Dear Prof. Zhou:

Being a student in your course of Classical Electrodynamics, I am glad to write this letter to you.

Here I try to present my own interpretation of the questions you raised in yesterday's class. Unfortunately, my answer goes totally different against your lecture and that of John David Jackson in his great book between Page165 and Page167. I will make this note in my way nevertheless and hope that you can point out some mistakes.

It may be more convenient to begin with the electrostatic energy in dielectric media. To any electrostatic system, the total energy can be expressed in the following form where ρ_t is the total charge density and φ_t is the potential they create.

$$W = \frac{1}{2}\int \rho_t(\mathbf{r})\varphi_t(\mathbf{r})d\mathbf{r} \quad (1.1)$$

Thus, even in dielectric media the total energy takes the form of (1.2).

$$W = \frac{1}{2}\varepsilon_o\int \nabla\mathbf{E}(\mathbf{r})\varphi_t(\mathbf{r})d\mathbf{r} = -\frac{1}{2}\varepsilon_o\int \mathbf{E}(\mathbf{r})\nabla\varphi_t(\mathbf{r})d\mathbf{r} = \frac{1}{2}\varepsilon_o\int \mathbf{E}^2(\mathbf{r})d\mathbf{r} \quad (1.2)$$

Now I need to give my understanding of energy density of the macroscopic medium (1.3).

$$W^{'} = \frac{1}{2}\int \mathbf{DE}d\mathbf{r} = -\frac{1}{2}\int \mathbf{D}\nabla\varphi_t d\mathbf{r} = \frac{1}{2}\int \nabla\mathbf{D}\varphi_t d\mathbf{r} = \frac{1}{2}\int \rho_f\varphi_t d\mathbf{r} \quad (1.3)$$

It's clear that $W^{'}$ in (1.3) is just a part of W associated with the free charge density. And I want to give (1.3) a physical interpretation. Consider a picture as follows where P_i stands for polarization charges and Q is a collection of free charges. $W^{'}$ is the energy we need to form Q part by part from some remote areas with the P_i s always lying in fixed directions as they are at the right time in the *graph. On the contrary,* W *is the energy consumed to arrange the whole picture including the* P_i *s.*

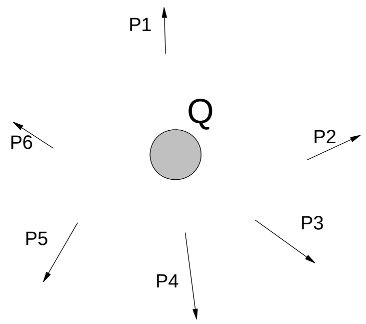

Figure 1.1

The discussion is just the same for magnetic energy. The total energy of a magnetic field is defined as the work done in establishing the distribution of currents and fields. One can easily get the mathematical expression following Jackson's steps in his book between Page212 and Page213 but carefully replacing $\nabla\times\mathbf{H}=\mathbf{J}$ *with* $\frac{1}{\mu_o}\nabla\times\mathbf{B}=\mathbf{J}$ *. Here they would come to the result the same as (1.4).*

$$W=\frac{1}{2\mu_o}\int\mathbf{B}^2(\mathbf{r})d\mathbf{r} \quad (1.4)$$

$$W^{'}=\frac{1}{2}\int\mathbf{BH}d\mathbf{r} \quad (1.5)$$

I will still use a picture to demonstrate the physical meaning of (1.5). To create a current and magnetic field in the picture with the energy of (1.5), one could just fix m_i *in the directions which are finally suitable to them and gradually start the current* J *at an infinitesimal rate.*

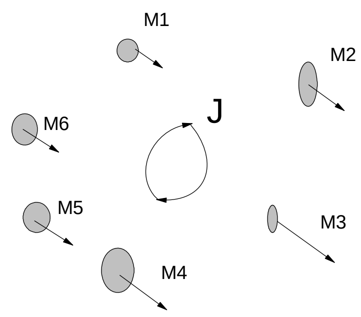

Figure 1.2

It is time to sum them up and calculate the energy for polarization and magnetization. The total electromagnetic energy of the field and the medium forever takes the form of (1.6). Meanwhile, (1.7) is just the energy we need to create the free charge and free current with all the polarization and magnetization done some time before. Simply subtracting (1.7) from (1.6) sees (1.8) — the electromagnetic energy absorbed by the medium. Luckily, (1.7) is consistent with Jackson's result in Page167 and Page214.

$$W = \frac{1}{2}\int \varepsilon_o \mathbf{E}^2(\mathbf{r}) + \frac{1}{\mu_o}\mathbf{B}^2(\mathbf{r})d\mathbf{r} \quad (1.6).$$

$$W^{'} = \frac{1}{2}\int \mathbf{ED}(\mathbf{r}) + \mathbf{BH}(\mathbf{r})d\mathbf{r} \quad (1.7)$$

$$W_{pm} = W - W^{'} = \int -\frac{1}{2}\mathbf{PE} + \frac{1}{2}\mathbf{MB}d\mathbf{r} \quad (1.7)$$

My words could end here for all the physics has done in these seven equations and two figures, but somehow I need to answer your two questions to complete the note. 1/2 is due to the construction of the electromagnetic field as shown in (1.1) and (1.4). Reasons for the minus sign in front of $\mathbf{PE}$ *in (1.7) comes out simply in the* *contradiction of* $\rho_P = -\nabla \mathbf{P}$ *and* $\rho = \nabla \varepsilon_o \mathbf{E}$. *One could expect a higher system energy if* $\mathbf{E}$ *and* $\mathbf{P}$ *lie in different directions because* $\mathbf{E}$ *starts from positive charge to negative charge while* $\mathbf{P}$ *is just the opposite.*

Yours sincerely

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