(1+n)lR_1 + (1-l)R_2}{\rho\{C^*_u + [(1+n)(1-y) - nx\xi]l\}}$ and when $C < \overline{C},\ V(C) = 0$.

We can find $\Pi_1,\ \Pi_2$ in Eq. (5), $\Omega_1,\ \Omega_2,\ C^*_{pre}$ in Eq. (6), and $\Omega_3,\ \Omega_4,\ C^*_{com}$ in Eq. (7), using the boundary conditions as follows: (1) At $\overline{C},\ V(C) = 0$ (2) $V(C)$ is continuous at $\widehat{C}$ (3) $V_C(C)$ is continuous at $\widehat{C}$ (4) $V_C = 1$ at C^* (5) $V_{CC} = 0$ at C^*.

The first boundary condition states that Bank 1 will be liquidated if the capital level is lower than the regulatory requirement, i.e., $C < \overline{C}$. Condition (2) and (3) obtains because the sample paths for C across the $\widehat{C}$ boundary are continuous. When $C > \overline{C}$, the level and change of Bank 1's value function can be continuously adjusted by changing dividend policy in the neighborhood of $\widehat{C}$. In Condition (4) and (5), C^* is the desired long-run or target level of capitalization at which all earnings are paid out. At C^* any increment to capital is paid immediately as a dividend (referred to as barrier control). Values of capital holdings above C^* cannot be obtained because of the continuous sample paths of the assumed diffusion process. The bank always wishes to retain a buffer of capital to reduce the expected cost of not meeting the regulatory capital requirement. Therefore, the optimal policy is to pay dividends at as high a level as possible when C exceeds C^*, but otherwise to retain all earnings. Condition (4) arises because control is instantaneous at the C^* boundary. Bank 1's value prior to the crisis is equal to the optimal capital level, C^*, when the capital is chosen optimally. Condition (5) is a consequence of an optimally selected C^*. Otherwise, the value function could be increasing at C^* by a small shift of C^* in the direction that $V_c < 1$.

Proof. See Appendix B. □

In Fig. 2, we present pictures of the value function ($V(C)$, $U(C)$ and $W(C)$) to show the changes in the capital that follow different actions (continuation, liquidation) based on the solution of the dynamic models Eqs. (A1-2) and (A1-3) for $U(C)$ and Eqs. (A1-5) and (A1-6) for $W(C)$ in Appendix A. The solid, dashed and dot-dashed lines represent $V(C)$, $U(C)$ and $W(C)$ respectively. The figures in the four panels differ horizontally by parameter range and vertically by type of bailout. $C^*_{pre}(C^*_{com})$ is the optimal capital ratio selected by Bank 1 prior to the crisis with the anticipated preferred stock (common stock) bailout. $\widehat{C}$ is the required capital for Bank 1 to take over Project G. The difference between $\widehat{C}$ and $C^*_{pre}(\widehat{C}$ and $C^*_{com})$ is the minimum amount of capital that must be provided by the government for Bank 1 to continue the project with the anticipated preferred stock (common stock) bailout. Panel (a) $\left(\overline{C} < \widehat{C} < C^*_{pre} < \widehat{\widehat{C}}\right)$ and Panel (c) $\left(\overline{C} < \widehat{C} < C^*_{com} < \widehat{\widehat{C}}\right)$ show the case when Bank 1 has sufficient capital to take over Project G without bailout, while Panel (b) $\left(\overline{C} < C^*_{pre} < \widehat{C} < \widehat{\widehat{C}}\right)$ and Panel (d) $\left(\overline{C} < C^*_{com} < \widehat{C} < \widehat{\widehat{C}}\right)$ show the case when bailout is necessary.

Several observations can be made from Fig. 2. First, Bank 1's post-crisis value functions, $U(C)$ and $W(C)$, are non-linear functions of the capital ratio post crisis. Second, the shareholder's value $U(C)$ when Bank 1 takes over Project G is always higher than $W(C)$ when Bank 1 liquidates Project G, no matter whether the firm receives bailout or not, or bailout takes the form of preferred or common stock. The comparison of $U(C)$ and $W(C)$ determines that Bank 1 will choose to continue Project G and the target level of capital prior to the crisis. Therefore, our subsequent analysis focuses on the case of continuation of the project. Third, the pre-crisis target level of capital, $C^*_{pre}(C^*_{com})$, which is endogenously determined by solving Eq. (3), depends on payoffs under these different scenarios.

5. Bank optimal capital holding, interbank contagion, and government bailout

Since we cannot obtain a closed-form solution for the optimal capital holding for Bank 1, we use simulations to examine the impact of a number of parameters on Bank 1's optimal capital holding and whether interbank contagion will emerge.[19] These factors include exogenous variables and factors related to Bank 1's exposure to Project G. For each case, we compare the required capital level when Bank 1 takes over Project G, $\widehat{C}$, with the optimal level of ex-ante capital holding under bailout in the form of common stock and preferred equity, C^*_{com} and C^*_{pre}. We also show the bailout amounts of common stock and preferred equity that are needed to maximize shareholder's value for continuation of Project G, K^*_{pre} and K^*_{com}, where $K^*_{pre} = C^*_u + [(1+n)(1-y) - nx\xi]l - C^*_{pre}$, $K^*_{com} = C^*_u + [(1+n)(1-y) - nx\xi]l - C^*_{com}$ and C^*_u is the new desired capital level C^* when Bank 1 takes over Project G.

Below are the baseline parameter values:

σ_1	R_1	σ_2	R_2	ω_{12}	ρ	A
0.02	0.04	0.01	0.02	0.05	0.05	1

Table 2
The impact of shock intensity on optimal capital holding, contagion and bailout amounts $\lambda = 0.5$, $l = 0.3$, $n = 0.8$, $x = 0.1$, $y = 0.98$, $\tau = 0.10$, $\xi = 0.50$.

ϕ	0.1	0.3	0.5	0.7	0.9
$\widehat{C}$	0.12172	0.12172	0.12172	0.12172	0.12172
C^*_{pre}	**0.115931**	**0.114752**	**0.114026**	**0.113492**	**0.113068**
C^*_{com}	**0.11694**	**0.117928**	0.139893	0.140582	0.141071
K^*_{pre}	0.0297867	0.0309655	0.031692	0.0322255	0.03265
K^*_{com}	0.0287782	0.0277893	0.00582449	0.00513538	0.00464646

Table 3
The impact of the anticipated government bailout probability on optimal capital holding, contagion and bailout amounts $\phi = 0.5$, $l = 0.10$, $n = 1$, $x = 0.15$, $y = 0.96$, $\tau = 0.08$, $\xi = 0.50$.

λ	0.1	0.3	0.5	0.7	0.9
$\widehat{C}$	0.08786	0.08786	0.08786	0.08786	0.08786
C^*_{pre}	0.104008	0.103964	0.103919	0.103873	0.103826
C^*_{com}	0.10401	0.103972	0.103933	0.103893	0.103852
K^*_{pre}	0.00259455	0.00263822	0.00268305	0.0027291	0.00277646
K^*_{com}	0.00259191	0.00263012	0.00266922	0.00270925	0.00275025

5.1. The impact of shock intensity, bailout policy, and regulatory capital requirement on contagion and bailout amounts

Table 2 presents simulation results to illustrate the relationship between the anticipated shock intensity and Bank 1's optimal capital holding, possibility of contagion, and bailout amounts. To show the economic magnitude, we assume that the total assets of Bank 1, A, are \$100 billion. The capital holding required to take over Project G, $\widehat{C}$, is \$12.172 billion. It is apparent that C^*_{com} is always higher than or equal to C^*_{pre}. Contagion occurs for a wider range of values of ϕ in anticipation of preferred stock bailout than common stock bailout (we use numbers in bold to indicate contagion). For example, when $\phi = 0.9$, C^*_{com} is \$14.1071 billion, exceeding the required capital ratio to take over Project G, while C^*_{pre} is \$11.3068 billion, lower than $\widehat{C}$. That is, Bank 1 is willing to set aside \$2.8 billion more for an anticipated common stock bailout. Intuitively, if Bank 1 views a shock and a common stock bailout as likely, it will keep more capital in order to

Table 4
The impact of the regulatory capital ratio on optimal capital holding, contagion and bailout amounts $\phi = 0.5$, $\lambda = 0.5$, $l = 0.1$, $n = 2$, $x = 0.15$, $y = 0.95$, $\xi = 0.50$.

τ	0.08	0.09	0.10	0.11	0.12
$\widehat{C}$	0.0948	0.10665	0.1185	0.13035	0.1422
C^*_{pre}	0.11154	**0.104859**	**0.114848**	**0.124838**	**0.134828**
C^*_{com}	0.111553	0.1234	**0.117999**	**0.126985**	**0.136416**
K^*_{pre}	0.0034052	0.0219367	0.023797	0.0256574	0.0275179
K^*_{com}	0.0033923	0.00339517	0.0206462	0.0235102	0.0259295

[19] We used Mathematica to generate simulation results. Due to space constraint, we only report a subset of simulation results. However, the code is available upon request for interested readers to investigate other cases. Please also refer our working paper version for more detailed simulation results and discussions.

Table 5
Bank 1's bargaining power, x, and the optimal capital holding $\phi = 0.5$, $\lambda = 0.5$, $l = 0.2$, $n = 1$, $y = 1$, $\tau = 0.10$, $\xi = 0.50$.

x	0	0.05	0.10	0.15	0.20
$\widehat{C}$	0.12	0.115	0.11	0.105	0.1
C^*_{pre}	**0.113809**	**0.113832**	0.126471	0.12158	0.117935
C^*_{com}	**0.117594**	0.131471	0.126476	0.121584	0.117935
K^*_{pre}	0.027084	0.0220611	0.00442152	0.00431263	0.00295751
K^*_{com}	0.023299	0.0044222	0.0044167	0.00430878	0.00295751

avoid contagion when the crisis materializes. This underscores the importance of keeping capital buffer in anticipation of interbank contagion. To help Bank 1 to reach the new optimal capital level that maximizes its shareholder's value, the government needs to inject 3.265 billion $\left(K^*_{pre}\right)$ and 0.46 billion $\left(K^*_{com}\right)$ in the case of preferred and common stock bailout, respectively. Bank 1's more ex ante capital holding for an anticipated common equity bailout allows a lower level of government recapitalization.

Contagion is more likely to occur if Bank 1 underestimates probability of crisis. For example, when ϕ is 0.9 (C^*_{com} is 14.1071 billion) but Bank 1 estimates the shock intensity to be 0.3, C^*_{com} (11.7928 billion) will be lower than $\widehat{C}$, thus contagion occurs. The government needs to inject an amount of 2.779 billion rather than 0.465 billion if Bank 1 correctly estimated the shock intensity of 0.9. An unexpected external shock is more likely to cause interbank contagion and large bailout requirements, as shown in recent financial crisis.

Table 3 illustrates the impact of the anticipated probability of the government bailout. When the probability of bailout goes from 0.1 to 0.9, C^*_{pre} decreases by \$18.2 million from \$10.4008 billion to \$10.3826 billion, while C^*_{com} decreases by a smaller amount of \$15.8 million from \$10.401 billion to \$10.3852 billion. The higher the anticipated probability of the government bailout, the lower the ex ante capital holding Bank 1 will maintain, and the greater amount of bailout will be needed, reflecting the moral hazard problem.

Next, we examine in Table 4 the impact on contagion and bailout amounts if the public policy on capital requirement is changed. The recently finalized Basel III requires banks to hold 4.5% of common equity (up from 2% in Basel II) and 6% of Tier I capital (up from 4%) of risk-weighted assets. Basel III also introduces an additional capital conservation buffer of 2.5%, which is designed to ensure that banks build up capital buffers outside periods of stress which can be drawn down as losses are incurred and to avoid breaches of minimum capital requirements. Our simulation shows that increasing the absolute regulatory minimum will not necessarily reduce contagion, in fact, this could increase contagion. But imposing the conservation buffer as Basel III could help banks to increase resilience.

As shown in Table 4, when τ is 0.08, both C^*_{pre} and C^*_{com} are greater than 0.08, no contagion will occur. However, if the authority increases τ (0.09, 0.10, 0.11 or 0.12), contagion could occur under the anticipated preferred stock bailout because $\widehat{C}$ goes up by a higher level than C^*_{pre}. Similarly, contagion will occur when τincreases to 0.10, 0.11 or 0.12 under the anticipated common stock. As shown in the last two rows, K^*_{pre} and K^*_{com} increase with τ, suggesting that simply increasing minimum capital requirement is not a cure-all

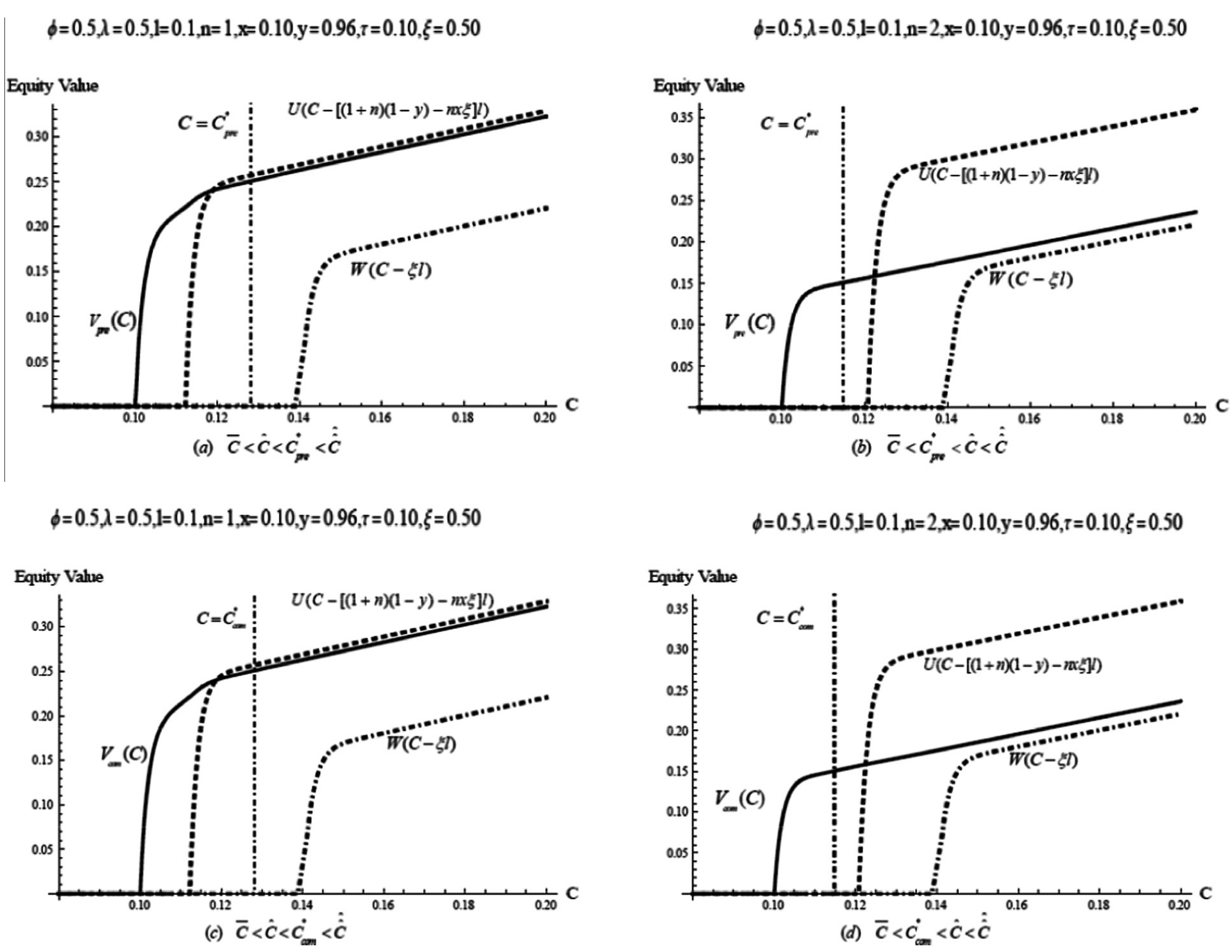

Fig. 2. Bank 1's pre-crisis and post-crisis value functions. *Note:* (a) and (b) shows Bank 1's pre-crisis and post-crisis value functions with the anticipated preferred stock bailout; (c) and (d) shows Bank 1's pre-crisis and post-crisis value functions with the anticipated preferred stock bailout.

solution. Instead, it may increase the burden of the government when crisis happens. However, if Bank 1 holds an additional 2.5% of the conservation capital buffer, which increases its capital holding to 12.5% when τ is 10%, the bank can take over Project G and avoid contagion since it is higher than $\widehat{C}$ (0.1185). Government bailout is not necessary as Bank 1 can draw down its capital buffer to avoid loss.

5.2. The impact of Bank 1's exposure to Project G on contagion and bailout amounts

Next, we show how a range of factors related to Bank 1's exposure to Project G affect Bank 1's optimal capital holding, possibility of contagion and bailout amounts.

Fig. 3 displays the relationship between C^* and the investment ratio, l, as l changes from 0 to 1. When l is small enough (e.g., less than 0.15), Bank 1 will have sufficient capital to avoid contagion ($C^* > \widehat{C}$). Once l becomes large enough (e.g., greater than 0.2), C^* drops discretely. Within this range, C^* is increasing in l, indicating that Bank 1 holds more capital buffer if it invests more of its assets in Project G. However, $\widehat{C}$ (the dot-dashed line) increases at a faster rate than C^*_{com} or C^*_{pre}, indicating contagion when l is large.

The relationship between l and bailout amounts is shown in Fig. 4. As l increases, bailout amount K^* increases but at different rates in two intervals. The interval to the left of jump discontinuity corresponds to the case where Bank 1 can survive without government bailout as $C^* > \widehat{C}$. To maximize shareholder's value, Bank 1 only needs a tiny amount of bailout. The right interval shows that the bailout amounts are considerably higher when $C^* < \widehat{C}$. Moreover, K^*_{com} is always lower than K^*_{pre}. Intuitively, more capital buffer held by Bank 1 with the anticipated common equity helps the government to save bailout budget.

Next, if the relative investment ratio in Project G of Bank 2 over Bank 1, n, increases within a certain range (e.g., lower than or equal to 1), Bank 1's ex ante optimal capital holding increases as well and no contagion occurs. However, as n increases further (n = 1.5, 2, or 2.5), i.e., Bank 2 holds a greater fraction of distressed loan than Bank 1, Bank 1's capital holding is lower than $\widehat{C}$ and contagion happens. Fig. 5 shows the relationship between bailout amounts and n. The bailout amounts are insignificant if Bank 2's investment ratio is smaller or close to Bank 1's investment ratio in Project G. However, when n becomes large enough, K^*_{pre} and K^*_{com} jump up discretely. Intuitively, it is more expensive for the government to help Bank 1 to take over the large fraction of loan held by Bank 2.

In addition, we examine how Bank 1's bargaining power, x, affects contagion possibility and bailout amounts. As shown in Table 5, when x increases from 0.10 to 0.20, Bank 1 will hold lower

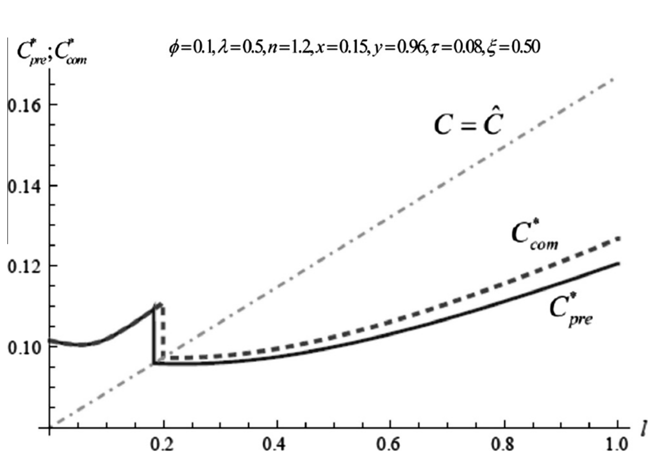

Fig. 3. The impact of Bank 1's investment ratio on the optimal capital holding.

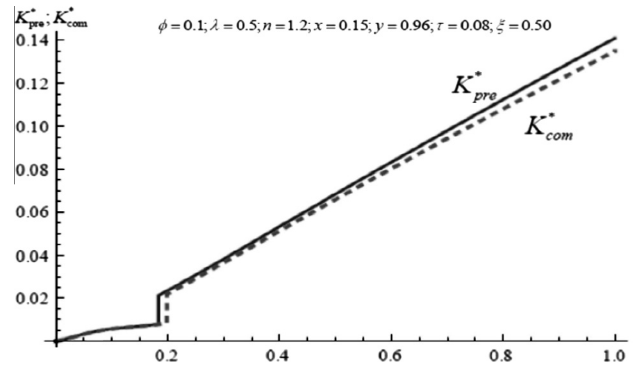

Fig. 4. The impact of Bank 1's investment ratio, l, on government bailout amounts.

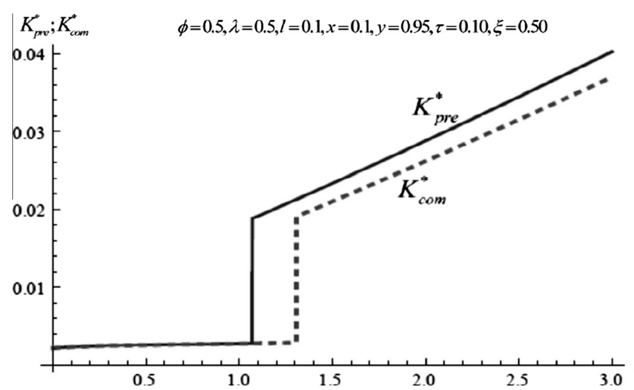

Fig. 5. The impact of two banks' relative investment ratio, n, on government bailout amounts.

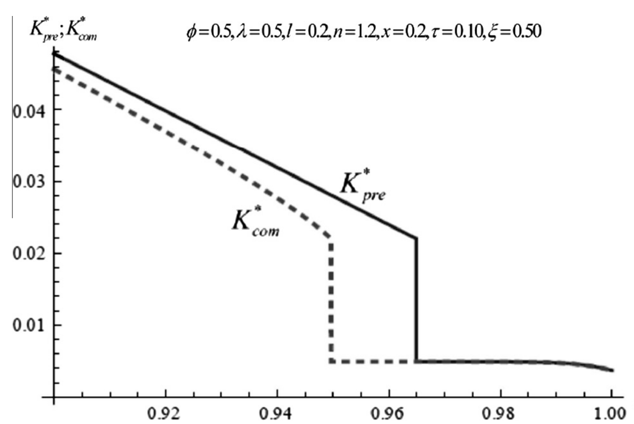

Fig. 6. The impact of the mark-to-market value of Project G, y, on government bailout amounts.

amounts of capital prior to the crisis. If Bank 1 seriously overestimates its bargaining power, contagion will occur. For example, if Bank 1 estimates its bargaining power to be 0.2, its optimal capital holding ratio is 11.7935 billion. However, if its actual bargaining power is 0, the regulatory capital requirement is 12 billion, which exceeds the bank's capital holding ratio and contagion will happen. There is a large drop of bailout amount, K^*, once x exceeds a certain level, and the decline occurs at a lower value of x for K^*_{com} than K^*_{pre}. Presumable, a bank with weak bargaining power will rely on more capital injection from the government.

Finally, when Bank 1 underestimates the loss of the loan value due to marking-to-market, contagion could happen. The bailout amounts are inversely related to the mark-to-market value of Project G, y, as shown in Fig. 6. If y drops slightly (the right interval to the jump discontinuity), bailout amounts are quite low as Bank 1 does not need government bailout to survive. In comparison with the preferred equity bailout, the amount of common stock injection stays at a lower range for a larger decline of market value. However, if the loan suffers a greater loss of market value as during the recent financial crisis (the left interval), government has to inject considerably more capital, regardless of the form of bailout.

6. Conclusion

Building on the inventory buffer model of bank capital, we analyze the impact of capital constraints on the choices of a bank to takeover or liquidate an exposure held jointly with another bank that fails upon a shock. The choices available to the healthy bank depend on the amount of capital it holds at the time of the crisis and the size of its exposure in the joint project, among other things. Contagion will occur if the bank's capital holding is below the smaller amount of regulatory capital required to take over or liquidate the project. Employing the stochastic dynamic programming, we further analyze the post-crisis value functions under liquidation and under continuation, and characterize the ex-ante optimal holding of bank capital in anticipation of government bailout, allowing for possible bank actions after the shock. Then we use simulations to investigate the relationships between a number of public policy and banks' investment parameters and the level of ex-ante capital holding, possibility of contagion, and government bailout amounts.

We have the following interesting findings. First, banks are less likely to hold sufficient capital prior to the crisis to continue or liquidate the joint project if they seriously underestimates the risk of failure of its partner bank, or if they have a greater fraction of distressed loan in its total assets, a smaller investment relative to its partner bank, a weaker bargaining power over the joint project, or a higher mark-to-market value loss of the impaired joint project. Contagion is more likely to happen and bailout is necessary. Second, the ex-ante optimal capital holding decreases with the anticipated probability of bailout, reflecting the risk of moral hazard. Recapitalization with common stock rather than preferred stock dilutes existing shareholder interests and gives the bank a greater incentive to hold capital to cope with potential contagion, thereby reducing moral hazard. Third, an increase in regulatory capital minimum does not necessarily reduce the possibility of contagion, while the requirement of conservation capital buffer as in Basel III could increase the bank's resilience.

Our study adds to the bank contagion literature by focusing on disturbances arising from the asset side. We model a new mechanism of interbank contagion arising from banks' joint exposure to a common asset. The inventory buffer model of bank capital allows us to model banks' precautionary risk management behaviors before crisis happens. Furthermore, our paper contributes to the growing bailout literature by explicitly examining the relationship between bank capital holding and government bailout policy design.

Although our model is a based on one failed bank and one partner bank, it can be generalized to one failed bank and many partner banks. Moreover, there are many failed banks during a financial crisis, each associated with a number of partner banks. The aggregate losses may lead to the failure of an otherwise healthy bank. Our model provides a possible operational tool for estimating what percentage of banks will fail owing to interbank linkages and inadequate capital. It is important for the government to estimate the severity of contagion before making a bailout decision.

Our findings have important economic and policy implications. They should add to our understanding of bank risk management, such as capital buffer management and diversification strategy. Because low capital holding plays a key role in promoting contagion, banks should take into account the potential risk of external shocks to their counterparty banks and increase capital buffer during good times in preparation for bad times. Our study should also be useful for policymakers to design regulation and bailout policies to reduce contagion, control moral hazard, and reduce the size and frequency of bailouts in the long run.

Acknowledgments

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Appendix A. Derivation of *U*(*C*) and *W*(*C*)

We present the equations of motion, the required boundary conditions and resulting close form solutions for the value function $U(C)$, if Bank 1 takes over Project G, and $W(C)$, if Bank 1 liquidates Project G.

If Bank 1 takes over Project G, its capital changes according to the following equation:

$$dC = ((1+n)lR_1 + (1-l)R_2 - \theta)dt + (1+n)\sigma_1 l dZ_1 + \sigma_2(1-l)dZ_2 \quad \text{(A1-1)}$$

Bank 1 chooses a value of θ to maximize $U(C)$ on the basis of its capital level, C^*_U, subject to Eq. (A1-1) and $\theta \geqslant 0$, $U(C) = 0$ if $C < \widehat{C}_U = (1 + [n - (1+n)(1-y)]l)\tau$.

This is a standard problem of optimal balance sheet management. $U(C)$ has a general analytical solution: $U(C) = \Psi \exp(m(C - \widehat{C}_U)) + \Phi$.

We use the three boundary conditions below to derive the values for m, Ψ, Φ, and C^*_U:

(i) Continuity of V at the liquidation threshold $C = \widehat{C}_U$.
This boundary condition obtains because the sample paths for C across the liquidation boundary are continuous.

(ii) $U_C = 1$ at C^*_U, i.e., continuity of U_C at the boundary C^*_U, using the value function for $C > C^*_U$, is given by $U(C) = U(C^*_U) + C - C^*_U$.
This condition arises because control is instantaneous at the C^*_U boundary. This boundary condition will apply regardless of whether C^*_U is chosen optimally because diffusion paths across the boundary are continuous.

(iii) $U_{CC} = 0$ at C^*_U, i.e., continuity of U_{CC} at the boundary C^*_U.
This is a consequence of an optimally selected C^*_U. Otherwise, the value function could be increasing at C^*_U by a small shift of C^*_U in the direction that $U_C < 1$. Using the three boundary conditions, we solve for the equity value as follows.

$$U(C) = \Psi_1 \exp(m_{1U}(C - \widehat{C}_U)) + \Psi_2 \exp(m_{2U}(C - \widehat{C}_U)) \quad \text{(A1-2)}$$

$$C^*_U - \widehat{C}_U = \frac{\ln m^2_{2U} - \ln m^2_{1U}}{m_{1U} - m_{2U}} \quad \text{(A1-3)}$$

$$U(C_U^*) = \frac{\overline{R}}{\rho} = \frac{(1+n)lR_1 + (1-l)R_2}{\rho}$$

$$\Psi_1 = -\Psi_2 = \left[m_{1U}\exp\left(m_{1U}\left(C_U^* - \widehat{C}_U\right)\right) - m_{2U}\exp\left(m_{2U}\left(C_U^* - \widehat{C}_U\right)\right)\right]^{-1}$$

where

$$m_{1U} = \frac{-\overline{R} + \sqrt{\overline{R}^2 + 2\rho\bar{\sigma}^2}}{\bar{\sigma}^2}, \quad m_{2U} = \frac{-\overline{R} - \sqrt{\overline{R}^2 + 2\rho\bar{\sigma}^2}}{\bar{\sigma}^2},$$
$$\overline{R} = (1+n)lR_1 + (1-l)R_2,$$
$$\bar{\sigma}^2 = ((1+n)l)^2\sigma_1^2 + (1-l)^2\sigma_2^2 + 2[(1+n)l](1-l)\omega_{12}\sigma_1\sigma_2$$

We summarize the value of $U(C)$ for different ranges of C as follows:

$U(C) = 0$ when $C < \widehat{C}_U$;

$U(C) = \Psi_1\exp(m_{1U}(C - \widehat{C}_U)) + \Psi_2\exp(m_{2U}(C - \widehat{C}_U))$ when $\widehat{C}_U < C < C_U^*$;

$U(C) = U(C_U^*) + C - C_U^*$ when $C > C_U^*$.

If Bank 1 liquidates Project G, its capital changes according to the following equation:

$$dC = (1-l)R_2dt + (1-l)\sigma_2 dZ_2 \tag{A1-4}$$

Bank 1 chooses a value of θ to maximize $W(C)$ on the basis of its capital level, C_W^*, subject to Eq. (A1-4) and $\theta \geqslant 0$, $W(C) = 0$ if $C < \widehat{C}_W = (1-l)\tau$.

$W(C)$ has a general analytical solution: $W(C) = \Psi\exp(m(C - \widehat{C}_W)) + \Phi$.

We use the three boundary conditions below to derive the values for m, Ψ, Φ, and C_W^*:

(i) Continuity of V at the liquidation threshold $C = C_W^*$. This boundary condition obtains because the sample paths for C across the liquidation boundary are continuous.

(ii) $W_C = 1$ at C_W^*, i.e., continuity of U_C at the boundary C_W^*, using the value function for $C > C_W^*$, is given by $W(C) = W(C_W^*) + C - C_W^*$. This condition arises because control is instantaneous at the C_W^* boundary. This boundary condition will apply regardless of whether C_W^* is chosen optimally because diffusion paths across the boundary are continuous.

(iii) $W_{CC} = 0$ at C_W^*, i.e., continuity of W_{CC} at the boundary C_W^*. This is a consequence of an optimally selected C_W^*. Otherwise, the value function could be increasing at C_W^* by a small shift of C_W^* in the direction that $W_C < 1$. Using the three boundary conditions, we solve for the equity value as follows.

$$W(C) = \Psi_1\exp(m_{1W}(C - \widehat{C}_W)) + \Psi_2\exp(m_{2W}(C - \widehat{C}_W)) \tag{A1-5}$$

$$C_W^* - \widehat{C}_W = \frac{\ln m_{2W}^2 - \ln m_{1W}^2}{m_{1W} - m_{2W}} \tag{A1-6}$$

$$W(C_W^*) = \frac{\overline{R}}{\rho} = \frac{(1-l)R_2}{\rho}$$

$$\Psi_1 = -\Psi_2 = \left[m_1\exp\left(m_1\left(C_W^* - \widehat{C}_W\right)\right) - m_2\exp\left(m_2\left(C_W^* - \widehat{C}_W\right)\right)\right]^{-1}$$

where

$$m_{1W} = \frac{-\overline{R} + \sqrt{\overline{R}^2 + 2\rho\bar{\sigma}^2}}{\bar{\sigma}^2}, \quad m_{2W} = \frac{-\overline{R} - \sqrt{\overline{R}^2 + 2\rho\bar{\sigma}^2}}{\bar{\sigma}^2},$$
$$\overline{R} = (1-l)R_2, \quad \bar{\sigma}^2 = (1-l)^2\sigma_2^2$$

We summarize the value of $W(C)$ for different ranges of C as follows: $W(C) = 0$ when $C < \widehat{C}_W = (1-l)\tau$;

$W(C) = \Psi_1\exp(m_{1W}(C - C_W^*)) + \Psi_2\exp(m_{2W}(C - C_W^*))$ when $\widehat{C}_W < C < C_W^*$;

$W(C) = W(C_W^*) + C - C_W^*$ when $C > C_W^*$.

Appendix B. Solving for Bank 1's optimal capital ratio

This appendix provides an outline of proof for our analytical solution in Section (4.3). First, we prove the following four properties to simplify the model.

Property 1. *The shareholders value of Bank 1 if it receives a bailout amount in the form of preferred equity and takes over Project G will always exceed the bailout amount, i.e.,*

$$\max_{K_1 > \widehat{C} - C}[U(C - [(1+n)(1-y) - nx\xi]l + K_1) - K_1] = \frac{(1+n)lR_1 + (1-l)R_2}{\rho} - \left(C_U^* - C + [(1+n)(1-y) - nx\xi]l\right) > 0$$

Proof. If Bank 1 receives a bailout amount of K_1, from Appendix A, we know that when $\widehat{C} < C - [(1+n)(1-y) - nx\xi]l + K_1 < C_U^*$, $U(C - [(1+n)(1-y) - nx\xi]l + K_1) - K_1$ is reached at $C - [(1+n)(1-y) - nx\xi]l + K_1 = C_U^*$, while if $C - [(1+n)(1-y) - nx\xi]l + K_1 > C_U^*$, $U(C - [(1+n)(1-y) - nx\xi]l + K_1) - K_1 = U(C_U^*) + C - [(1+n)(1-y) - nx\xi]l - C_U^* > 0$. □

Property 2. *The shareholders value of Bank 1 if it receives a bailout amount in the form of preferred equity and liquidates Project G will always exceed the bailout amount, i.e.,*

$$\max_{K_2 > \widehat{\widehat{C}} - C}[W(C - \xi l + K_2) - K_2] = \frac{(1-l)R_2}{\rho} - \left(C_W^* - C + \xi l\right) > 0$$

Proof. If Bank 1 receives a bailout amount of K_2, from Appendix A, we know that when $\widehat{C} < C - \xi l + K_2 < C_W^*$, $W(C - \xi l + K_2) - K_2$ is reached at $C - \xi l + K_2 = C_W^*$, while if $C - \xi l + K_2 > C_W^*$, $W(C - \xi l + K_2) - K_2 = W(C_W^*) + C - \left(C_W^* + \xi l\right) > 0$. □

Property 3. *If $\overline{C} < \widehat{C} < \widehat{\widehat{C}}$, the shareholders value of Bank 1 if it takes over Project G without government bailout will always be higher than the value if it liquidates the project, i.e., If $\widehat{C} = (1 + [n - (1+n)(1-y)]l)\tau - [nx\xi - (1+n)(1-y)]l < \widehat{\widehat{C}} = (1-l)\tau + \xi l$ and $C > \overline{C}$, we have $U(C - [(1+n)(1-y) - nx\xi]l) \geqslant W(C - \xi l)$.*

Proof. As $U_{CCC} \geqslant 0$, $U_{CC} \leqslant 0$, $U_C \geqslant 1$ for $\widehat{C}_U < C < C_U^*$, $W_{CCC} \geqslant 0$, $W_{CC} \leqslant 0$, $W_C \geqslant 1$ for $\widehat{C}_W < C < C_W^*$ and $U(C_U^*) = \frac{(1+n)lR + (1-l)R_2}{\rho} > W(C_W^*) = \frac{(1-l)R_2}{\rho}$.

We show $U(C - [(1+n)(1-y) - nx\xi]l) > W(C - \xi l)$ for $C > \widehat{\widehat{C}} \equiv (1-l)\tau + \xi l$.

If $\widehat{\widehat{C}} = (1-l)\tau + \xi l > C > \widehat{C} = (1 + [n - (1+n)(1-y)]l)\tau - [nx\xi - (1+n)(1-y)]l$, $W(C - \xi l) = 0$, $U(C - [(1+n)(1-y) - nx\xi]l) > 0$, So $U(C - [(1+n)(1-y) - nx\xi]l) > W(C - \xi l)$. If $\widehat{C} = (1 + [n - (1+n)(1-y)]l)\tau - [nx\xi - (1+n)(1-y)]l > C > \overline{C} = \tau$, $W(C - \xi l) = U(C - [(1+n)(1-y) - nx\xi]l) = 0$.

Hence, $U(C - [(1+n)(1-y) - nx\xi]l) \geqslant W(C - \xi l)$ for $C > \overline{C}$. □

Property 4. *The shareholders value of Bank 1 if it takes over Project G with government bailout (either in the form of preferred equity or common stock) will always be higher than the value if it liquidates the project, i.e.,em* If $\widehat{C} = (1 + [n - (1+n)(1-y)]l)\tau - [nx\xi - (1+n)(1-y)]l < \widehat{\widehat{C}} = A(1-l)\tau + \xi l$, for $\overline{C} < C < \widehat{C}$, we have

$$U(C_U^*) + C - [(1+n)(1-y) - nx\xi]l - C_U^* > W(C_W^*) + C - \xi l - C_W^*,$$
$$\frac{CU(C_U^*)}{C_U^*} > \frac{CW(C_W^*)}{\xi l_1 + C_W^*}$$

According to the above properties, Bank 1's value function is always higher if it takes over rather than liquidates Project G given anticipated government bailout. Therefore, the HJB Eq. (3) in Section (4.2) can be simplified as follows:

$$\rho V = \max_\theta \left\{ \begin{array}{l} \theta + [lR_1 + (1-l)R_2 - \theta]V_c \\ + \frac{1}{2}\left[\sigma_1^2 l^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{cc} \end{array} \right\} + \Lambda(C) \quad \text{(A2-1)}$$

where $\Lambda(C)$ takes different forms depending on the relationships between C, $\overline{C}, \widehat{C}$, and $\widehat{\widehat{C}}$:

$$\phi[U(C - [(1+n)(1-y) - nx\xi]l) - V(C)]$$
$$\text{if } \widehat{C} < \widehat{\widehat{C}} < C \quad \text{or } \widehat{C} < C < \widehat{\widehat{C}}$$

$$\lambda\phi\left\{\left[\frac{(1+n)lR_1 + (1-l)R_2 A - (C_U^* - C + A[(1+n)(1-y) - nx\xi]l)}{\rho}\right] - V(C)\right\}$$
$$-(1-\lambda)\phi V(C) \quad \text{if } \overline{C} < C \leqslant \widehat{C} \quad \text{and bailout in the form of preferred equity} \quad \text{(iii)}$$

and

$$\lambda\phi\left[\frac{C}{C_U^*}\frac{((1+n)lR_1 + (1-l)R_2)A}{\rho}\right] - (1-\lambda)\phi V(C) \quad \text{if } \overline{C} < C \leqslant \widehat{C} \quad \text{and bailout in the form of common stock} \quad \text{(iv)}$$

Eq. (4) can be rewritten as Eq. (A2-2):

$$V(C) = 0 \quad \text{if } C \leqslant \overline{C} = \tau \quad \text{(A2-2)}$$

The first-order condition for Eqs. (A2-1) to (A2-2) is:

$$V_C = 1 \quad \text{(A2-3)}$$

The optimal policy pre-crisis is once again a buffer capital rule, paying no dividends if $C < C^*$ and otherwise to make sufficient dividend payments to maintain $C \geqslant C^*$ for some target level of buffer capital C^*.

Next we analyze the value function of Bank 1 to find analytical solutions for the optimal capital holding before the shock occurs.(i) and (ii) $\widehat{C} < C$ The dividend is zero. Maximizing Eq. (A2-1) yields the following differential equation for V:

$$\frac{1}{2}\left\{\left[\sigma_1^2(l)^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{cc}\right\} + [lR_1 + (1-l)R_2]V_C - (\rho+\phi)V + \phi U(C - [(1+n)(1-y) - nx\xi]l) = 0, \quad \widehat{C} < C < C^* \quad \text{(A2-4)}$$

Eq. (A2-4) has the general analytical solution

$$V(C) = \Pi_1 \exp(m_{1V}(C - \overline{C})) + \Pi_2 \exp(m_{2V}(C - \overline{C})) - \frac{\phi\Psi_1}{\frac{1}{2}\sigma_V^2 m_{1U}^2 + R_V m_{1U} - (\rho+\phi)}\exp(m_{1U}(C - \widehat{C})) - \frac{\phi\Psi_2}{\frac{1}{2}\sigma_V^2 m_{2U}^2 + R_V m_{2U} - (\rho+\phi)}\exp(m_{2U}(C - \widehat{C})) \quad \text{(A2-5)}$$

where

$$m_{1V} = \frac{-R_V + \sqrt{R_V^2 + 2(\rho+\phi)\sigma_V^2}}{\sigma_V^2},$$
$$m_{2V} = \frac{-R_V - \sqrt{R_V^2 + 2(\rho+\phi)\sigma_V^2}}{\sigma_V^2},$$
$$\sigma_V^2 = \sigma_1^2(l)^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12},$$
$$R_V = lR_1 + (1-l)R_2$$

(iii) $\overline{C} < C < \widehat{C}$ and the government will inject capital in the form of preferred equity.

Let's define $\widetilde{C} = C_U^* + [(1+n)(1-y) - nx\xi]l - \frac{(1+n)lR_1 + (1-l)R_2}{\rho}$. The dividend is zero. (A2-1) becomes

$$\frac{1}{2}\left[\sigma_1^2 l^2 + \sigma_2^2(1-l)^2 + 2l(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{CC} + [lR_1 + (1-l)R_2]V_C - (\rho+\phi)V + \lambda\phi C - \lambda\phi\widetilde{C} = 0 \quad \text{(A2-6)}$$

The value function has the following analytical solution:

$$V(C) = \Omega_1 \exp(m_{1v}(C - \overline{C})) + \Omega_2 \exp(m_{2v}(C - \overline{C})) + \frac{R_V A\lambda\phi}{(\rho+\phi)^2} + \frac{\lambda\phi}{\rho+\phi}(C - \widetilde{C}) \quad \text{(A2-7)}$$

When $C < \overline{C}$, $V(C) = 0$.

We solve for Π_1, Π_2, Ω_1, Ω_2, C_{pre}^* using boundary conditions as follows:

(1) At $\overline{C}, V(C) = 0$

$$V(\overline{C}) = \Omega_1 + \Omega_2 + \frac{R_V\lambda\phi}{(\rho+\phi)^2} + \frac{\lambda\phi}{\rho+\phi}(\overline{C} - \widetilde{C}) = 0 \quad \text{(A2-8)}$$

(2) $V(C)$ is continuous at $\widehat{C}$

$$\Omega_1 \exp(m_{1V}(\widehat{C} - \overline{C})) + \Omega_2 \exp(m_{2V}(\widehat{C} - \overline{C})) + \frac{R_V\lambda\phi}{(\rho+\phi)^2} + \frac{\lambda\phi}{\rho+\phi}(\widehat{C} - \overline{C})$$
$$= \Pi_1 \exp(m_{1V}(\widehat{C} - \overline{C})) + \Pi_2 \exp(m_{2V}(\widehat{C} - \overline{C})) - \frac{\phi\Psi_1}{\frac{1}{2}\sigma_V^2 m_{1U}^2 + R_V m_{1U} - (\rho+\phi)}\exp(m_{1U}(0)) - \frac{\phi\Psi_2}{\frac{1}{2}\sigma_V^2 m_{2U}^2 + R_V m_{2U} - (\rho+\phi)}\exp(m_{2U}(0)) \quad \text{(A2-9)}$$

(3) $V_C(C)$ is continuous at $\widehat{C}$

$$V_C(\widehat{C}) = \Pi_1 m_{1V}\exp(m_{1V}(\widehat{C} - \overline{C})) + \Pi_2 m_{2V} \times \exp(m_{2V}(\widehat{C} - \overline{C})) - \frac{\phi\Psi_1 m_{1U}}{\frac{1}{2}\sigma_V^2 m_{1U}^2 + R_V m_{1U} - (\rho+\phi)} \times \exp(m_{1U}(0)) - \frac{\phi\Psi_2 m_{2U}}{\frac{1}{2}\sigma_V^2 m_{2U}^2 + R_V m_{2U} - (\rho+\phi)} \times \exp(m_{2U}(0))$$
$$= \Omega_1 m_{1V}\exp(m_{1V}(\widetilde{C} - \overline{C})) + \Omega_2 m_{2V} \times \exp(m_{2V}(\widetilde{C} - \overline{C})) + \frac{\lambda\phi}{\rho+\phi} \quad \text{(A2-10)}$$

(4) $$V_C = 1 \quad \text{at } C_{pre}^* \quad \text{(A2-11)}$$

(5) $$V_{CC} = 0 \quad \text{at } C_{pre}^* \quad \text{(A2-12)}$$

(iv) $\overline{C} < C < \widehat{C}$ and the government will inject capital in the form of common stock.

The dividend is zero, (A2-1) becomes

$$\frac{1}{2}\left[\sigma_1^2 l^2+\sigma_2^2(1-l)^2+2l(1-l)\sigma_1\sigma_2\omega_{12}\right]V_{CC} + [lR_1+(1-l)R_2]V_C-(\rho+\phi)V+\lambda\phi\frac{C}{C_U^*}\frac{(1+n)lR_1+(1-l)R_2}{\rho}=0 \quad (A2\text{-}13)$$

The value function has the analytical solution:

$$V(C)=\Omega_3\exp(m_{1V}(C-\overline{C}))+\Omega_4\exp(m_{2V}(C-\overline{C})) +\frac{R_V\lambda\phi\Omega_5}{(\rho+\phi)^2}+\frac{\lambda\phi\Omega_5}{\rho+\phi}C \quad (A2\text{-}14)$$

where $\Omega_5=\frac{(1+n)lR_1+(1-l)R_2}{\rho\left\{C_u^*+[(1+n)(1-y)-nx\xi]l\right\}}$

When $C<\overline{C},\ V(C)=0$.

We solve for $\Pi_1,\ \Pi_2,\ \Omega_3,\ \Omega_4,\ C^*_{com}$ using boundary conditions as follows:

(1) At $\overline{C},\ V(C)=0$

$$V(\overline{C})=\Omega_3+\Omega_4+\frac{R_V\lambda\phi\Omega_5}{(\rho+\phi)^2}+\frac{\lambda\phi\Omega_5}{\rho+\phi}\overline{C}=0 \quad (A2\text{-}15)$$

(2) $V(C)$ is continuous at $\widehat{C}$

$$\begin{aligned}&\Pi_1\exp(m_{1V}(\widehat{C}-\overline{C}))+\Pi_2\exp(m_{2V}(\widehat{C}-\overline{C}))\\&-\frac{\phi\Psi_1}{\frac{1}{2}\sigma_V^2m_{1U}^2+R_Vm_{1U}-(\rho+\phi)}\exp(m_{1U}(0))\\&-\frac{\phi\Psi_2}{\frac{1}{2}\sigma_V^2m_{2U}^2+R_Vm_{2U}-(\rho+\phi)}\exp(m_{2U}(0))\\&=\Omega_3\exp(m_{1V}(\widehat{C}-\overline{C}))+\Omega_4\exp(m_{2V}(\widehat{C}-\overline{C}))\\&\quad+\frac{R_V\lambda\phi\Omega_5}{(\rho+\phi)^2}+\frac{\lambda\phi\Omega_5}{\rho+\phi}\widehat{C}\end{aligned} \quad (A2\text{-}16)$$

(3) $V_C(C)$ is continuous at $\widehat{C}$

$$\begin{aligned}&\Pi_1m_{1V}\exp(m_{1V}(\widehat{C}-\overline{C}))+\Pi_2m_{2V}\exp(m_{2V}(\widehat{C}-\overline{C}))\\&-\frac{\phi\Psi_1m_{1U}}{\frac{1}{2}\sigma_V^2m_{1U}^2+R_Vm_{1U}-(\rho+\phi)}\exp\\&(m_{1U}(-[(1+n)(1-y)-nx\xi]l))\\&-\frac{\phi\Psi_2m_{2U}}{\frac{1}{2}\sigma_V^2m_{2U}^2+R_Vm_{2U}-(\rho+\phi)}\exp\\&(m_{2U}(-[(1+n)(1-y)-nx\xi]l))=\Omega_3m_{1V}\exp(m_{1V}(\widehat{C}-\overline{C}))\\&+\Omega_4m_{2V}\exp(m_{2V}(\widehat{C}-\overline{C}))+\frac{\lambda\phi\Omega_5}{\rho+\phi}\end{aligned} \quad (A2\text{-}17)$$

(4) $$V_C=1 \quad \text{at } C^*_{com} \quad (A2\text{-}18)$$

(5) $$V_{CC}=0 \quad \text{at } C^*_{com} \quad (A2\text{-}19)$$

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