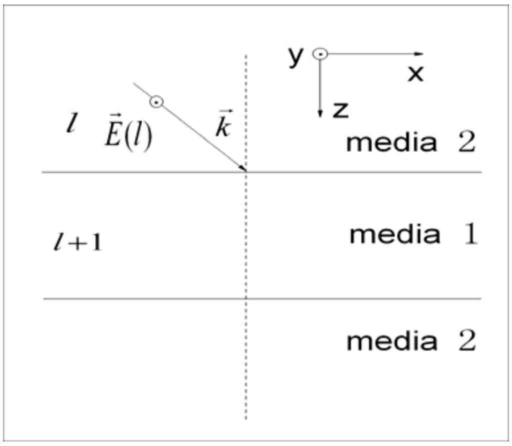
Note for reflection phase in case of total reflection

（1）Transfer matrix of layered media

TE wave



Electric and magnetic field must be continuous at interfaces:

$$E_+(l)+E_-(l)=E_+(l+1)+E_-(l+1)$$

$$H_x(l)=H_x(l+1)$$

$$\Rightarrow -E_+(l)+E_-(l)=\frac{k_z(l+1)\mu_{xx}(l)}{k_z(l)\mu_{xx}(l+1)}[-E_+(l+1)+E_-(l+1)]$$

Fields at interfaces can be connected by a transfer matrix :

$$\begin{pmatrix} E_+(l+1) \\ E_-(l+1) \end{pmatrix} = M^{l,l+1} \begin{pmatrix} E_+(l+1) \\ E_-(l+1) \end{pmatrix}$$

The transfer matrix is :

$$M^{l,l+1}=\frac{1}{2}\begin{pmatrix} 1+\Delta & 1-\Delta \\ 1-\Delta & 1+\Delta \end{pmatrix}$$

$$\Delta=\frac{k_z(l)\mu_{xx}(l+1)}{k_z(l+1)\mu_{xx}(l)}$$

Where $k_z=\sqrt{\varepsilon_l\mu_l\left(\frac{\omega}{c}\right)^2-\beta^2}=\left(\frac{\omega}{c}\right)\sqrt{\varepsilon_l\mu_l\ (1-\sin\theta^2)}=k_l\cos\theta$

For the special case of normal incidence,

$$\Delta=\frac{\sqrt{\varepsilon_l\mu_l}\,\mu_{l+1}}{\sqrt{\varepsilon_{l+1}\mu_{l+1}}\,\mu_l}=\sqrt{\frac{\frac{\mu_{l+1}}{\varepsilon_{l+1}}}{\frac{\mu_l}{\varepsilon_l}}}=\frac{Z_{l+1}}{Z_L}$$

$$M^{l,l+1}=\frac{1}{2}\begin{pmatrix}1+\frac{Z_{l+1}}{Z_L} & 1-\frac{Z_{l+1}}{Z_L}\\ 1-\frac{Z_{l+1}}{Z_L} & 1+\frac{Z_{l+1}}{Z_L}\end{pmatrix}$$

For oblique incidence at single interface:

$$\begin{pmatrix}t\\0\end{pmatrix}=M^{l,l+1}\begin{pmatrix}1\\r\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1+\Delta & 1-\Delta\\1-\Delta & 1+\Delta\end{pmatrix}\begin{pmatrix}1\\r\end{pmatrix}$$

$$r=\frac{\sqrt{\frac{\varepsilon_1}{\mu_1}}k_{z1}-\sqrt{\frac{\varepsilon_2}{\mu_2}}k_{z2}}{\sqrt{\frac{\varepsilon_1}{\mu_1}}k_{z1}+\sqrt{\frac{\varepsilon_2}{\mu_2}}k_{z2}}=\frac{Z_2\cos\theta_1-Z_1\cos\theta_2}{Z_2\cos\theta_1+Z_1\cos\theta_2}\ .$$

TM wave:

Because all the results for TE and TM wave are reciprocal, except for that ε and μ are reverse. We can obtain the result of TM mode.

$$M^{l,l+1}=\frac{1}{2}\begin{pmatrix}1+\Delta & 1-\Delta\\1-\Delta & 1+\Delta\end{pmatrix}$$

$$\Delta=\frac{k_z(l)\varepsilon_{xx}(l+1)}{k_z(l+1)\varepsilon_{xx}(l)}\ 。$$

$$r=\frac{\sqrt{\frac{\mu_1}{\varepsilon_1}}k_{z1}-\sqrt{\frac{\mu_2}{\varepsilon_2}}k_{z2}}{\sqrt{\frac{\mu_1}{\varepsilon_1}}k_{z1}+\sqrt{\frac{\mu_2}{\varepsilon_2}}k_{z2}}=\frac{Z_1\cos\theta_1-Z_2\cos\theta_2}{Z_1\cos\theta_1+Z_2\cos\theta_2}\ 。$$

(2) reflection phase of total reflection.

We can see that for partial reflection r is real, indicating that there is no phase shift between incident wave and reflected wave.

TE mode:

we denote that $\kappa = \sqrt{\varepsilon_1 \mu_1 \left(\frac{\omega}{c}\right)^2 - \beta^2} = k_l \cos\theta_l$, for total reflection $\varepsilon_2 \mu_2 \left(\frac{\omega}{c}\right)^2 - \beta^2 < 0, \gamma = \sqrt{\beta^2 - \varepsilon_2 \mu_2 \left(\frac{\omega}{c}\right)^2}$,the expression " r " became:

$$r = \frac{\frac{\mu_2 \kappa}{i\mu_1 \gamma} - 1}{\frac{\mu_2 \kappa}{i\mu_1 \gamma} + 1} = \frac{\frac{i\mu_2 \kappa}{\mu_1 \gamma} + 1}{\frac{i\mu_2 \kappa}{\mu_1 \gamma} - 1} = \frac{\frac{\mu_1 \gamma}{\mu_2 \kappa} + i}{\frac{\mu_1 \gamma}{\mu_2 \kappa} - i} = \frac{(\frac{\mu_1 \gamma}{\mu_2 \kappa})^2 - 1 + 2\frac{\mu_1 \gamma}{\mu_2 \kappa} i}{(\frac{\mu_1 \gamma}{\mu_2 \kappa})^2 + 1} = e^{i\theta}$$

$$\theta = \tan^{-1}\left(\frac{2\frac{\mu_1 \gamma}{\mu_2 \kappa}}{(\frac{\mu_1 \gamma}{\mu_2 \kappa})^2 - 1}\right) = -2\tan^{-1}\left(\frac{\mu_1 \gamma}{\mu_2 \kappa}\right)$$

theta is the reflection phase ,for regular material ,theta has the range of (-180,0), it means that the wave has lost a certain phase while being reflected. for PEC, theta is -180, that's what is called "half wave lost". for

PMC theta is 0.

However, for meta-material which μ_2 <0, we can obtain arbitrary reflection phase. For the special case of $\mu_2 \to 0^-, \theta \to 180$,this means we can get a “half wave ” phase gain from reflection. That’s what we want in the cases of waveguides and resonators.

For normal incidence $$\theta = -2\tan^{-1}\left(\frac{\sqrt{\frac{\mu_1}{\varepsilon_1}}}{\sqrt{\frac{-\mu_2}{\varepsilon_2}}}\right),$$

For TM wave we can achieve the reciprocal result.

Note by

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