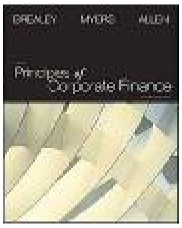
Principles of Corporate Finance

By Zhang Xiaorong

2: How to Calculate Present Values



2-1

Topics Covered

- Present Value and Future Value
- Net Present Value
- NPV Rule and IRR Rule
- Opportunity Cost of Capital
- Valuing Long-Lived Assets
- PV Calculation Short Cuts
- Compound Interest

2-2

Time Value of Money

- Time is valuable for money
  - 1 Dollar today is more valuable than 1 Dollar "tomorrow"
  - Consumption foregone
  - Inflation (or deflation?)
  - Timing
  - Risk

2-3

Time Value of Money

- Time

	Now	Next Year
A：	$1000	$0
B：	$0	$1000

- Uncertainty

A： $0 / $2000

B： $1000

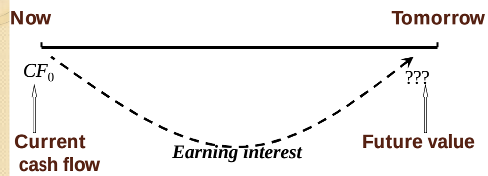
1-4

Present Value

1.Single period discount for future cash flow

2-5

Future Value



2. Can get future value of today's cash flow

2-6

Present and Future Value

Future Value

Amount to which an investment will grow after earning interest

Present Value

Value today of a future cash flow.

Present and Future Value

Present Value

Value today of a future cash flow.

Future Value

Amount to which an investment will grow after earning interest

Discount Rate

"Interest rate" used to compute present values of future cash flows.

Discount Factors and Rates

Discount Rate

Interest rate used to compute present values of future cash flows.

Discount Factor

Present value of $1 future payment.

Future Values

Future Value of $100 = FV

$$FV = \$100 \times (1 + r)$$

Future Values

$$FV = \$100 \times (1 + r)$$

Example - FV

What is the future value of $400,000 if interest is compounded annually at a rate of 5% for one year?

$$FV = \$400{,}000 \times (1 + .05)^1 = \$420{,}000$$

Present Value

Present Value = PV

$$\text{PV} = \text{discount factor} \times C_1$$

Present Value

Discount Factor = DF = PV of $1

$$DF = \frac{1}{(1+r)}$$

Discount Factors can be used to compute the present value of any cash flow.

2-13

Valuing an Office Building

Step 1: Forecast cash flows

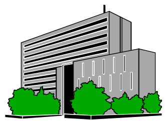
Cost of building = C_0 = 370

Sale price in Year 1 = C_1 = 420

Step 2: Estimate opportunity cost of capital

If equally risky investments in the capital market offer a return of 5%, then

Cost of capital = r = 5%



2-14

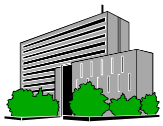
Valuing an Office Building

Step 3: Discount future cash flows

$$PV = \frac{C_1}{(1+r)} = \frac{420}{(1+.05)} = 400$$

Step 4: Go ahead if PV of payoff exceeds investment

$$NPV = 400 - 370 = 30$$



2-15

Net Present Value

NPV = PV - required investment

$$NPV = C_0 + \frac{C_1}{1+r}$$

2-16

Valuing an Office Building

- Decompose the cost
  - Cost of building: 320
  - Cost of land: 50

Question-1: The company does not have to pay for the land. Why is 50 included as part of the cost?

2-17

Valuing an Office Building

Question-2: Where do we get the discount rate of 5%?

- The discount rate is the market rate of return given by the investments at the same risk level as that of your investment.
- It considers both TIMING and RISK. (Take it as given till chapter 7)
- Compare apple to apple, not to orange.

2-18

Risk and Present Value

- Higher risk projects require higher rates of return
- Higher required rates of return cause lower PVs

$$\text{PV of } C_1 = \$420 \text{ at } 5\%$$
$$\text{PV} = \frac{420}{1+.05} = 400$$

2-19

Risk and Present Value

$$\text{PV of } C_1 = \$420 \text{ at } 12\%$$
$$\text{PV} = \frac{420}{1+.12} = 375$$

$$\text{PV of } C_1 = \$420 \text{ at } 5\%$$
$$\text{PV} = \frac{420}{1+.05} = 400$$

2-20

Risk and Net Present Value

$$\text{NPV} = \text{PV - required investment}$$

$$\text{NPV} = 375{,}000 - 370{,}000$$
$$= \$5{,}000$$

2-21

Decision Rules for Investment

- Two important rules
  - Accept investments that offer rates of return *in excess of their opportunity cost of capital*
  - Accept investments that have positive net present value

2-22

Rate of Return Rule

- Accept investments that offer rates of return in excess of their *opportunity cost of capital*

Example

In the project listed below, the foregone investment opportunity is 12%. Should we do the project?

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{420{,}000 - 370{,}000}{370{,}000} = .135 \text{ or } 13.5\%$$

2-23

Net Present Value Rule

- Accept investments that have positive net present value

Example

Suppose we can invest \$50 today and receive \$60 in one year. Should we accept the project given a 10% expected return?

$$\text{NPV} = -50 + \frac{60}{1.10} = \$4.55$$

2-24

Opportunity Cost of Capital

- Resources are limited, so is capital.
- To invest in the specified project means giving up other opportunities of investment.
- The opportunity cost of capital is *the* ***highest*** *rate of return among the alternatives.*
- If your investment gives a higher rate of return than the opportunity cost of capital, you are using the capital in the most efficient way.

2-25

Opportunity Cost of Capital

Example

You may invest $100,000 today. Depending on the state of the economy, you may get one of three possible cash payoffs:

Economy	Slump	Normal	Boom
Payoff	$80,000	110,000	140,000

$$\text{PV} = \frac{110{,}000}{1.15} = \$95{,}650$$

$$\text{NPV} = \$95{,}650 - 100{,}000 = \$-4{,}350$$

2-26

Opportunity Cost of Capital

Example - continued

You notice that one stock in the market has the same risk as that of your investment. The stock is trading for $95.65. Next year's price, given a normal economy, is forecasted at $110.

The stocks expected payoff leads to an expected return.

2-27

Opportunity Cost of Capital

Example - continued

The stocks expected payoff leads to an expected return.

$$\text{Expected return} = \frac{\text{expected profit}}{\textit{investment}}$$

$$= \frac{110 - 95.65}{95.65} = .15 \text{ or } 15\%$$

2-28

Opportunity Cost of Capital

Example - continued

Discounting the expected payoff at the expected return leads to the PV and NPV of the project

$$\text{PV} = \frac{110{,}000}{1.15} = \$95{,}650$$

$$\text{NPV} = \$95{,}650 - 100{,}000 = \$-4{,}350$$

2-29

Opportunity Cost of Capital

Example - continued

Notice that you come to the same conclusion if you compare the expected project return with the cost of capital.

$$\text{Expected return} = \frac{\text{expected profit}}{\textit{investment}} = \frac{110{,}000 - 100{,}000}{100{,}000} = .10 \text{ or } 10\%$$

Expected return on the investment is 10%,
less than the expected return on the stock,
or the opportunity cost of capital 15%.

2-30

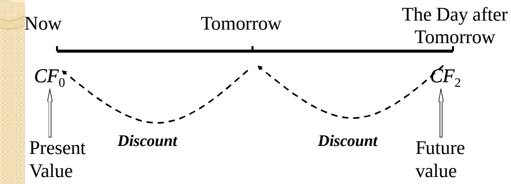
Opportunity Cost of Capital

– Distinguish between Opportunity Cost of Capital and Cost of Borrowing

You are going to start your own business by taking an investing project. You can get bank loan at 8%, or you can get the seed fund from your parents, who are rich and generous and do not require any return. The market rate of return of such projects is 12% on average. What's the discount rate of your project?

2-31

PV of a Long-lived Asset



Multi-period discount

2-32

PV of a Long-lived Asset

Discount Factor = DF = PV of $1

$$DF = \frac{1}{(1+r)^t}$$

Discount Factors can be used to compute the present value of any cash flow.

2-33

PV of a Long-lived Asset

$$PV = DF \times C_1 = \frac{C_1}{1+r_1}$$

$$DF = \frac{1}{(1+r)}$$

Discount Factors can be used to compute the present value of any cash flow.

2-34

PV of a Long-lived Asset

$$PV = DF \times C_t = \frac{C_t}{(1+r)^t}$$

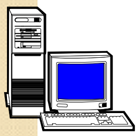
Replacing "1" with "t" allows the formula to be used for cash flows that exist at any point in time

2-35

PV of a Long-lived Asset

Example

You just bought a new computer for $3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?



2-36

PV of a Long-lived Asset

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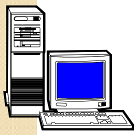


$$PV = \frac{3000}{(1.08)^2} = \$2{,}572.02$$

2-37

PV of a Long-lived Asset

PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$

2-38

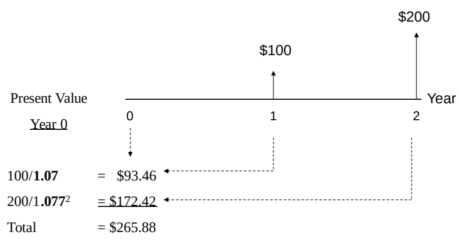
PV of a Long-lived Asset

PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{100}{(1+.07)^1} + \frac{200}{(1+077)^2} = 265.88$$

2-39

PV of a Long-lived Asset



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PV of a Long-lived Asset

Given two dollars, one received a year from now and the other two years from now, the value of each is commonly called the Discount Factor.

Assume r_1 = 20% and r_2 = 7%.

$$DF_1 = \frac{1.00}{(1+.20)^1} = .83$$

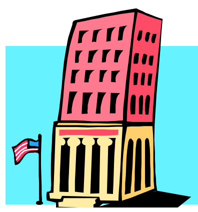
$$DF_2 = \frac{1.00}{(1+.07)^2} = .87$$

2-41

PV of a Long-lived Asset

Example

*Assume that the cash flows from the construction and sale of an office building is as follows. **Given a 5% required rate of return**, create a present value worksheet and show the net present value.*



Year 0	Year 1	Year 2
−170,000	−100,000	+320,000

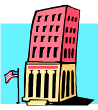
2-42

PV of a Long-lived Asset

Example - continued

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return, create a present value worksheet and show the net present value.

Period	Discount Factor	Cash Flow	Present Value
0	1.0	−170,000	−170,000
1	$\frac{1}{1.05} = .952$	−100,000	−95,238
2	$\frac{1}{(1.05)^2} = .907$	+320,000	+290,249
		NPV = Total =	$25,011



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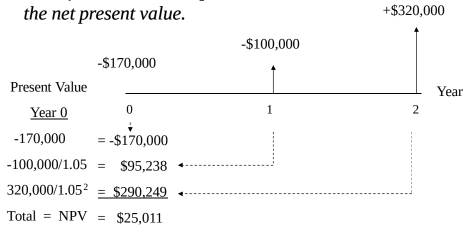
PV of a Long-lived Asset

Example - continued

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return, create a present value worksheet and show the net present value.



2-44

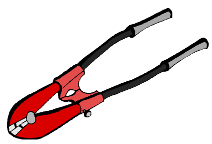
When Cash Flows Can Be Added Up

- General Rules
  - Present values can be added up;
  - (Future) cash flows at the same period can be added up;
  - But cash flows at different periods can not.

2-45

Short Cuts

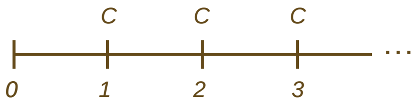
- Sometimes there are shortcuts that make it very easy to calculate the present value of an asset that pays off in different periods. These tools allow us to cut through the calculations quickly.



2-46

Short Cuts: Perpetuity

Perpetuity - Financial concept in which a cash flow is *theoretically received forever*.



2-47

Short Cuts: Perpetuity

Perpetuity - Financial concept in which a cash flow is *theoretically received forever*.

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{PV}$$

2-48

Short Cuts: Perpetuity

Perpetuity - Financial concept in which a cash flow is *theoretically received forever*.

$$\text{PV of Cash Flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

$$PV_0 = \frac{C_1}{r}$$

Short Cuts: Perpetuity

$$\text{PV of Cash Flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

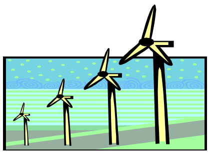
$$PV = \frac{C_1}{1+r} + \frac{C_1}{(1+r)^2} + \frac{C_1}{(1+r)^3} + \ldots + \frac{C_1}{(1+r)^n} + ..$$

$$= \lim_{n\to\infty}\left[\frac{C_1}{r} - \frac{C_1}{r(1+r)^n}\right] = \frac{C_1}{r}$$

Example of Perpetuity

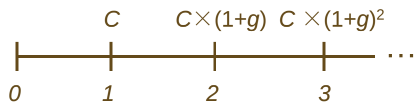
- Imagine you deposit US$100 Million in bank at the annual rate of 10%. At the end of every year, you withdraw the interest of US$10Million, and leave the principal in the bank. How long can you go on with this withdrawing? (Suppose the rate does not change)

  Forever!



Short Cuts: Growing Perpetuity

Growing Perpetuity - cash flow is theoretically growing constantly at g (g<r) and received forever.



Short Cuts: Growing Perpetuity

Growing Perpetuity - cash flow is theoretically growing constantly at g (g<r) and received forever.

$$\text{PV of Cash Flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

$$PV = \frac{C_1}{r-g}$$

Short Cuts: Growing Perpetuity

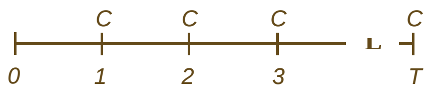
$$\text{PV of Cash Flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

$$PV = \frac{C_1}{1+r} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3} + \ldots + \frac{C_1(1+g)^{n-1}}{(1+r)^n} + ..$$

$$= \frac{C_1}{1+r}\left[\frac{1+g}{1+r} + \left(\frac{1+g}{1+r}\right)^2 + \left(\frac{1+g}{1+r}\right)^3 + \ldots\right] = \frac{C_1}{1+r}\lim_{n\to\infty}\left[\frac{1-\left(\frac{1+g}{1+r}\right)^n}{1-\frac{1+g}{1+r}}\right] = \frac{C_1}{r-g}$$

Short Cuts: Annuity

Annuity - An asset that pays a fixed sum each year for a specified number of years.



Short Cuts: Annuity

Annuity - An asset that pays a fixed sum each year for a specified number of years.

Asset	Year of Payment	Present Value
Perpetuity (first payment in year 1)	1 2…..t t + 1	$\frac{C}{r}$
Perpetuity (first payment in year t + 1)		$\left(\frac{C}{r}\right)\frac{1}{(1+r)^t}$
Annuity from year 1 to year t		$\left(\frac{C}{r}\right)-\left(\frac{C}{r}\right)\left(\frac{1}{(1+r)^t}\right)$

Short Cuts: Annuity

Annuity - An asset that pays a fixed sum each year for a specified number of years.

$$\text{PV of annuity} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

Short Cuts: Annuity Due

Annuity Due - An asset that pays a fixed sum each year for a specified number of years, **the first cash flow occurring in year 0**.

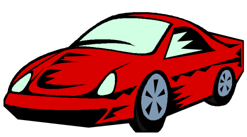
$$\text{PV of annuity due} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] \times (1+r)$$

$$= C + C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^{t-1}}\right]$$

Annuity Example

Example

Tiburon Autos offers an "easy payment" scheme on a new Toyota of $5,000 a year, paid at the end of each of the next 5 years, with no cash down. What is the car really costing you?



Annuity Example

Tiburon Autos offers an "easy payment" scheme on a new Toyota of $5,000 a year, paid at the end of each of the next 5 years, with no cash down. What is the car really costing you?

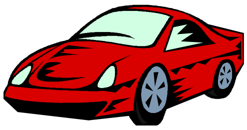
$$\text{Cost} = 5{,}000 \times \left[\frac{1}{.07} - \frac{1}{.07(1+.07)^5}\right]$$

$$= 5{,}000 \times 4.100 = \$20{,}501$$

Annuity Example

Example

You agree to lease a car for 4 years at $300 ***per month****. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?*



Annuity Example

Example - continued

You agree to lease a car for 4 years at $300 per month. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?

$$\text{Lease Cost} = 300 \times \left[\frac{1}{.005} - \frac{1}{.005(1+.005)^{48}} \right]$$

$$Cost = \$12{,}774.10$$

Annuity Example: Amortized Loans

本金P:	100,000	年利率：	10.00%	还款年限N：	10
年份	本期期初贷款余额	本期还款额	本期偿还利息	本期偿还本金	本期期末余额
1	100,000	16,275	10,000	6,275	93,725
2	93,725	16,275	9,373	6,902	86,823
3	86,823	16,275	8,682	7,592	79,231
4	79,231	16,275	7,923	8,351	70,880
5	70,880	16,275	7,088	9,187	61,693
6	61,693	16,275	6,169	10,105	51,588
7	51,588	16,275	5,159	11,116	40,472
8	40,472	16,275	4,047	12,227	28,245
9	28,245	16,275	2,825	13,450	14,795
10	14,795	16,275	1,480	14,795	0

Annuity Example: Amortized Loans

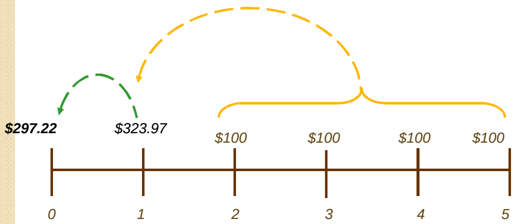
年利率10%，第6年起年利率下跌至8%					
年份	本期期初贷款余额	本期还款额	本期偿还利息	本期偿还本金	本期期末余额
1	100,000	16,275	10,000	6,275	93,725
2	93,725	16,275	9,373	6,902	86,823
3	86,823	16,275	8,682	7,592	79,231
4	79,231	16,275	7,923	8,351	70,880
5	70,880	16,275	7,088	9,187	61,693
6	61,693	15,451	4,935	10,516	51,177
7	51,177	15,451	4,094	11,357	39,820
8	39,820	15,451	3,186	12,266	27,554
9	27,554	15,451	2,204	13,247	14,307
10	14,307	15,451	1,145	14,307	0

Annuity Example: Amortized Loans

年利率10%，第6年起年利率上升至12%					
年份	期期初贷款余额	本期还款额	本期偿还利息	本期偿还本金	本期期末余额
1	100,000	16,275	10,000	6,275	93,725
2	93,725	16,275	9,373	6,902	86,823
3	86,823	16,275	8,682	7,592	79,231
4	79,231	16,275	7,923	8,351	70,880
5	70,880	16,275	7,088	9,187	61,693
6	61,693	17,114	7,403	9,711	51,982
7	51,982	17,114	6,238	10,876	41,106
8	41,106	17,114	4,933	12,182	28,924
9	28,924	17,114	3,471	13,643	15,281
10	15,281	17,114	1,834	15,281	0

One More Annuity Example

What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?



One More Annuity Example

What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$327.97$$

$$PV_0 = \frac{\$327.97}{1.09} = \$297.22$$

2-67

Compound Interest vs. Simple Interest

Example of Simple Interest

Annual interest rate of time deposit in Bank of China:

1-year	3.0%
2-years	3.75%
3-years	4.25%
5-years	4.75%

2-68

Compound Interest vs. Simple Interest

- Compound interest
  - Interest of this year is accrued to the principal on which interest is calculated next year.
  - $FV = C(1+r)^n$
- Simple interest
  - Interest of this year is not accrued to the principal.
  - $FV = C(1+n \times r)$

2-69

Compound Interest

Suppose that Jay Ritter invested in the initial public offering of the Modigliani company. Modigliani pays a current dividend of $1.10, which is expected to grow at 42-percent per year for the next five years.

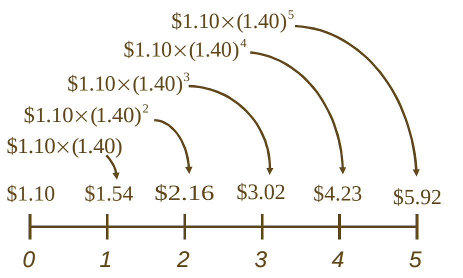
What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

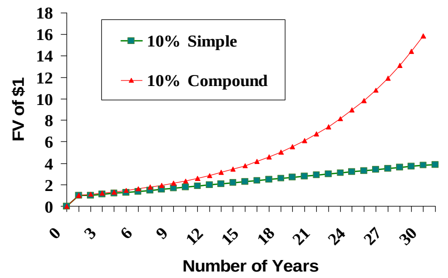
2-70

Compound Interest



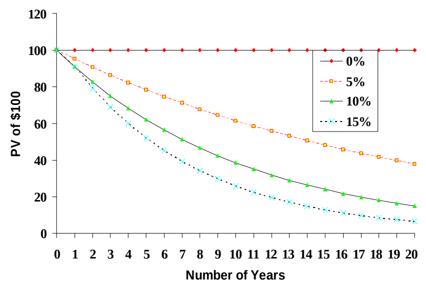
2-71

Compound Interest



2-72

Compound Interest



2-73

Simple Interest

– Simple interest when time period is less than one year

Annual interest rate of time deposit in Bank of China:

3-month	2.6%
6-month	2.8%
12-month	3.0%

2-74

EAR and APR

– APR (Annual Percentage Rate)
  - Stated interest rate in financial institutions
  - What investors really earn depends on the frequency of interest payments, i.e. , semi-annually, quarterly, monthly or continuously.

– EAR (Effective Annual Rate)
  - Rate that investors actually earn their interests.
  - EAR differs under the same APR if interest payment frequency differs.

2-75

EAR and APR

– From APR to EAR

$$EAR = (1 + \frac{APR}{n})^n - 1$$

– From EAR to APR

$$APR = \left[(1 + EAR)^{1/n}\right] * n - 1$$

2-76

Compound Interest

Periods per year	Interest per period	APR	Value after one year	Annually compounded interest rate
1	6%	6%	1.06	6.000%
2	3	6	1.03^2 = 1.0609	6.090
4	1.5	6	1.015^4 = 1.06136	6.136
12	.5	6	1.005^{12} = 1.06168	6.168
52	.1154	6	1.001154^{52} = 1.06180	6.180
365	.0164	6	1.000164^{365} = 1.06183	6.183

2-77

Compound Interest

Example

Suppose you are offered an automobile loan at an APR of 6% per year. What does that mean, and what is the true rate of interest, given ***monthly*** *payments?*

2-78

Compound Interest

Example - continued

*Suppose you are offered an automobile loan at an APR of 6% per year. What does that mean, and what is the true rate of interest, given monthly payments? **Assume $10,000 loan amount.***

$$\text{Loan Pmt} = 10{,}000 \times (1.005)^{12}$$
$$= 10{,}616.78$$
$$EAR = 6.1678\%$$

2-79

Continuous Compounding

At a given annual rate (APR), if cash flow comes in evenly, what's the EAR?

$$EAR = (1+\frac{APR}{n})^n - 1$$

As n approaches infinity,

$$EAR = (1+\frac{APR}{n})^n - 1$$
$$= \lim_{n \to \infty}(1+\frac{APR}{n})^n - 1$$
$$= e^{APR} - 1$$

2-80

Continuous Compounding

Example

Your are investing $1000 on a project which is continuously compounded at an annual rate of 5%. What's the EAR and FV of cash flow in the end of year 1 and year 2?

$$EAR = e^{APR} - 1 = e^{0.05} - 1 = 5.127\%$$
$$FV_1 = 1000*(1+5.127\%) = 1{,}051.27$$
$$FV_2 = 1000*(1+5.127\%)^2 = 1{,}105.17$$
$$FV_n = 1000*e^{0.05n}$$

2-81

Continuous Compounding

Example

1) *What's the present value of a perpetuity ($100 in each year end) at an annually compounded rate of 18.5%?*

$$PV = 100/0.185 = \$540.54$$

2) *What's the PV of a perpetuity for which each year $100 comes in but continuously and evenly at an annually compounded rate of 18.5%?*

$$EAR = e^{APR} - 1, APR = \ln(EAR+1)$$
$$APR = \ln(1+18.5\%) = 16.97\%$$
$$PV = 100/16.97\% = \$588.24$$

2-82

Continuous Compounding

Example

What's the PV of a donation that $1 billion comes in evenly in each year for 20 years? The annually compounded rate is 10%.

$$EAR = e^{APR} - 1, APR = \ln(EAR+1)$$
$$APR = \ln(1+10\%) = 9.53\%$$
$$\text{PV of Perpetuity 1 at year 0} = 1/9.53\% = 10.493$$
$$\text{PV of Perpetuity 2 at year 20} = 1/9.53\% = 10.493$$
$$\text{PV of Perpetuity 2 at year 0} = 10.493/(1+10\%)^{20} = 1.660$$
$$\text{PV of Annuity at year 0} = 10.493 - 1.660 = 8.933$$

2-83