

Suppose

$$\rho(z) = \begin{cases} 0, & z < 0 \\ g(z), & 0 \leq z < l \\ 0, & z \geq l \end{cases}$$

Therefore, we have

$$E = \begin{cases} 0, & z < 0 \\ \int_0^z \frac{\rho(z)}{\epsilon_0} dz, & 0 \leq z \leq l \\ \int_0^l \frac{\rho(z)}{\epsilon_0} dz, & z \geq l \end{cases}$$

Then

$$\begin{aligned} \bar{F} &= \int dS e_n \int_0^l \rho(z) E dz \\ &= \int dS e_n \int_0^l \rho(z) \left(\int_0^z \frac{\rho(z')}{\epsilon_0} dz' \right) dz \end{aligned}$$

Suppose

$$T(z) = \int_0^z \frac{\rho(z')}{\epsilon_0} dz'$$

Then

$$\rho(z) = \frac{dT}{dz}$$

and

$$\begin{aligned} \bar{F} &= \int dS e_n \int_0^l \frac{dT}{dz} T(z) dz \\ &= \frac{1}{2\epsilon_0} \int dS e_n T^2(z) \Big|_0^l \\ &= \frac{1}{2\epsilon_0} \int dS e_n [T^2(l) - T^2(0)] \\ &= \frac{1}{2\epsilon_0} \int dS e_n T^2(l) \\ &= \frac{\sigma^2}{2\epsilon_0} \int dS e_n \end{aligned} \quad .\text{QED}$$