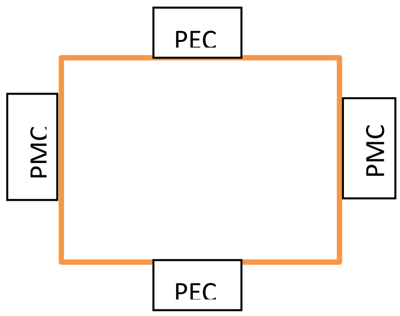
I. Properties of Waveguide with PEC-PMC-PEC-PMC Boundary

In waveguide along z direction, we always perform Maxwell equations as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right)\begin{pmatrix}\vec{E} \\ \vec{B}\end{pmatrix} = 0$$



For the special boundary PEC-PMC-PEC-PMC WG, with length a, b, a, b, respectively, we have boundary conditions

$$E_z = E_x = 0\Big|_{y=0,b} \qquad B_z = B_y = 0\Big|_{x=0,a}$$

And we consider TE wave, changing the second one into E or B condition

$$E_x = 0\Big|_{x=0,a} \qquad B_y = 0\Big|_{y=0,b}$$

Simply we just calculate, with variable separation

$$E_x = E_0 \sin\frac{n\pi y}{b}\sin\frac{m\pi x}{a} \qquad B_y = \frac{k_g E_0}{k_0 c}\sin\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$$

With the original Maxwell equation $\nabla\cdot B = 0$ $\nabla\times c^2 B = \frac{\partial E}{\partial t}$ we derive

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -i\frac{k_0}{c}E_z = 0 \qquad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0$$

Then

$$E_y = -\sqrt{\frac{k_c^2 b^2}{n^2\pi^2} - 1}\,E_0\cos\frac{n\pi y}{b}\cos\frac{m\pi x}{a} \quad ,$$

$$B_x = \sqrt{\frac{k_c^2 b^2}{n^2\pi^2} - 1}\,\frac{k_g E_0}{k_0 c}\cos\frac{n\pi y}{b}\cos\frac{m\pi x}{a}$$

And easily

$$B_z = i\frac{k_0^2 - k_g^2}{k_0 c}\frac{b}{n\pi}E_0\cos\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$$

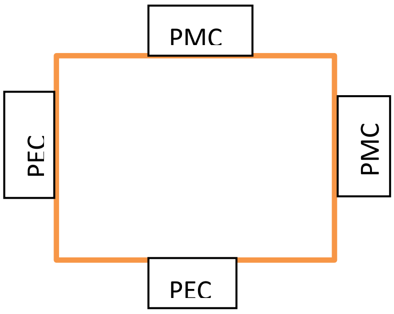
These field solutions are just similar like the classic electrodynamics ones in the text books (all omit the propagating terms $e^{i(k_g z-\omega t)}$), and it's as nice as we thought.

But Wait! When we inspect the mode in the waves, we find that we could make m=0, then B_z=0. While E_x and B_y go to 0 , E_y and B_x are still be OK, into

$$E_y = -\sqrt{\frac{k_c^2 b^2}{n^2\pi^2} - 1}E_0 \cos\frac{n\pi y}{b} \quad , \quad B_x = \frac{k_0^2 - k_g^2}{k_0 c}\frac{b}{n\pi}E_0 \cos\frac{n\pi y}{b}$$

This is neat! And we come to realize that the TEM wave could exist in this special boundary WG as the basis mode.

II. Properties of Waveguide with PEC-PEC-PMC-PMC Boundary



After we have go through with some tough calculation in the previous chapter, we feel familiar with this model, just different boundary condition. For TE wave

$$E_x = 0\big|_{y=0} \quad E_y = 0\big|_{x=0} \quad B_x = 0\big|_{y=b} \quad B_y = 0\big|_{x=a}$$

And make them

$$E_x = 0\big|_{y=0,x=a} \qquad \frac{\partial E_x}{\partial x} = 0\big|_{y=b,x=0}$$

Follow the same tricks previously, saving time in these tedious calculations, giving the field

$$E_x = E_0 \sin\frac{n\pi y}{2b}\cos\frac{m\pi x}{2a}$$

$$E_y = -\sqrt{\frac{k_c^2 b^2}{n^2\pi^2} - 1}E_0 \cos\frac{n\pi y}{2b}\sin\frac{m\pi x}{2a}$$

$$B_y = \frac{k_g}{k_0 c}E_0 \sin\frac{n\pi y}{2b}\cos\frac{m\pi x}{2a}$$

$$B_x = \frac{k_g}{k_0 c}\sqrt{\frac{k_c^2 b^2}{n^2\pi^2} - 1}E_0 \cos\frac{n\pi y}{2b}\sin\frac{m\pi x}{2a}$$

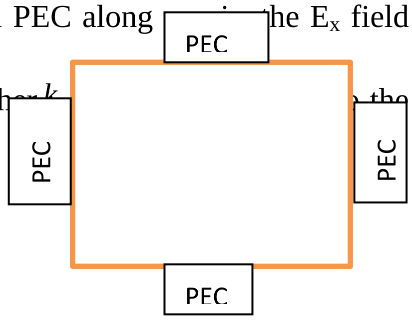
$$B_z = i\frac{k_0^2 - k_g^2}{k_0 c}\frac{b}{n\pi}E_0 \cos\frac{n\pi y}{2b}\cos\frac{m\pi x}{2a}$$

The key point here is not whether the WG could sustain a TEM wave, but when we focus on the wave vector $k_x = \frac{m\pi x}{2a}$ and $k_y = \frac{n\pi y}{2b}$ compared to the TEM ones last chapter

$k_x = \frac{m\pi x}{a}$ $k_y = \frac{n\pi y}{b}$ we find that the wave vectors become half, and corresponding cut-off frequency is half now. So this special boundary WG could be applied to lowering the cut-off frequency and adaptable for longer wavelength waves.

III. Standing Wave Method for Solving WG Field

A EM wave in the WG must have propagating wave along the WG direction, and standing wave transversely. So between the boundaries of two parallel PEC along ... the E_x field must be knots at boundaries, got $\frac{n\lambda}{2} = \frac{n}{2}\frac{2\pi}{k_y} = b$ and furth... the 0 value at boundaries, there must be $\sin\frac{n\pi y}{b}$ terms.



Similarly, for the two parallel PEC along y axis, E_x field would have $k_x = \frac{m\pi}{b}$ and $\cos\frac{m\pi x}{a}$ terms, for E_x reaching maximum value at boundaries.

After this analysis, we have already had the x, y dependent terms, $E_x = E_0 \sin\frac{n\pi y}{b}\cos\frac{m\pi x}{a}$ only parameters to be defined. For E_y it is just the same, as the good geometry with good symmetry, a 90° rotate of E_x. $E_y = E_{0y}\cos\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$

For B field to PEC, it has "opposite" properties compared to E field, shown as B_x field has maximum value at x axis boundaries and 0 value at y axis boundaries, with the wave vectors unchanged. We got $B_x = B_{0x}\cos\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$

For PMC we would not tediously reiterate the same kinds of tricks again.

What we care about is, from the **Standing Wave Method** we could avoid complex calculation to get the Field distribution.

For example, in the first chapter model, we could got $E_x = E_0 \sin\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$ and $B_y = B_0 \sin\frac{n\pi y}{b}\sin\frac{m\pi x}{a}$ that simple by analyze the standing wave. The corresponding terms of them are just the same, which sinuate the support of TEM.

Another example in the second chapter, we can even got $k_x = \frac{m\pi x}{2a}$ and $k_y = \frac{n\pi y}{2b}$

at the first sight, while the physics between is almost there. The special WG could break the limit of half wavelength restrain for conventional WG, reaching a one quarter wavelength working scale, helping developing our miniaturizing in material.