Handout on Calculus:

<u>Total derivatives</u>

$$d / dx[F(x) + G(x)] = d / dx[F(x)] + d / dx[G(x)]$$

$$example\ : \frac{d}{dx}[3x^2 + 5x^3] = \frac{d}{dx}(3x^2) + \frac{d}{dx}(5x^3) = 6x + 15x^2$$

$$d / dx[F(x).G(x)] = F(x)d / dx[G(x)] + G(x)d / dx[F(x)]$$

$$example\ : \frac{d}{dx}[5x^{1/3} * 3x^2] = 5x^{1/3}\frac{d}{dx}(3x^2) + 3x^2\frac{d}{dx}(5x^{1/3}) = 5x^{1/3} * 6x + 3x^2 * \frac{5}{3}x^{-2/3}$$

$$d / dx[F(x) / G(x)] = \frac{[G(x)d / dx[F(x)] - F(x)d / dx[G(x)]]}{\{G(x)\}^2}$$

$$example\ : \frac{d}{dx}[3x^2 / 5x^{1/3}] = \frac{5x^{1/3}\frac{d}{dx}(3x^2) - 3x^2\frac{d}{dx}(5x^{1/3})}{(5x^{1/3})^2}$$

$$d / dx[F(G(x)] = dF / dG.\{d / dx[G(x)\}]$$

$$example\ : \frac{d}{dx}[3x^2]^{1/} = \frac{1}{2}(3x^2)^{(1/2)-1}\frac{d}{dx}(3x^2) = \frac{6x}{2 * (3x^2)^{1/2}}$$

In the last function which is the case of function of a function, identify there are two functions in $(3x^2)^{1/3}$. The F function is the cube root function and G function is the $3x^2$. In this simple case indeed we could have made it one function which is $3x^{2/3}$. Consider the more complex case of the following function of a function case.

To find the derivative of $\ln(\frac{3x}{x+2})$. Here the first function is the Log function and the second function is $(\frac{3x}{x+2})$. Hence the derivative is given as: $\frac{1}{\frac{3x}{x+2}}\frac{(x+2)*3-3x}{(x+2)^2}$. **In deriving the derivatives in the examples above, derivatives of some common functional forms have been used which are given below.**

Derivatives of some common functional forms

$$\frac{d}{dx}(x^a) = ax^{a-1}$$

$$example: \frac{d}{dx}(3x^2) = 6x$$

$$\frac{d}{dx}(k/x) = -k/x^2, k = constant$$

$$example: \frac{d}{dx}(3/x) = \frac{-3}{x^2}$$

$$\frac{d}{dx}(\ln x) = 1/x$$

$$\frac{d}{dx}(a^x) = a^x \ln a, a = constant$$

$$example: \frac{d}{dx}(3^x) = 3^x \ln 3$$

$$\frac{d}{dx}(e^x) = e^x$$

Partial Derivatives

When a function has more than one arguments and we want to look at the change with respect to one of the arguments only we use partial derivatives (denoted with ∂). In the calculation of marginal product of the factor labor in the short run, remember we used this notation. In the short run we were looking at the returns to factor labor keeping the other argument of the production function **fixed** i.e. capital. Similar logic applies when we calculated the own price elasticity of demand since we kept all other determinants of demand viz. prices of other goods, income, tastes etc **fixed.** In other words the partial derivative of the function with respect to a certain argument say x, measures the change in the value of the function when keeping y fixed (where y is the other argument or arguments) variable x is varied by a very small amount.

If F is a function that varies with x and y then we write the function as F(x,y). Now if we have to look at the derivative with respect to x only we denote it as $\partial F(x,y)/\partial x$ and if we want to look at the derivative when only y changes we denote it as $\partial F(x,y)/\partial y$. In taking these two derivatives the other argument (in first case y and in second case x) are to be treated as constants. The formulas of the total derivatives listed above go through same.

$$F(x,y) = 6x + 5y + 12xy + 10x^2y^3$$

Example: $$\frac{\partial F(x,y)}{\partial x} = 6 + 12y + 10y^3 * 2x = 6 + 12y + 20y^3x$$

$$\frac{\partial F(x,y)}{\partial y} = 5 + 12x + 10x^2 * 3y^2 = 5 + 12x + 30x^2y^2$$