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Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio

By HAIM LEVY*

The pioneering work of Harry Markowitz (1952, 1959) and James Tobin in portfolio theory has led to the development of a theory of the pricing of capital assets under uncertainty. This theory, well-known in the literature as the capital asset pricing model (*CAPM*), was developed independently by William Sharpe, John Lintner (1965a), and Jack Treynor. Two basic related properties implied by the *CAPM* are: (a) that all investors hold in their portfolio all the risky securities available in the market, and (b) that investors hold the risky assets in the same proportions, as these assets are available in the market, independent of the investors' preference.[1] This latter property of the *CAPM* makes it possible to draw many conclusions regarding the equilibrium risk-return relationship of risky assets.

Properties (a) and (b) contradict the market experience as established in all empirical research. First, investors differ in their investment strategy and do not necessarily adhere to the same risky portfolio. Second, the typical investor usually does not hold many risky assets in his portfolio. Indeed, in a recent study, Marshall Blume, Jean Crockett, and Irwin Friend found that, in the tax year 1971, individuals held highly undiversified portfolios. The sample, which included 17,056 individual income tax forms, revealed that 34.1 percent held only one stock paying dividends, 50 percent listed no more than two, and only 10.7 percent listed more than ten. Though only firms paying cash dividends were included in this statistic, it is obvious that most individuals held a relatively small number of stocks in their portfolio. Another source of data which confirms these findings is the Federal Reserve Board's 1967 survey of the Financial Characteristics of Consumers. This survey covered all households whether or not they filed income tax forms. According to this survey, the average number of securities in the portfolio was 3.41.[2]

The fact that properties (a) and (b) do not conform to reality is not a sufficient cause for rejecting the theoretical results of the *CAPM*. One could also accept the *CAPM* results on positive grounds. If the theoretical model does indeed explain the price behavior of risky assets, one could argue that investors behave *as if* properties (a) and (b) were true, in spite of the fact that these properties obviously do not prevail in the market. Unfortunately, we can *not* justify the theoretical results of the *CAPM* on positive grounds.

To illustrate the latter difficulty, let us return in greater detail to the *CAPM*. According to the *CAPM*, the expected return on asset i, $E(R_i)$ is related to the expected return on the market portfolio $E(R_m)$ as follows:

$$(1) \qquad E(R_i) - r = [E(R_m) - r]\beta_i$$

where r is the risk-free interest rate, β_i is the risk index of the ith security (the "systematic risk") and is defined as $Cov(R_i, R_m)/Var(R_m)$, and R_m is the rate of return on a portfolio which consists of all available risky assets and is called the "market portfolio."

Although the *CAPM* is formulated in terms of *ex ante* parameters, it is common to employ *ex post* data rather than *ex ante* values in empirical studies. Thus, we first

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[1]Lintner (1969) extends the *CAPM* to the case of disagreement of investors with regards to expected parameters. I assume in this model that investors agree with regard to future parameters but the model presented in this paper can be easily extended to the case of disagreement.

[2]For more details of these findings and their analysis, see Blume and Friend (1975).

run a time-series regression,

$$(2) \qquad R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

and estimate the systematic risk $\hat{\beta}_i$ of each asset i (where R_{it} and R_{mt} are the rates of return of the ith asset and the market portfolio, respectively, in year t). In the second step, in order to examine the validity of the *CAPM*, we run a cross-section regression,

$$(1') \qquad \overline{R}_i - r = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{\beta}_i + u_i$$

where $\overline{R}_i$ is the average return on the ith risky asset, $\hat{\beta}_i$ is the estimate of the ith asset systematic risk, taken from the time-series regression, and u_i is a residual term. If the *CAPM* is valid one should obtain (see equation (1)) in equilibrium, $\hat{\gamma}_0 = 0$ and $\hat{\gamma}_1 = \overline{R}_m - r$, where $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are the regression coefficients estimated by (1′), and $\overline{R}_m$ is the average observed rate of return on the market portfolio (for example, average rate of return on Standard and Poor's index).

Unfortunately, in virtually all empirical research,[3] it emerges that $\hat{\gamma}_0$ is significantly positive and $\hat{\gamma}_1$ is much below $\overline{R}_m - r$. For rates of return of *individual* stocks the correlation coefficient of (1′) is very low if one employs monthly rates of return, and only 20–25 percent with annual rates of return.[4]

Finally, in virtually all empirical studies, formulation (3) increases the correlation coefficient,

$$(3) \qquad \overline{R}_i - r = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{\beta}_i + \hat{\gamma}_2 \hat{S}^2_{e_i}$$

where i stands for the ith security and $\hat{S}^2_{e_i}$ is the residual variance around the time-series regression (2), i.e., the variance of the residuals e_{it}. In this formulation the estimate γ_2 happens to be significantly positive, contrary to the expected results from the *CAPM* since, if the *CAPM* is correct, one should find that $\gamma_2 = 0$. Moreover, in most cases, the contribution of $\hat{S}^2_{e_i}$ to the coefficient of correlation is even more important than the contribution of the systematic risk, $\hat{\beta}_i$.

In this paper I try to narrow the gap between the theoretical model and the empirical findings by deriving a new version of the *CAPM* in which investors are assumed to hold in their portfolios some given number of securities. Obviously, investors' portfolios differ in the proportions of risky assets and even in the types of risky assets that they hold. This, of course, is consistent with investors' behavior as established in previous empirical research. I denote the modified model as *GCAPM* (general capital asset pricing model), since the *CAPM* emerges as a special case.

The derivation of the *GCAPM* under these conditions is given in Section II. In the third section I show that the modified model explains the discrepancy between the theoretical results of the *CAPM* and the empirical findings mentioned above. Some empirical results are presented which confirm that the systematic risk β_i plays no role in explaining price behavior, once the variance is taken into account, (Section IV). Concluding remarks are given in Section V.

I. Equilibrium in an Imperfect Market: The *GCAPM*

William Sharpe and Lintner (1965a) have shown that, if there is no constraint on the number of securities to be included in the investors' portfolio, all investors will hold some combination of m, the market portfolio of risky assets, and the riskless asset bearing interest rate r (see Figure 1).

Now, suppose that, as a result of transaction costs, indivisibility of investment, or even the cost of keeping track of the new financial development of all securities, the kth investor decides to invest only in n_k securities. Under this constraint he will have some interior efficient set (of risky assets), say, $A'B'$, and the investor will divide his portfolio between some risky portfolio k

[3] See Fisher Black, Michael Jensen, and Myron Scholes; George Douglas; Lintner (1965b); Merton Miller and Scholes.

[4] I emphasize that the low correlation is obtained when equation (1′) is regressed using individual stock. In order to minimize the measurement errors, it is common to use in (1′) portfolios rather than individual stocks. This portfolio technique increases the correlation coefficient dramatically. However, in spite of the possible errors, individual stocks should be used since the *CAPM* defines equilibrium prices of individual stocks.

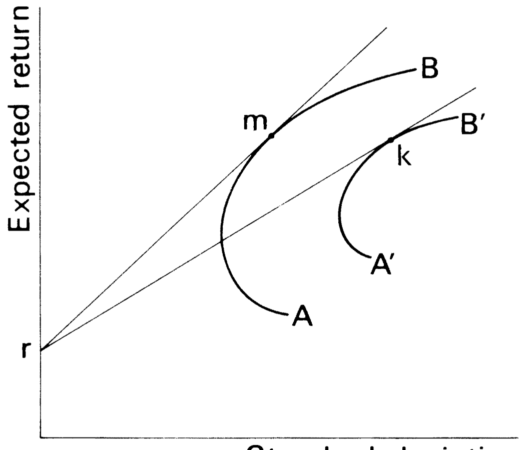

FIGURE 1

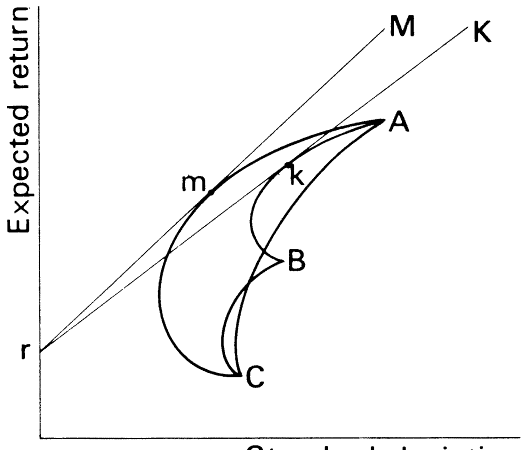

FIGURE 2

and the riskless asset. Obviously, the investor's welfare will decrease if no more than n_k securities may be included in the portfolio, since for a given expected return, he will be exposed to higher risk (see Figure 1).

In the specific case in which all investors hold the same number of risky assets n_k in equilibrium, all these interior efficient sets will be tangent to the same straight line. To illustrate, suppose that $n_k = 2$ for all k and that there are $n = 3$ risky assets available in the market. Figure 2 shows this possibility using A, B, and C to indicate the three risky securities.

Without any constraints, all investors hold portfolio m (i.e., the market portfolio), and all efficient portfolios lie on line rmM. Now suppose that all investors decide to include only two risky assets in their portfolio. Investors who hold securities A and B are faced with opportunity line rkK. If all investors decide to include two risky assets in their portfolio, this situation will not represent an equilibrium situation, since no one will purchase security C (see Figure 2). Hence the price of security C will decline, and its expected return will increase, until we get a new efficient curve between B and C (or C and A) which will be tangent to line rkK. In this case, however, the market may be cleared out. Note that not all two securities' efficient sets need to be tangent to the market line rkK. A sufficient condition for the market to be cleared out, in this example, is for two out of three efficient sets given in Figure 2 (i.e., AB, BC, AC) to be tangent to the line rkK. In other words, each of the three assets must be included in some two-asset portfolio which is tangent to the straight line.

In the more realistic case, which will be dealt with below, the kth investor has the constraint of investing in no more than n_k risky assets when n_k varies among investors

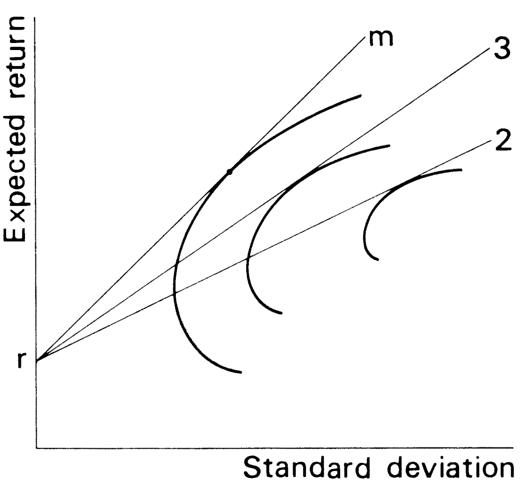

FIGURE 3

mainly as a function of the size of their wealth. In this case there are many interior efficient sets (see Figure 3), and the existence of many market lines does not contradict the possibility that the market may be in equilibrium.

In this case, *rm* is the opportunity line without any constraint on the number of securities in the portfolio; *r*2 is the market line with the constraint that no more than two securities are included in the portfolio; *r*3 is the line with the constraint of no more than three securities in the portfolio, etc. Obviously, the same security may be held in proportion of 20 percent of one portfolio, 5 percent of a second portfolio, etc. We derive below the equilibrium prices of risky assets for the general case in which the constraint on n_k varies from investor to investor. Again, a necessary condition for equilibrium in the stock market is that each security be included in at least one of the chosen unlevered portfolios from the above efficient sets.

Let us turn now to the derivation of the risk-return relationship under the constraint that not all risky assets are held in the investors' portfolio. We assume that there are K investors (or groups of investors), and the kth investor wealth is T_k dollars. Furthermore, assume that the kth investor invests only in n_k risky assets while there are in the market $n > n_k$ risky assets. Thus, the kth investor minimizes the portfolio's variance subject to the constraint that the number of securities in his portfolio cannot exceed n_k. More specifically, one has to differentiate partially with respect to x_{ik} and λ_k the Lagrangian function

$$L = \sum_{i=1}^{n_k} x_{ik}^2 \sigma_i^2 + 2 \sum_{\substack{i=1 \\ j>i}}^{n_k} x_{ik} x_{jk} \sigma_{ij} + 2\lambda_k \left[\mu_k - \sum_{i=1}^{n_k} x_{ik}\mu_i - \left(1 - \sum_{i=1}^{n_k} x_{ik}\right) r \right]$$

subject to the constraint that no more than n_k securities will be included in the optimal portfolio, where

σ_i^2 = the variance of the ith security return (per \$1 of investment)

σ_{ij} = the covariance between returns of securities i and j

μ_k = the portfolio expected return

x_{ik} = the proportion invested in the ith security by the kth investor

r = riskless interest rate

λ_k = Lagrange multiplier appropriate for the kth investor

Suppose that the investor selects n_k assets out of the n available assets to be included in his optimal portfolio. Then by differentiating the Lagrangian function we obtain the following $n_k = 1$ equations, which provide the optimal diversification strategy among the n_k risky assets

$$(4) \quad x_{1k}\sigma_1^2 + \sum_{j=2}^{n_k} x_{jk}\sigma_{1j} = \lambda_k(\mu_1 - r)$$

$$x_{2k}\sigma_2^2 + \sum_{\substack{j=1 \\ j\neq 2}}^{n_k} x_{jk}\sigma_{2j} = \lambda_k(\mu_2 - r)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$x_{nk}\sigma_{n_k}^2 + \sum_{\substack{j=1 \\ j\neq n_k}}^{n_k} x_{jk}\sigma_{n_k j} = \lambda_k(\mu_{n_k} - r)$$

$$\mu_k = \sum_{i=1}^{n_k} x_{ik}\mu_i + \left(1 - \sum_{i=1}^{n_k} x_{ik}\, r\right)$$

Thus, the optimal investment strategy of the kth investor is given by the vector x_{1k}, $x_{2k}, \ldots, x_{nk}$ which solves the above equations. We multiply the first equation by x_{1k}, the second equation by x_{2k}, etc., and then sum up the first n_k equations to obtain

$$\sigma_k^2 = \lambda_k\left(\sum_{i=1}^{n_k} x_{ik}\mu_i - \sum_{i=1}^{n_k} x_{ik} r\right) = \lambda_k\left[\sum_{i=1}^{n_k} x_{ik}\mu_i + \left(1 - \sum_{i=1}^{n_k} x_{ik}\right) r - r\right] = \lambda_k(\mu_k - r)$$

Hence,

$$(5) \qquad \frac{1}{\lambda_k} = \frac{\mu_k - r}{\sigma_k^2}$$

where μ_k and σ_k^2 are the expected return and variance of the kth investor's optimal portfolio. Using (4) and (5) the kth investor will be in equilibrium if and only if

(6) $$\mu_i = r + \frac{\mu_k - r}{\sigma_k^2} cov(R_i R_k)$$

where R_i and R_k are the rates of return on the ith security and on the portfolio chosen by the kth investor. Equation (6) can be rewritten as

(6′) $$\mu_i = r + (\mu_k - r)\beta_{ki}$$

where β_{ki} is the systematic risk of the ith asset in the kth investor's optimal portfolio R_k and is defined as $\beta_{ki} = Cov(R_i, R_k)/\sigma_k^2$.

It is important to note that the equilibrium relationship given in equations (6) and (6′) is independent of the borrowing or lending policy of the kth investor.[5] Thus, without loss of generality, we can assume that

$$\sum_{i=1}^{n_k} x_{ik} = 1$$

and this will not affect the solution of the optimal investment. In the rest of the paper we assume that μ_k and σ_k^2 are the parameters of the optimal *unlevered* portfolio chosen by the kth investor. This is tantamount to the assumption that

$$\sum_{i=1}^{n_k} x_{ik} = 1$$

In order to examine the impact on equilibrium price determination, of not holding all assets in the portfolio we need to use some algebra. Since $R_k = \Sigma_{j=1}^{n_k} x_{jk} R_j$, equation (6) can be rewritten as

(7) $$\frac{v_{i1} - v_{i0}}{v_{i0}} = r + \frac{(\mu_k - r)}{\sigma_k^2} \cdot \left[x_{ik}\sigma_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n_k} x_{jk}\sigma_{ij} \right]$$

when v_{i1} and v_{i0} stand for the expected market value of firm i at the end of the period, and for the equilibrium present value, respectively. Hence,

(8) $$v_{i1} - v_{i0}(1 + r) = \frac{(\mu_k - r)}{\sigma_k^2} \cdot \left[v_{i0}x_{ik}\sigma_i^2 + v_{i0} \sum_{\substack{j=1 \\ j \neq i}}^{n_k} x_{jk}\sigma_{ij} \right]$$

Let us denote

σ_i^{*2} = the expected variance of the return on *one share* of the ith firm at the end of the investment period

σ_{ij}^* = the expected covariance of the return of *a share* of firm i and *a share* of firm j

N_i = the number of outstanding shares of firm i

P_{i0} = the equilibrium price of a share of firm i

P_{i1} = the expected price of a share of firm i at the end of the period

[5] To be more specific suppose that an investor who owns T_k dollars decides to borrow or lend $(\Sigma_i^{n_k} x_{ik} - 1)$ per each dollar that he owns. Then, if, R_k is the return (per one dollar) on his optimal portfolio solely from risky assets, the return on his selected portfolio (including the borrowing or lending) denoted by R_k^* will be

$$R_k^* = \left(\sum_{i=1}^{n_k} x_{ik}\right) R_k - \left(\sum_{i=1}^{n_k} x_{ik} - 1\right) r$$

and hence

$$\mu_k^* = \left(\sum_{i=1}^{n_k} x_{ik}\right) \mu_k - \left(\sum_{i=1}^{n_k} x_{ik}\right) r + r,$$

$$\sigma_k^{2*} = \left[\sum_{i=1}^{n_k} x_{ik}\right]^2 \sigma_k^2$$

and, $$cov^*(R_i R_k) = \left[\left(\sum_{i=1}^{n_k} x_{ik}\right)\right] cov(R_i R_k)$$

Rewriting (6) in terms of R_k^* we obtain

$$\mu_i = r + \frac{\sigma_k^* - r}{\sigma_k^{2*}} cov^*(R_i R_k)$$

or

$$\mu_i = r + \frac{\sum_{i=1}^{n_k} x_{ik}(\mu_k - r) + r - r}{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2 \sigma_k^2} \cdot \left(\sum_{i=1}^{n_k} x_{ik}\right) cov(R_i R_k)$$

and finally

$$\mu_i = r + \frac{\mu_k - r}{\sigma_k^2} cov(R_i R_k)$$

where μ_k and σ_k^2 are the expected return and variance of the optimal portfolio of the kth investor when he neither borrows nor lends money.

Thus,

$$\sigma_i^{*2} = \sigma_i^2 P_{i0}^2, \ \sigma_{ij}^* = \sigma_{ij} P_{i0} P_{j0}$$

and equation (8) can be rewritten in terms of market price per share,

$$(9) \quad N_i P_{i1} - N_i P_{i0}(1+r) = \frac{(\mu_k - r)}{\sigma_k^2} \cdot \left[N_i P_{i0} x_{ik} \sigma_i^2 + N_i P_{i0} \sum_{\substack{j=1 \\ j \neq i}}^{n_k} x_{ik} \sigma_{ij} \right]$$

Dividing by N_i yields

$$(10) \quad P_{i1} - P_{i0}(1+r) = \frac{(\mu_k - r)}{\sigma_k^2} \cdot \left[P_{i0} x_{ik} \sigma_i^2 + P_{i0} \sum_{\substack{j=1 \\ j \neq i}}^{n_k} x_{jk} \sigma_{ij} \right]$$

Now recall that the proportions invested by the kth investor x_{ik} and x_{jk} in the ith and jth assets, respectively, have been given by $x_{ik} = N_{ik} P_{i0} / T_k$, and $x_{jk} = N_{jk} P_{j0} / T_k$, where N_{ik} and N_{jk} stand for the number of shares of firm i and j in the kth investor's portfolio, and T_k is the total amount of dollars invested by him in risky assets. Thus, the substitution of x_{ik} and x_{jk} in equation (10) yields,

$$(11) \quad P_{i1} - P_{i0}(1+r) = \frac{(\mu_k - r)}{T_k \sigma_k^2} \cdot \left[P_{i0}^2 N_{ik} \sigma_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n_k} N_{jk} P_{i0} P_{j0} \sigma_{ij} \right]$$

By substituting for σ_i^* and σ_{ij}^* (variance and covariances in terms of one share rather than one dollar), and multiplying and dividing by T_k, we obtain,

$$(12) \quad P_{i1} - P_{i0}(1+r) = \frac{T_k(\mu_k - r)}{T_k^2 \sigma_k^2} \cdot \left[N_{ik} \sigma_i^{*2} + \sum_{\substack{j=1 \\ j \neq 1}}^{n_k} N_{jk} \sigma_{ij}^* \right]$$

Equation (12) should apply to the kth investor, but *only for securities which are included in his portfolio.*

Now, in order to have price equilibrium in terms of the aggregate demand for the ith stock we use the same technique as employed by Lintner (1965a) with only one distinction: Lintner was allowed to sum up his equations for *all* investors. In our model, we are allowed to sum them up only for investors k who hold the security under consideration in their portfolios, since equation (4) (from which we derive equation (12)) includes the ith security only for investors k who hold it. After multiplying equation (12) by $T_k^2 \sigma_k^2$ and summing up *only for investors k who hold security i*, we obtain

$$(13) \quad [P_{i1} - P_{i0}(1+r)] \sum_k T_k^2 \sigma_k^2 = \sum_k T_k(\mu_k - r) \left[N_{ik} \sigma_i^{*2} + \sum_{\substack{j=1 \\ j \neq i}}^{n_k} N_{jk} \sigma_{ij}^* \right]$$

The equilibrium price of share i, P_{i0}, is given by

$$(14) \quad (1+r) P_{i0} = P_{i1} - \left[\sum_k \left(T_k(\mu_k - r) \cdot \left[N_{ik} \sigma_i^{*2} + \sum_{\substack{j=1 \\ j \neq i}}^{n_k} N_{jk} \sigma_{ij}^* \right] \right) \right] \div \sum_k T_k^2 \sigma_k^2$$

In order to derive a more comparable form for the equilibrium price as implied by the *CAPM* we multiply and divide by $[\Sigma_k T_k \cdot (\mu_k - r)]$ to obtain

$$(15) \quad (1+r) P_{i0} = P_{i1} - \frac{\left[\sum_k T_k(\mu_k - r) \right]}{\sum_k T_k^2 \sigma_k^2} \cdot \frac{\sum_k \left(T_k(\mu_k - r) \left[N_{ik} \sigma_1^{*2} + \sum_{\substack{j=1 \\ j \neq i}}^{n_k} N_{jk} \sigma_{ij}^* \right] \right)}{\left[\sum_k T_k(\mu_k - r) \right]}$$

where P_{i0} is the equilibrium price of stock i as suggested by this model. The price of risk is given by $[\Sigma\, T_k(\mu_k - r)] / \Sigma\, T_k^2 \sigma_k^2$ and is relevant only for investors who hold security i. Obviously, investors who do not hold security i are faced by a different price of risk. Moreover, the same investor may face two (or more) different prices of risk, one appropriate for security i and one for security j. This may occur since the group of investors who hold security i is not necessarily identical to the group of investors who hold security j. Thus, the term $\Sigma T_k \cdot (\mu_k - r)/\Sigma T_k^2\sigma_k^2$ (price of risk) is a function of the security under consideration, and is relevant only to investors who decide to hold this security in their portfolio.

The equilibrium formula given by equation (15) has very important implications for the empirical findings of the *CAPM*. To demonstrate, assume that all investors who hold security i hold also security j (namely, only two risky assets) and these investors purchase all the available securities of these two firms. For simplicity only, and without loss of generality, assume that $\mu_k - r$ is a constant (say $= A$) and that $T_k/\Sigma T_k = \alpha$ for all these investors. Thus (15) reduces to

$$(15') \quad (1 + r)P_{i0} = P_{i1} - \frac{\sum_k T_k(\mu_k - r)}{\sum_k T_k^2\sigma_k^2} \cdot \frac{\left[\sum_k T_k N_{ik}\sigma_i^{*2} + \sum_k T_k N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k}$$

On the basis of the above simplifying assumptions, we obtain from (15′)

$$(1 + r)P_{i0} = P_{i1} - \frac{\sum_k T_k(\mu_k - r)\alpha \sum_k\left[N_{ik}\sigma_i^{*2} + \sum_k N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k^2\sigma_k^2}$$

or

$$(15'') \quad (1 + r)P_{i0} = P_{i1} - \frac{\sum_k T_k(\mu_k - r)\alpha[N_i\sigma_i^{*2} + N_j\sigma_{ij}^*]}{\sum_k T_k^2\sigma_k^2}$$

since $\Sigma_k N_{ik} = N_i$, $\Sigma_k N_{jk} = N_j$, where N_i and N_j are the number of outstanding shares of i and j, respectively.

It can readily be seen from (15″) that the equilibrium price P_{i0} is a function of the ith security variance and of only *one* covariance, that is, its covariance with security j. Obviously, in such a case, we would expect that the ith security variance will play a central role in its equilibrium price determination, quite contrary to the result of the traditional *CAPM*. On the other hand, the traditional β_i (see equation (1)) has little to to with the determination P_{i0}, since β_i includes all the covariances (see equation (7)) while in the above example we have only one covariance. Note that few assumptions have been made in order to simplify the analysis. However, even when investors hold stocks of three or four companies, we still obtain the same result; the ith security variance is much more important in price determination than one would expect from the analysis of traditional *CAPM*. Empirical support to this theoretical result is given in Section IV.

For the specific case in which *all investors hold security* i, we sum up equation (12) for *all* investors k. Hence $\Sigma_k T_k(\mu_k - r)$ is the total aggregate excess dollar return of all investors' portfolios, which is equal to $T_0(\mu_m - r)$, where μ_m is the expected return on the market portfolio and $T_0 = \Sigma_k T_k$. However, $\Sigma_k T_k^2\sigma_k^2$ is not necessarily equal to $T_0^2\sigma_m^2$, and hence one does not have, even in the above specific case, the interpretation of the aggregate risk in the market as obtained when a perfect market is assumed. However, equation (15) can be written as

$$(1 + r)P_{i0} = P_{i1} - \frac{\left[\sum_k T_k(\mu_k - r)\right]}{T_0^2\sigma_m^2} \frac{T_0^2\sigma_m^2}{\sum_k T_k^2\sigma_k^2} \cdot \frac{\sum_k T_k(\mu_k - r)\left[N_{ik}\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq i}}^{n_k} N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k(\mu_k - r)}$$

If all investors hold security i, then $\Sigma_k T_k \cdot (\mu_k - r) = T_0(\mu_m - r)$ and the second term on the right-hand side is the market price of risk γ, when the *CAPM* is derived without constraint on the number of securities in the portfolio (see Lintner 1965a, p. 600).

Hence,

$$(16)\quad (1+r)P_{i0} = P_{i1} - \gamma\left(\frac{T_0^2\sigma_m^2}{\sum T_k^2\sigma_k^2}\right) \cdot \frac{\sum_k T_k(\mu_k - r)\left[N_{ik}\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq 1}}^{n_k} N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k(\mu_k - r)}$$

or

$$(17)\quad (1+r)P_{i0} = P_{i1} - \gamma_1 \frac{\sum_k T_k(\mu_k - r)\left[N_{ik}\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq i}}^{n_k} N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k(\mu_k - r)}$$

where $$\gamma_1 = \frac{\gamma T_0^2\sigma_m^2}{\sum_k T_k^2\sigma_k^2}$$

Equation (17) is very similar to the classic relationship of the *CAPM* (see equation (20′)). The only two differences are: (a) now the securities' risk is given as the weighted average of the risks of each investor when the weights are $T_k(\mu_k - r)$, so that, the larger the investor's wealth (T_k), the greater his impact on price determination, and (b) the market price of risk γ_1 is defined somewhat differently from the well-known γ, as defined by Lintner (1965a). Thus, the classic *CAPM* may be the approximate equilibrium model for stocks of firms which are held by many investors (for example, AT&T), but not for small firms whose stocks are held by a relatively small group of investors.

If we relax the constraint that the kth investor holds only n_k securities, then each investor holds the market portfolio and hence[6] $\mu_k - r = \mu_m - r$, and $\sigma_m^2 = \sigma_k^2$, where μ_m and σ_m^2 are the expected rate of return and variance of the market portfolio, respectively.

[6]Recall that without loss of generality we deal only with the optimal unlevered portfolio. The basic equilibrium equation (equation (6)) and hence all the other results derived from it are unchanged no matter if we deal with the levered or the unlevered portfolio. See fn. 5.

On the basis of these assumptions we obtain the classic *CAPM* formula as a special case of the *GCAPM* suggested in this paper. In this case, equation (16) reduces to

$$(18)\quad (1+r)p_{i0} = p_{i1} - \gamma\frac{T_0^2}{\sum_k T_k^2} \cdot \frac{\sum_k T_k\left[N_{ik}\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq 1}}^{n_k} N_{jk}\sigma_{ij}^*\right]}{\sum_k T_k}$$

But since the relaxation of the imperfection induces all investors to have the same investment strategy in risky assets, (see Sharpe and Lintner, 1965a) all of them hold all the risky assets $n_k = n$ and, also $N_{ik}/N_i = T_k/T_0$ and hence $N_{ik} = N_iT_k/T_0$ and $N_{jk} = N_jT_k/T_0$. By substituting the last reults in equation (18) we derive

$$(19)\quad (1+r)p_{i0} = p_{i1} - \gamma\frac{T_0^2}{\sum_k T_k^2}\,\frac{\sum_k T_k}{\sum_k T_k} \cdot \left[\frac{T_kN_i}{T_0}\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{T_kN_j}{T_0}\sigma_{ij}^*\right]$$

or

$$(20)\quad (1+r)p_{i0} = p_{i1} - \gamma\frac{T_0^2}{T_0\sum_k T_k^2} \cdot \frac{\sum_k T_k^2\left[N_i\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq i}}^{n} N_j\sigma_{ij}^*\right]}{\sum_k T_k}$$

Since $\Sigma_k T_k = T_0$, equation (20) reduces to the well-known equilibrium equation of the traditional *CAPM* (see Lintner 1965a, p. 600),

$$(20')\quad (1+r)p_{i0} = p_{i1} - \left[N_i\sigma_i^{*2} + \sum_{\substack{j=1\\ j\neq i}}^{n} N_j\sigma_{ij}^*\right]$$

Finally, I would like to emphasize the basic difference between equations (15) and (17). Equation (15), which I advocate, represents the most general form, and hence only σ_{ij} of securities included in the kth investors' portfolios, are taken into account. However, if we assume unrealistically that security i is included in *all* investors' portfolios (equation (17)) then for an equilibrium price determination we must take into account the covariances σ_{ij} of all securities available in the market since we sum up in equation (17) for all k.

II. The Implication for the Empirical Findings

Recent empirical evidence indicates that the traditional *CAPM* does not explain the empirical data as well as might be expected. Douglas, using annual and quarterly data, shows that there is a significant relationship between the mean rate of return of a stock and its standard deviation—a fact which contradicts the *CAPM*. Lintner (1965b) regresses annual rates of return of 301 stocks over the period 1954–63. He estimates the systematic risk from time-series and then regresses the mean rate of return on the systematic risk and on the estimate of the residual variance (see equation (3)). His results, too, indicate that the theoretical model does not provide a satisfactory description of price behavior. Using annual data, Merton Miller and Myron Scholes confirm the basic results of Lintner and suggest possible explanations for the deviation between the model and the empirical evidence. Black, Jensen, and Scholes using monthly data also show that the model does not provide a satisfactory description of price behavior in the stock market.

In recent papers David Levhari and I have investigated the effect of the assumed investment horizon on the estimates of the systematic risk as well as on the other results implied by the *CAPM*. We have found that the investment horizon plays a crucial role in any econometric research and, particularly, in empirical work which tests the *CAPM*. However, in analyzing horizons ranging from one to twenty-four months, we have also found that the coefficient of the residual variance (γ_2 in equation (3)) remains significantly positive. In most cases, too, the residual variance explains price behavior even better than the estimates of the systematic risk (i.e., γ_1 in equation (3)). I demonstrate below that the fact that investors hold portfolios with only a few risky assets, rather than the market portfolio, provides a possible explanation for the three discrepancies between the theoretical model and the empirical findings obtained by various researchers.

Suppose that an investor holds a portfolio k whose random return is R_k, while the random return on the market portfolio is R_m. The expected return on R_k can be smaller or greater than the expected return of R_m. However, since R_k includes only a few securities while R_m consists of all securities available in the market, one would expect that the variance of R_m would be smaller than the variance of most selected portfolios, k. The relationship between R_k and R_m can be described as follows:

$$R_m = R_k + \psi \tag{21}$$

(alternatively, one can define this relationship in the form $R_m = a + bR_k + \psi$, see Miller and Scholes), where ψ is an error term. Let us now analyze the impact of the error in the variables given in (21), on empirical evidence related to the *CAPM*.

In the empirical research, the time-series regression is formulated as follows:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_t \tag{22}$$

where $\hat{\beta}_i$ derived from (22) is the estimate of the ith security systematic risk. Since the investors hold portfolio R_k rather than R_m, the true relationship is given by

$$R_{it} = \alpha^*_{ik} + \beta^*_{ik} R_{kt} + u_t \tag{23}$$

where β^*_{ik} is the kth investor's true systematic risk. We shall see that using (22) rather than (23) causes a certain bias in the estimate of the systematic risk. The estimate of $\hat{\beta}_i$ is given by

$$\hat{\beta}_i = \frac{cov\,(R_i, R_m)}{var\,(R_m)} = \frac{cov\,(R_i, R_k + \psi)}{var\,(R_k + \psi)} = \frac{cov\,(R_i, R_k) + cov\,(R_i, \psi)}{\sigma^2_k + \sigma^2_\psi + 2\,cov\,(R_k, \psi)} \tag{24}$$

If we divide by σ^2_k and assume that the errors are distributed independently of the true values (R_i and R_k), then the last term in the numerator, as well as the last term in the denominator, will tend to zero as the sample size increases indefinitely. Thus, $\hat{\beta}_i = cov\,(R_i, R_k)/(1 + \sigma_\psi^2/\sigma_k^2)\sigma_k^2$. But since $cov\,(R_i, R_k)/\sigma_k^2 = \beta_{ik}^*$ we finally obtain

$$\hat{\beta}_i = \frac{\beta_{ik}^*}{1 + \sigma_\psi^2/\sigma_k^2} \tag{25}$$

Hence[7]

$$\hat{\beta}_i < \beta_{ik}^* \tag{26}$$

for all investors k, and hence $\hat{\beta}_i < \beta_i^*$ where β_i^* is a weighted average of β_{ik}^*. (I shall define this weighted average later on; see equation (35).)

Let us now investigate the impact of this bias in measuring the systematic risk, on the cross-section regression which is essential to an examination of the validity of the *CAPM* (see equation (1′)). Since $\hat{\beta}_i$ is biased, one can write $\hat{\beta}_i$ as follows,

$$\hat{\beta}_i = \beta_i^* + \phi_i \tag{27}$$

where ϕ_ι is an error term. Most empirical works carry out the cross-section regression in the following manner (see equation (1′)):

$$\overline{R}_i - r = \gamma_0 + \gamma_1\hat{\beta}_i + e_i \tag{28}$$

while the true relationship is given by

$$\overline{R}_i - r = \gamma_0^* + \gamma_1^*\beta_i^* + e_i^* \tag{29}$$

where $\overline{R}_i$ is the average rate of return of the ith asset, r is the riskless interest rate, and $\hat{\beta}_i$ is the estimate of the systematic risk obtained from the time-series regression. Thus

$$\hat{\gamma}_1 = \frac{cov\,(\overline{R}_i, \hat{\beta}_i)}{\sigma^2(\hat{\beta}_i)} = \frac{cov(\overline{R}_i, \beta_i^* + \theta_i)}{\sigma^2(\beta_i^*) + \sigma^2(\theta_i) + 2cov(\beta_i^*, \theta_i)} = \frac{cov(\overline{R}_i, \beta_i^*) + cov\,(\overline{R}_i, \theta_i)}{\sigma^2(\beta_i^*) + \sigma^2(\theta_i) + 2cov(\beta_i^*, \theta_i)}$$

Dividing by $\sigma^2(\beta_i^*)$ and assuming that the error θ_i is uncorrelated with the values $\overline{R}_i$ and β_i^*, we obtain,

$$\hat{\gamma}_1 = \frac{cov(\overline{R}_i, \beta_i^*)/\sigma^2(\beta_i^*)}{1 + \sigma^2(\theta_i)/\sigma^2(\beta_i^*)}$$

or

$$\hat{\gamma}_1 = \frac{\hat{\gamma}_1^*}{1 + \sigma^2(\theta_i)/\sigma^2(\beta_i^*)}$$

and hence[8]

$$\hat{\gamma}_1 < \gamma_1^* \tag{30}$$

This may explain the result of most empirical studies where $\hat{\gamma}_1$ is below the value predicted by the *CAPM*.

It has also been found in all empirical research that $\hat{\gamma}_0 > 0$, while, according to the *CAPM*, γ_0 should equal zero. This bias may be explained as follows: from equation (28) the estimate of γ_0 is given by

$$\hat{\gamma}_0 = \overline{\overline{R}} - \hat{\gamma}_1\overline{\hat{\beta}}$$

where $\overline{\overline{R}}$ is the average of the variables $\overline{R}_i - r$, and $\overline{\hat{\beta}}$ is the average of the estimates of the systematic risks $\hat{\beta}_i$ of all risky assets. However, the true relationship should be (from equation (29))

$$\hat{\gamma}_0 = \overline{\overline{R}} - \hat{\gamma}_1^*\hat{\beta}^*$$

since according to the above assumptions $\hat{\gamma}_1 < \hat{\gamma}_1^*$ and $\hat{\beta}_i < \hat{\beta}_i^*$, also $\hat{\gamma}_1\overline{\hat{\beta}} < \hat{\gamma}_1^*\overline{\hat{\beta}}^*$, hence we obtain the result $\hat{\gamma}_0 > \hat{\gamma}_0^* = 0$.

Apparently, the most disturbing empirical result is that $\hat{\gamma}_2$ (see equation (3)) is significantly greater than zero. The latter result, however, can be explained by the model presented in this paper. According to the *CAPM*, investors diversify in many securities, and hence, the residual variance $\hat{S}_{e_i}^2$ should have no impact on the risk-return equilibrium relationship. The individual security's variance as well should have no impact on this relationship since the contribution of the individual risk is about $(1/n)\sigma^2(R_i)$ when n is the number of securities available in the market.[9] However, if

[7] In deriving (26) it is assumed that the errors are distributed independently of R_i and R_k. However, it is easy to verify that it is sufficient to require that u and Ψ are distributed independently and that the regression coefficient of R_k on Ψ_j is greater than -1.

[8] Equation (30) is valid even under less restrictive assumptions (see fn. 7).

[9] For simplicity's sake we assume that the investor diversify equally his resources among all securities. (See Miller and Scholes.)

one assumes that investors hold undiversified portfolios which contain stocks of three or four companies (i.e., $n_k = 3, 4$) and that the ith security is not included in all portfolios, then the variance (and hence the residual variance) should have a strong impact on the risk-return relationship. Although we have already analyzed the role of the variance in price determination (see equation (15)), we can find a more transparent example by looking once again at equation (6). Rewriting (6) we obtain

$$(31) \qquad \mu_i - r = \frac{\mu_k - r}{\sigma_k^2} Cov(R_i, R_k)$$

Assuming, once again, for the sake of simplicity only, that the typical investor who holds security i will diversify equally between three stocks, we obtain

$$\mu_i - r = \frac{\mu_k - r}{\sigma_k^2} \cdot \left[Cov\left(R_i, \frac{1}{3} R_i + \frac{1}{3} R_{i-1} + \frac{1}{3} R_{i+1} \right) \right]$$

where i, $i - 1$, and $i + 1$ stand for the three securities included in the portfolio. Thus

$$(32) \quad \mu_i - r = \frac{\mu_k - r}{\sigma_k^2} \left[\frac{1}{3} \sigma_{R_i}^2 + \frac{1}{3} Cov(R_i, R_{i-1}) + \frac{1}{3} Cov(R_i, R_{i+1}) \right]$$

It is obvious from (32) that variance plays a central role in explaining the risk-return relationship. Moreover, one would expect that the individual variance would have greater impact on price determination than the β_i (as defined in equation (1)) since β_i has very little to do with the stock's risk when the portfolios include only a small number of different securities. Indeed, Douglas found that the coefficient of the variance is more important than the coefficient of the β in most periods covered in his empirical research.

To design a precise empirical study to test the model suggested in this paper is not an easy task since equation (6) includes a factor β_{ki} which varies from investor to investor. One has first to find a solution to the optimization problem with the constraint on the number of securities n_k, and also to know the amount invested by each investor in the stock market. To illustrate the difficulties involved in such an empirical test, let us reexamine equation (6′). When we multiply equation (6′) by T_k and sum up only for investors k who *hold security i*, we obtain

$$(33) \quad \mu_i \sum_k T_k = r \sum_k T_k + \sum_k T_k(\mu_k - r)\beta_{ki}$$

or

$$(34) \quad \mu_i = r + \sum_k T_k(\mu_k - r)\beta_{ki} / \sum_k T_k$$

By defining β_i^* as the weighted average, $\beta_i^* = \Sigma_k T_k(\mu_k - r)\beta_{ki} / \Sigma_k T_k(\mu_k - r)$ and $\Sigma_k T_k(\mu_k - r)/\Sigma_k T_k = \gamma_{1i}^*$ we can rewrite (34) as

$$(35) \qquad \mu_i = r + \gamma_{1i}^* \beta_i^*$$

where γ_{1i}^* varies from one security to another.

Equation (35) can then be used in order to test empirically the risk-return relationship as suggested in this paper. However, I would like to mention a few characteristic results as well as difficulties in testing this equation empirically: (a) Since $\hat{\beta}_i < \beta_{ik}$ for all k, $\hat{\beta}_i < \beta_i^*$ is also true, since β_i^* is a weighted average of β_{ik}. (b) $\gamma_{1i} = \Sigma T_k(\mu_k - r)/\Sigma T_k$ when we sum up only for investors k who hold security i. Thus, γ_{1i}^* varies from security to security, and any cross-section regression will provide an estimate of some average of all these γ_{1i}^*. (c) In order to test the *CAPM* in the present framework, one has to estimate first β_i^*, that is, to have information, not only on the selected portfolio by each investor k, but also on the relative size of his investment, $T_k/\Sigma T_k$. (d) Finally, it is worth mentioning that if all investors hold security i, $\gamma_{1i} = \Sigma_k T_k(\mu_k - r)/\Sigma_k T_k$, when we sum up for *all* investors k. Hence $\gamma_{1i} = \mu_m - r$, since in this case $\Sigma_k \cdot T_k = T_0$, and $\Sigma_k T_k(\mu_k - r) = T_0(\mu_m - r)$, where μ_m is the expected rate of return on the market portfolio.

Designing such an empirical research is beyond the scope of this paper. However, if the present form of the *CAPM* is correct,

TABLE 1—SECOND-PASS REGRESSION WITH MONTHLY DATA

$\bar{R}_i = \gamma_0$	+	$\gamma_1\hat{\beta}_i$	+	$\gamma_2\hat{S}^2_{e_i}$	+	$\gamma_3\hat{\sigma}^2_i$	R^2
0.00894 (0.00096) $t = 9.3$		0.00196 (0.00094) $t = 2.1$					0.04
0.00985 (0.00057) $t = 17.3$						0.18369 (0.08956) $t = 2.0$	0.04
0.00999 (0.00053) $t = 19.0$				0.21916 (0.11129) $t = 2.0$			0.04
0.00914 (0.00099) $t = 9.2$		0.00117 (0.00136) $t = 0.86$				0.10404 (0.12865) ($t = 0.81$)	0.05
0.00899 (0.00096) $t = 9.3$		0.00136 (0.00110) $t = 1.2$		0.13736 (0.12909) $t = 1.1$			0.05

then, in spite of the fact that we do not have a perfect empirical procedure to test it, we expect the variance itself, σ_i^2, to provide a better explanation of price behavior than the traditional systematic risk, β_i.

III. The Empirical Findings

The monthly rates of return of a sample of 101 stocks traded on the New York Stock Exchange (NYSE) were calculated for the period 1948–68, that is, for each security there are 240 observations. Thus, if R_{i1}, $R_{i2}, \ldots, R_{i240}$ were the monthly rates of return, on the ith security, one can calculate the bimonthly rates of return, $R^*_{i1}, R^*_{i2}, \ldots, R^*_{i120}$ by substituting $(1 + R_{i1})\ (1 + R_{i2}) = 1 + R^*_{i1}$, $(1 + R_{i3})(1 + R_{i4}) = 1 + R^*_{i2}$ etc., where R^*_i $(i = 1, 2, \ldots, 120)$ are the rates of return for an investment horizon of two months. Note that, by using a horizon of two months, we subdivided the period 1948–68 to 120 time units rather than to 240 time units, without changing the length of the period covered by the empirical research: namely, twenty years. Similarly, if we had used annual rates of return, we would have only 20 observations. As a proxy to the market portfolios I used the Fisher Arithmetic Index, which assumes an equal investment in each of the NYSE stocks.

In this paper we examine the following linear regressions, with monthly data, semiannual data and annual data:

$$\bar{R}_i - r = f(\hat{\beta}_i)$$
$$\bar{R}_i - r = f(\hat{S}^2_{e_i})$$
$$\bar{R}_i - r = f(\hat{\sigma}^2_i)$$
$$\bar{R}_i - r = f(\hat{\beta}_i, \hat{S}^2_{e_i})$$
$$\bar{R}_i - r = f(\hat{\beta}_i, \hat{\sigma}^2_i)$$

where $\bar{R}_i$ is the average rate of return on the ith security, r is the rate of return on riskless assets,[10] and $\hat{\beta}_i$ is the systematic risk estimated from the time-series regressions; $\hat{S}^2_{e_i}$ is the residual variance (taken also from the time-series regressions) and $\hat{\sigma}^2_i$ stands for the estimate of the ith security variance.

These regressions are run for three different investment horizons (one, six, and twelve months) since it has been shown that

[10]The rates of return on Treasury Bills as well as on government bonds were taken from various issues of the *Federal Reserve Bulletin*. The sample of shares was taken from the return file of the CRSP tape. Note that in estimating *Beta*, and in the cross-section regression we employ the same set of data. This may cause some statistical bias. However, I believe that by a division of the period to two superiods (one for estimating *Beta* and the other for the cross-section regression) one may lose many observations, which is undesirable. Moreover, the *Beta* may change from period to period which decreases the reliability of this procedure.

TABLE 2—SECOND-PASS REGRESSION WITH SEMIANNUAL DATA

$\bar{R}_i = \gamma_0$	+	$\gamma_1\hat{\beta}_1$	+	$\gamma_2\hat{S}^2_{e_i}$	+	$\gamma_3\hat{\sigma}^2_i$	R^2
0.0493		0.0219					0.19
(0.0048)		(0.0046)					
t = 10.2		t = 4.7					
0.0583						0.2630	0.21
(0.0030)						0.0517	
t = 19.4						t = 5.1	
0.0603				0.3378			0.17
(0.0029)				(0.0747)			
t = 20.8				t = 4.5			
0.0528		0.0099				0.1771	0.23
(0.0050)		(0.0072)				(0.0808)	
t = 10.6		t = 1.4				t = 2.2	
0.0494		0.0151		0.2164			0.24
(0.0047)		(0.0052)		(0.0834)			
t = 10.6		t = 2.9		t = 2.6			

in the cross-section regression, the estimate of the systematic risk and the other parameters, (for example, R^2) are very sensitive to the assumed investment horizon (see Levhari and the author). Tables 1, 2, and 3 summarize the empirical findings for one-month, six-month, and twelve-month horizons, respectively. In Table 1, most of the regression coefficients are insignificant, and the coefficient of correlation is very low (less than 5 percent), indicating that the assumption of a one-month horizon is a very poor assumption.[11]

Moving to Table 2, we still obtain a low R^2. However, even from this table one can see that the simple regression $\bar{R}_i - r = f(\hat{\sigma}^2_i)$ yields a better (or at least not a worse) explanation than the regression $\bar{R}_i - r = f(\hat{\beta}_i)$, which is implied by the *CAPM*. Using the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{S}^2_{e_i})$ we find that the coefficient of $\hat{\beta}_i$ as well as the coefficient of $\hat{S}^2_{e_i}$ are statistically significant. The fact that the coefficient of $\hat{S}^2_{e_i}$ is significant is quite obvious from the above analysis. It is particularly obvious from the fact that each investor holds only a few securities in his portfolio, since $\hat{S}^2_{e_i}$ serves as a proxy to $\hat{\sigma}^2_i$. Indeed the R^2 between $\hat{S}^2_{e_i}$ and $\hat{\sigma}^2_i$ in this sample is 0.80. The fact that the coefficient of $\hat{\beta}_i$ is significant can be explained by the fact that the estimate of $\hat{\beta}_i$ is also correlated with $\hat{\sigma}^2_i$. Thus, even though β_i has little to do with the security risk, the regression coefficient of β_i is positive, since β_i is positively correlated with a main component of the true risk (σ^2_i). (Indeed, as we shall see below, β_i plays no role in price determination.) Such seems to be the case for the present sample where the relation between $\hat{\beta}_i$ and $\hat{\sigma}^2_i$ is[12]

$$\hat{\beta}_i = \underset{(0.06)}{0.68} + \underset{(0.32)}{2.78}\,\hat{\sigma}^2_i$$

$$t = \quad 11.4 \qquad 8.63 \qquad\qquad R^2 = 0.43$$

When we run the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{\sigma}^2_i)$ we find that the coefficient of $\hat{\sigma}^2_i$ is positive and significant, while the coefficient of $\hat{\beta}_i$ becomes insignificant. Once again, the simple model $\bar{R}_i - r = f(\hat{\sigma}^2_i)$ can explain price behavior almost as well as any other suggested model.

[11] One can easily increase the R^2 by running rates of a group of securities $\bar{R}_i$ on the market portfolio. However, since the *CAPM* should hold for individual securities, I think that a high R^2 which is achieved by grouping neither confirms nor refutes the *CAPM*.

[12] Miller and Scholes have found that the estimates of the systematic risk β_i is also correlated with the residual variance. They found in their sample R^2 = 0.17 while similar regression of the present sample yields $R^2 = 0.14$.

TABLE 3—SECOND-PASS REGRESSION WITH ANNUAL DATA

$\bar{R}_i = \gamma_0$	+	$\gamma_1 \hat{\beta}_1$	+	$\gamma_2 \hat{S}^2_{e_i}$	+	$\gamma_3 \hat{\sigma}^2_i$	R^2
0.109 (0.009) t = 12.0		0.037 (0.008) t = 5.1					0.21
0.122 (0.005) t = 22.9						0.219 (0.029) t = 7.7	0.38
0.126 (0.005) t = 23.4				0.248 (0.036) t = 6.8			0.32
0.117 (0.008) t = 14.2		0.008 (0.009) t = 0.9				0.197 (0.038) t = 5.2	0.38
0.106 (0.008) t = 13.2		0.024 (0.007) t = 3.3		0.201 (0.038) t = 5.3			0.39

Table 3 deals with annual data and confirms the previous results. First, we note that our results are very similar to those obtained by Miller and Scholes (who also used annual data) in spite of the fact that a different sample of data is used. We find the R^2 of the regression $\bar{R}_i - r = f(\beta_i)$ to be 21 percent in comparison to 19 percent in their research; for the regression $\bar{R}_i - r = f(\hat{S}^2_{e_i})$ we obtain 32 percent in comparison to their 28 percent; and finally, for the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{S}^2_{e_i})$ we find R^2 to be equal to 39 percent in comparison to 34 percent that they obtain. With annual data, all the regression coefficients are positive and significant in my research as well as in Miller and Scholes' research. However, in Table 3, I present two more regressions which do not appear in Miller and Scholes' paper. These two regressions confirm the previous results of the semiannual data which can be summarized as follows: (a) The simple regression $\bar{R}_i - r = f(\sigma^2_i)$ yields R^2 of 38 percent. This is only 1 percent less than the more complicated regression $\bar{R}_i = f(\hat{\beta}_i, \hat{S}^2_{e_i})$ which has been employed in most empirical studies that test the validity of the *CAPM*. (b) When we run the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{\sigma}^2_i)$ rather than $\bar{R}_i - r = f(\hat{\beta}_i, \hat{S}^2_{e_i})$, we find that the conventional estimate of the systematic risk $\hat{\beta}_i$ adds nothing to the explanation of price behavior. The coefficient of the systematic risk is very small and statistically insignificant (t value = 0.9). (c) If one had to choose between the traditional *CAPM* (i.e., $\bar{R}_i - r = f(\hat{\beta}_i)$ and the simple model $\bar{R}_i - r = f(\hat{\sigma}^2_i)$, one would note that the latter performs much better, with R^2 = 38 percent compared to only R^2 = 21 percent for the previous model.

IV. Concluding Remarks

The assumption of the perfect indivisibility of an investment and of the absence of transaction costs in the stock market, induces a theoretical result which asserts that each investor holds in his portfolio all the securities available in the market. It is obvious that the above assumption does not conform to reality, since many investors hold stocks of only one company, and most individuals hold stocks of less than four companies. Nor can we accept *CAPM* on a positive ground since it performs quite poorly in explaining price behavior.

In this paper, I have relaxed the assumption of a perfect market, and hence, the kth investor holds stocks of n_k companies in his portfolio where n_k can be very small (i.e., 1, 2, etc.). We first derive an equilibrium relationship between the return and risk of each security. We have found that the well-known systematic risk of the traditional *CAPM*, β_i, has little to do with equilibrium price determination. On the other hand, β_i^*, which is a weighted average of the kth investor systematic risk β_{ik}, is the correct measure of the ith security risk. Since σ_i^2 is a major component of β_{ik}, it plays a crucial role in the risk measure of each stock, quite contrary to the equilibrium results of the capital asset pricing model. When we impose the assumption of a perfect market and assume that investors hold all the available risky assets, (i.e., $n_k = n$), we obtain the well-known form of the *CAPM* as a special case of the *GCAPM* developed in this paper. The suggested model developed here, based on the fact that individuals hold relatively undiversified portfolios, explains the empirical results of the cross-section regression which have been found in most empirical studies.

The empirical findings support the theoretical results. The simple regression $\bar{R}_i - r = f(\hat{\sigma}_i^2)$ performs much better than the regression $\bar{R}_i - r = f(\hat{\beta}_i)$. The fact that $\bar{R}_i$ and β_i are positively correlated is caused simply by the fact that β_i and σ_i^2 are positively correlated, and that β_i serves as a proxy to the true risk component σ_i^2. In the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{S}_{e_i}^2)$, the coefficient of $\hat{\beta}_i$ as well as of $\hat{S}_{e_i}^2$ is positive and significant. The latter results have been found in other studies as well as in this paper. However, we claim that the coefficients of $\hat{\beta}_i$ and $\hat{S}_{e_i}^2$ are upward biased since $\hat{\beta}_i$ as well as $\hat{S}_{e_i}^2$ are positively correlated with $\hat{\sigma}_i^2$. Indeed, when we ran the regression $\bar{R}_i - r = f(\hat{\beta}_i, \hat{\sigma}_i^2)$ we found that the regression coefficient of $\hat{\sigma}_i^2$ was significant whereas the coefficient of $\hat{\beta}_i$ did not differ significantly from zero. This confirms the notion that, in an imperfect market, $\hat{\beta}_i$ plays no role, or at least a negligible role in price determination.

I would like to mention that σ_i^2 plays a central role in the risk-return relationship, but it is not the only measure of the ith security risk. The variance is only one component in this risk, and an empirical test should be designed in order to examine the validity of the *CAPM* in its imperfect form. Designing a precise empirical test which examines the validity of the *CAPM* in an imperfect market, is not an easy task, and is beyond the scope of this paper.

Finally, I think that the true risk index of the ith security is determined in the market, somewhere between the ith variance $\hat{\sigma}_i$, and the more sophisticated index β_i, as implied by the *CAPM*. For securities which are widely held (i.e., AT&T) we expect that *Beta* will provide a better explanation for price behavior[13] (see equation (17)), while for most securities, which are not held by many investors we would expect that the variance σ_i^2 would provide a better explanation for price behavior.

[13]Blume and Friend (1974) who tested the *Beta* and another quality rating index as measures of risk come to the conclusion that the *Beta* index performs relatively better for stocks with large market values. On the assumption that large market values implies also that the stocks are held by relatively many investors, this finding is consistent with my theoretical argument.

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