Dear Prof. Zhou:

Being a student in your course of Classical Electrodynamics, I am glad to write this letter to you.

Here I try to present my own interpretation of the questions you raised in yesterday's class. Unfortunately, my answer goes totally different against your lecture and that of John David Jackson in his great book between Page165 and Page167. I will make this note in my way nevertheless and hope that you can point out some mistakes.

It may be more convenient to begin with the electrostatic energy in dielectric media. To any electrostatic system, the total energy can be expressed in the following form where ρ_t is the total charge density and φ_t is the potential they create.

$$W = \frac{1}{2}\int \rho_t(\mathbf{r})\varphi_t(\mathbf{r})d\mathbf{r} \quad (1.1)$$

Thus, even in dielectric media the total energy takes the form of (1.2).

$$W = \frac{1}{2}\varepsilon_o\int \nabla\mathbf{E}(\mathbf{r})\varphi_t(\mathbf{r})d\mathbf{r} = -\frac{1}{2}\varepsilon_o\int \mathbf{E}(\mathbf{r})\nabla\varphi_t(\mathbf{r})d\mathbf{r} = \frac{1}{2}\varepsilon_o\int \mathbf{E}^2(\mathbf{r})d\mathbf{r} \quad (1.2)$$

Now I need to give my understanding of energy density of the macroscopic medium (1.3).

$$W' = \frac{1}{2}\int \mathbf{DE}d\mathbf{r} = -\frac{1}{2}\int \mathbf{D}\nabla\varphi_t d\mathbf{r} = \frac{1}{2}\int \nabla\mathbf{D}\varphi_t d\mathbf{r} = \frac{1}{2}\int \rho_f\varphi_t d\mathbf{r} \quad (1.3)$$

It's clear that W' in (1.3) is just a part of W associated with the free charge density. And I want to give (1.3) a physical interpretation. Consider a picture as follows where P_i stands for polarization charges and Q is a collection of free charges. Subtracted by half of the energy of interaction between free charge and the polarizations, W' is the energy we need to form Q part by part from some remote *areas with the* P_i *s always lying in fixed directions as they are at the right time in the graph. On the contrary,* W *is the energy consumed to arrange the whole picture including the* P_i *s. This can be easily shown if we use* $\{q_i\}_{i\le S}$ *to denote the polarization charge and* $\{q_i\}_{S<i\le N}$ *for the free charge.*

$$\sum_{S<i\le N} q_i\left[(\sum_{k\le S}\frac{q_k}{|\mathbf{r}_i-\mathbf{r}_k|})+\sum_{S<j<i}\frac{q_j}{|\mathbf{r}_i-\mathbf{r}_j|}\right]=2\times\frac{1}{2}\sum_{k\le S<i\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{S<i\ne j\le N}\frac{q_j q_i}{|\mathbf{r}_i-\mathbf{r}_j|}$$

$$=\frac{1}{2}\sum_{k\le S<i\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{\substack{S<i\\ j\le N}}\frac{q_j q_i}{|\mathbf{r}_i-\mathbf{r}_j|}=\frac{1}{2}\sum_{k\le S<i\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{S<i} q_i\varphi_t \quad (1.4)$$

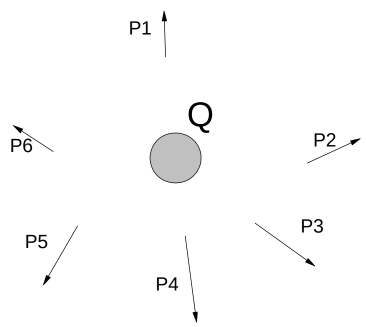

Figure 1.1

Now the work needed to put P_i *s from random states to positions and directions in the picture with the presence of* Q *is just the difference between* W *and* W' *plus half of the energy of interaction between free charge and the polarizations.*

$$\sum_{i\le S} q_i\left[(\sum_{S<k\le N}\frac{q_k}{|\mathbf{r}_i-\mathbf{r}_k|})+\sum_{j<i\le S}\frac{q_j}{|\mathbf{r}_i-\mathbf{r}_j|}\right]=2\times\frac{1}{2}\sum_{i\le S<k\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{i\ne j\le S}\frac{q_j q_i}{|\mathbf{r}_i-\mathbf{r}_j|}$$

$$=\frac{1}{2}\sum_{i\le S<k\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{\substack{i\le S\\ j\le N}}\frac{q_j q_i}{|\mathbf{r}_i-\mathbf{r}_j|}=\frac{1}{2}\sum_{i\le S<k\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+\frac{1}{2}\sum_{i\le S} q_i\varphi_t$$

$$=\frac{1}{2}\sum_{i\le S<k\le N}\frac{q_k q_i}{|\mathbf{r}_i-\mathbf{r}_k|}+W-W' \quad (1.5)$$

I am sorry to fail to note this interaction in the first version. If you go back to my previous letter, you can find my mistake which I came to realize just after sending it to you.

Here comes the problem, I think (1.3) is not the energy of the electric field. Thus it is plausible to think that $\frac{1}{2}(\mathbf{DE}+\mathbf{BH})$ *really means the energy of the electromagnetic field and the energy restored in the macroscopic medium. I am wondering if I still have some misunderstandings and waiting for your reply.*

Yours sincerely

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